Fiscal Trends and Self-Fulfilling Crises

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Abstract

This paper develops a second-generation currency crisis model with endogenously changing fundamentals. Previous second-generation models are static, e.g. Obstfeld (1994), or dynamic with exogenous paths of fundamentals, e.g. Obstfeld (1986). In our model, the government weighs the disutility of making fundamentals consistent with a peg against a penalty for floating. If the former dominates, the government runs expansionary policies, precipitating a crisis. For some parameters, self-fulfilling speculation affects when the crisis happens, but not whether it happens. For other values, there are “purely self-fulfilling” crises, where a peg that could have survived forever collapses if attacked in the first few periods.

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1. Introduction

In the last fifteen years, currency crises have struck in countries as diverse as Britain, Korea, Russia, and Argentina. Many of these crises appeared suddenly, surprising academics and policymakers alike. Especially in Europe (1992-1993) and Asia (1997-1998), many crises did not appear to conform to the deteriorating-fundamentals scenario envisioned by Krugman (1979), Flood and Garber (1984), and other first-generation crisis models.

This discrepancy between crises in the 1990s and first-generation models gave impulse to the development of a second generation of theories (Obstfeld (1986), (1994), (1996), Sachs, Tornell, and Velasco (1996), Ozkan and Sutherland (1998), and others). These theories propose multiple equilibria to explain why crises are often unpredictable. In second-generation models, for some fundamentals, the government finds it optimal to abandon a peg if private agents expect it to do so, but, for the same fundamentals, if agents expect the peg to be maintained, that expectation also turns out to fulfill itself. It is important to stress the words *for some fundamentals*, as this literature does not claim that fundamentals are unimportant. In second-generation models, there are also fundamentals so sound, that crises cannot happen, and so weak, that the only possible outcome is a crisis.

But second-generation models take fundamentals as exogenously given. This is an important limitation, because even though fundamentals are crucial for the ultimate survival or demise of currency pegs, the models do not explain what determines them, or how they change over time. The environments are either static (Obstfeld (1994), (1996)), or dynamic
with exogenously evolving fundamentals (Obstfeld (1986), Ozkan and Sutherland (1998)).

In reality, the time paths of fundamentals such as money, reserves and debt, result from government choices, which in turn are driven by trade-offs, such as keeping a peg versus raising seignorage. Besides, as the “Delaying the Inevitable” title by Lahiri and Végh (2003) suggests, the preemptive, long-run polices that determine these time paths can do much more to save a peg than short-run defense policies. Thus, given their relevance, it would be desirable for a model to derive these paths from government optimization rather than take them as given. This extension would allow for a more robust analysis of whether self-fulfilling speculation can dismantle pegs that would have been sustainable in the long run, since for every point in the space of fundamentals, we would know the path that the government would have (optimally) taken in the absence of an attack. In fact, this approach is prevalent in related fields, like sovereign default, where crises are often studied in setups in which the government chooses how fundamentals change over time, but multiple equilibria arise because of lack of commitment (see, for instance, Cole and Kehoe (1996), (2000)). To our knowledge, in the currency crises literature, these ingredients—optimal dynamics and self-fulfilling crises—have not been combined, neither in the papers of the 1990s, nor in more recent contributions.

1 Sachs et al. (1996) study a 2-period model where time-1 choices affect time-2 fundamentals. However, if the peg survives time 2, we do not know what happens in the long run, since the model ends. Moreover, they only consider the possibility of multiple equilibria in period 2, whereas we allow for multiple equilibria at any time.

2 In Lahiri and Végh (2003) interest-rate hikes can postpone a crisis, but not avoid it altogether, because fiscal policy is inconsistent with the peg. Also, it is worth noting that in the four years prior to the ERM 1992-93 crisis, countries that ultimately maintained their central parity, like France and Denmark, had very similar inflation as Germany and very small money growth. In contrast, countries that left ERM or devalued their central parity, such as Great Britain, Italy, and Spain, had substantially higher inflation than Germany, and much greater percentage increases in the money supply than France and Denmark.

3 The two influential papers by Burnside, Eichenbaum, Rebelo (2001, 2004) take the fiscal choices of the government as given. In both models, when a shock creates a shortfall in the government’s intertemporal budget constraint, it is assumed that the missing revenue will come from money creation. The 2004 paper generates
In an attempt to fill in this gap, this paper extends second-generation models by endogenizing fundamentals. First, we assume that the government can commit to a long-run plan given a fixed exchange rate, initial reserves, and the initial money supply. The government can cut spending, adjusting money and reserves so that they are consistent with the peg, or run expansionary policies and eventually abandon the peg. Depending on which is more painful—spending cuts or the penalty for leaving the peg—the space of fundamentals is split into a region where the government would rather adjust fundamentals (the “fix forever zone”), a region where reserves are so low that the currency must float immediately (the “float immediately zone”), and a region where the government runs expansionary policies but maintains the peg for some time (the “delay the penalty zone”).

We then remove commitment and study the possibility of self-fulfilling crises. The source of multiple equilibria in the model is the following: If households expect an immediate devaluation, they buy large amounts of foreign currency, seeking speculative profits. The government’s problem is affected by this in two ways: First, it foregoes interest on the reserves that households have acquired (or, if net reserves are negative, pays interest on borrowed reserves), and thus finds itself in a more difficult fiscal position. Second, households have a portfolio of domestic and foreign currency that is optimal if there is a devaluation, but suboptimal if not. These effects make it costly to fight a speculative attack. For fundamentals in the ‘crisis zone’ the cost of fighting the attack exceeds the benefit of keeping the peg (either for a few or for many periods), and the expectation of immediate floating becomes self-fulfilling. This enriches the set of equilibria in two ways. First, for crises that, under commitment, were bound to happen at a given time, speculation can make self-fulfilling crises in the same way as Obstfeld (1986), i.e. by assuming that the government runs expansionary policies if and only if there is a crisis.
the crash arrive earlier, and thus a range of possible crisis times emerges. This is consistent with the observation that markets cannot predict the timing of crises with accuracy, even in cases where fundamentals are clearly unsustainable. For some parameter values—i.e. for some values of the penalty for floating, tax rate, and weights of different goods in the utility function—the ‘fix forever zone’ and the ‘crisis zone’ do not intersect, and thus, all that self-fulfilling attacks can do is affect when an inevitable crisis happens, not whether a crisis ultimately happens. For other sets of parameter values, in particular those that place the frontier of the ‘fix forever zone’ close to the ‘float immediately’ zone, the ‘fix forever zone’ and the ‘crisis zone’ intersect. This gives rise to ‘purely’ self-fulfilling crises, where an exchange rate that would have survived in the long run collapses if attacked in the first few periods. In those cases, our model rationalizes the policies assumed in Obstfeld’s (1986) seminal paper, i.e. run restrictive policies if not attacked, but expansionary policies if attacked. The ‘crisis zone’ expands as the cost of defending against attacks grows. Hence, an increase in the interest rate at which reserves can be borrowed widens the scope for both kinds of self-fulfilling attacks. For pegs that are bound to collapse, the range of possible crisis times widens, and ‘purely’ self-fulfilling crises become possible for a larger set of initial fundamentals.

To the extent that, in our model, there are fundamentals for which a crisis cannot happen, others for which a crisis must happen, and a third category for which it may happen,

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4 Other mechanisms, besides multiple equilibria, also generate multiple crisis times. The most common one, already in Flood and Garber (1984), is random fundamentals, e.g. random domestic credit expansion. Pastine (2002) presents another model that generates a range of possible collapse times. In Pastine’s setup, this result is due to the fact that the government plays a mixed strategy.

5 In fact, perfect foresight implies much higher interest rates in the days preceding crises than those observed, as a depreciation rate of 1% per day corresponds to 3,678% per year. Not even the spectacular 500% rate set by the Sweden in September 1992 comes close to this figure.
our results confirm previous findings. However, our dynamic model paints a more complete and nuanced picture of self-fulfilling crises than previous models, since it explicitly distinguishes self-fulfilling attacks that simply accelerate inevitable crises from ‘purely’ self-fulfilling crises. Previous papers, such as Obstfeld (1986, 1994, 1996) and Sachs, Tornell, and Velasco (1996) typically presented the failure of markets to foresee crises and the existence of multiple equilibria as arguments in support of ‘purely’ self-fulfilling crises. In contrast, our model suggests that, in many circumstances, the inability of markets to anticipate crises may be due to multiple equilibria regarding the time of the crash. While some authors, notably Krugman (1996), have attributed this lack of foresight to myopic behavior, our explanation is consistent with rational expectations.

The fact that self-fulfilling attacks that accelerate inevitable crises arise for most parameter constellations, whereas ‘purely’ self-fulfilling crises require more restrictions does not mean, however, that we regard the latter as fragile or unlikely equilibria. In fact, there exist plausible parameters for which these crises are possible. Especially if the interest rate on foreign reserves (a key determinant of the cost of defending against an attack) is high, ‘purely’ self-fulfilling crises need not be rare. High interest rates on reserves sometimes reflect high interest rates in the anchor country, as Buiter, Corsetti and Pesenti (2001) discuss in the context of German monetary policy in the ERM 1992-93 crisis. Or, particularly in emerging countries, the cost of borrowing reserves often rises in times of exchange rate
uncertainty, because fears of default also rise. A case in point is Mexico in 1994, where, as Kehoe (1996) discusses, debt and currency crises came at the same time.6

The typical policy recommendation that emerges from multiple-equilibria models is to adjust fundamentals and avoid the crisis zone (Obstfeld (1994), Sachs et al. (1996), Cole and Kehoe (1996), (2000)). Our model generates this same recommendation only for parameters and initial fundamentals such that multiple equilibria would lead to ‘purely’ self-fulfilling crises. But our analysis is general enough to embed the opposite scenario. If floating is eventually optimal, the best policy is to move towards, not away from, the crisis zone. In these cases, optimal policy is reminiscent of Lahiri and Végh (2003), since, as they do, we derive an optimal time to float. We take one step further, though, and allow for multiple equilibria in this case as well. The existence of several possible floating times, coupled with the fact that, in our model, the government prefers floating later rather than sooner, offers an explanation for why policymakers often lie—quite obviously—in times of currency uncertainty. Through the lens of our model, denying intentions of ever abandoning the peg can be seen as an attempt to coordinate agents to attack as late as possible, or in the case of ‘purely’ self-fulfilling crises, to never attack.

On a methodological note, our model shows that the mechanisms generating self-fulfilling crises in second-generation models can be embedded in a general equilibrium model with microfoundations. While our model is a bit more complicated than most reduced-form models, it is also more comparable with other models in macroeconomics, and its

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6 Other well-known examples include Russia (1998), Brazil (1999), and Argentina (2001-02).
parameters may be calibrated more easily than the parameters in ad hoc loss functions. In fact, recent first-generation papers, such as Lahiri and Végh (2003) and Burnside Eichenbaum and Rebelo (2001, 2004) also advance in the direction of incorporating microfoundations.\(^7\)

The remainder of the paper is organized as follows: Sections 2 and 3 present the model and define equilibrium. Section 4 finds optimal policy under commitment, section 5 studies self-fulfilling crises. Section 6 concludes.

2. The Model

There is a small open economy populated by a continuum with measure one of identical, infinitely lived households and a government. There is a domestic good \(c_{d,t}\), a foreign good \(c_{f,t}\), and a public good \(G_t\).\(^8\) Every period, each household is endowed with \(y_t\) units of the domestic good. Trading takes place first in an assets market, where domestic currency is exchanged for foreign currency, and later in a goods market, where the domestic and the foreign currencies are traded, respectively, for the domestic and foreign goods. Preferences are represented by the utility function

\[
E_0 \sum_{t \geq 0} \beta^t \left[ \omega \log c_{d,t} + (1 - \omega) \log c_{f,t} + \gamma \log G_t \right],
\]

(1)

\(^7\) Third-generation models, such as Schneider and Tornell (2004), also introduce microfoundations. But they focus mostly on tradable vs. nontradable sector asymmetries and private-sector currency mismatch encouraged by bail-out guarantees. We agree that these factors have played important roles in recent emerging-markets crises, but we have focused on reserves and fiscal and monetary variables because they have also been important and because the first and second-generation models that we are trying to extend typically focus on these variables.

\(^8\) Throughout the paper, lower- and uppercase letters denote, respectively, individual and aggregate variables. In equilibrium, they are equal.
where \( 0 < \omega < 1, 0 < \beta < 1, \gamma > 0 \), and \( E_0 \) denotes expectation given time-0 information. At the beginning of the period, the household holds \( m_{d,t} \) units of domestic cash and \( m_{f,t} \) units of foreign cash. In the assets market, it exchanges currencies at the rate \( e_t \) subject to

\[
x_{d,t} + e_t x_{f,t} = 0, \tag{2}
\]

with \( m_{x,t} + x_{x,t} \geq 0 \), for \( i = d, f \). After the assets market closes, the household splits into a shopper and a seller. The shopper takes the reshuffled money holdings \( m_{d,t} + x_{d,t} \) and \( m_{f,t} + x_{f,t} \) to the goods market in order to buy goods subject to two separate cash-in-advance (CIA) constraints:

\[
p_i c_{d,t} \leq m_{d,t} + x_{d,t}, \tag{3}
\]

and

\[
c_{f,t} \leq m_{f,t} + x_{f,t}, \tag{4}
\]

where \( p_i \) is the price of the domestic good in domestic currency and the price of the foreign good in foreign currency is always equal to one. The seller sells the endowment for \( p_i y_t \) and pays taxes \( \tau p_i y_t \). The newly acquired money \((1 - \tau)p_i y_t\) cannot be used to make purchases during that period's goods market. Thus, next period's initial cash holdings \( m_{d,t+1} \) and \( m_{f,t+1} \) are

\[
m_{d,t+1} = (1 - \tau)p_i y_t + m_{d,t} + x_{d,t} - p_i c_{d,t}, \tag{5}
\]

and

\[
m_{f,t+1} = m_{f,t} + x_{f,t} - c_{f,t}. \tag{6}
\]
We next describe the government, whose objective is to maximize the welfare of households. At time zero, the government inherits a fixed exchange rate, and each period, its actions are to buy/sell reserves in the assets market, buy the domestic good in the goods market, and keep/abandon the peg. The government enters the assets market with reserves in the amount of $R_t / \beta$ units of foreign currency.\(^9\) There is a constant, exogenous inflow of $\eta$ units of foreign currency due to a demand for exports from the rest of the world and an outflow due to domestic demand for foreign currency $X_{f,d}$ which, by (2) equals $-X_{d,t} / e_t$. Hence, assets-market clearing is given by

$$R_{t+1} = \frac{R_t}{\beta} + \eta + \frac{X_{d,t}}{e_t}. \quad (7)$$

While the peg lasts, $R_{t+1}$ is dictated by (7) with $e_t$ equal to $\bar{e}$, whereas when the currency floats, the government can, by buying or selling reserves, i.e. by setting $R_{t+1}$, affect $e_t$ at its discretion. Under any exchange regime, reserves can never fall below an exogenous lower bound $A$, i.e. for all $t$, we must have $R_t \geq A$. In the goods market, the government buys the domestic good, and transforms it into the public good $G_t$ at a one-to-one rate. Government purchases are subject to the budget constraint

$$p_t(G_t - \tau Y_t) = M'_{d,t+1} - M_{d,t} + e_t \left( \frac{R}{\beta} - R_{t+1} \right). \quad (8)$$

\(^9\) Reserves are assumed net of foreign debt, and thus, can be negative. Also, $R_t$ is divided by $\beta$ because reserves earn interest, at a rate assumed (for simplicity) to equal $1/\beta - 1$. Thus, $R_t$ is reserves at the end of period $t - 1$, and $R_t / \beta$ reserves at the beginning of period $t$. 

9
As we explained above, while the exchange rate is fixed, \( R_{t+1} \) is dictated by the market. Further, we assume that the tax rate \( \tau \) is fixed, and hence, the government can only increase \( G_t \) by increasing \( M_{d,t+1}^f - M_{d,t} \). Clearly, this generates a trade-off, as sustained increases in money are incompatible with the fixed exchange rate.

Before the period ends, the government decides whether the exchange rate will remain fixed next period or not. There is an exogenous penalty for devaluation in the form of a drop of output from 1 to \( \alpha \), with \( \alpha \in (0,1) \). This penalty cannot be recovered, i.e. \( Y_t = \alpha \) implies \( Y_{t+1} = \alpha \). Thus, next period's output \( Y_{t+1} \) indicates the exchange rate regime:\(^{10}\)

\[
Y_{t+1} = \begin{cases} 
1 & \text{if } Y_t = 1 \text{ and } e_{t+1} = \overline{e}, \\
\alpha & \text{otherwise.}
\end{cases}
\] (9)

We introduce a sunspot variable \( \xi_t \) which equals 1 with probability \( \pi \) and 0 with probability \( (1-\pi) \). We consider the time of realization of the sunspot the beginning of the period, and hence define the aggregate state at time \( t \) as \( S_t = (M_{d,t}, M_{f,t}, R_t, Y_t, \xi_t) \). \( \xi_t = 1 \) represents the presence of fears of devaluation due to non-fundamentals factors, such as rumors. The sunspot is the only stochastic shock in the model, and is only relevant when, given fundamentals \( (M_{d,t}, M_{f,t}, R_t, 1) \), there are multiple equilibria, i.e. when the government abandons the peg at time \( t \) if and only if there is an attack.

\(^{10}\) Assuming that GDP falls after the crisis is an admittedly mechanical way of giving the government an incentive to keep the fixed exchange rate. GDP did fall after many crises, but not in others, a prominent example being the UK in 1992. Second-generation models, such as Obstfeld (1994, 1996), Ozkan and Sutherland (1998), Pastine (2002) also have exogenous penalties for abandoning a peg. While endogenizing such a penalty would make the theory more complete, it would also make it much more complicated.
Finally, within-period timing is as follows: First, $\xi_t$ is realized. Second, the assets market opens. Third, the goods market opens. Fourth, the government chooses $Y_{t+1}$, i.e. whether to devalue or not.

3. A Private Sector Equilibrium

Since the government can only implement allocations that are chosen by consumers in equilibrium, before solving the government’s problem, we must define the private sector equilibrium associated with a given policy. In the absence of commitment, a government policy is a description of government choices as functions of all variables that have been observed at each stage where the government moves. If we make the assumption that the government can commit to future actions, we can work with the much simpler definition: A policy is a sequence $\{R_{t+1}, G_t, M^s_{d,t+1}, Y_{t+1}\}_{t \geq 0}$ that is independent of household actions.

Since under commitment, sunspots are irrelevant, households have perfect foresight over all future variables, including prices $\{e_t, p_t\}_{t \geq 0}$. Thus, given initial money holdings $(m_{d,0}, m_{f,0})$, households choose $\{x_{d,t}, x_{f,t}, c_{d,t}, c_{f,t}, m_{d,t+1}, m_{f,t+1}\}_{t \geq 0}$ to solve

$$\max \sum_{t=0}^{\infty} \beta^t \left[ \omega \log c_{d,t} + (1 - \omega) \log c_{f,t} + \gamma \log G_t \right]$$

s.t. $\ (2), (3), (4), (5), (6), \quad c_{i,t} \geq 0, \quad m_{i,t+1} \geq 0, \quad \text{for } i = d, f, \quad m_{d,0} \text{ and } m_{f,0} \text{ given.}$

Given the household’s problem, we can now define a private sector equilibrium.

**Definition 1** Given initial fundamentals $(M_{d,0}, M_{f,0}, R_0, Y_0)$ a private sector equilibrium (PSE) for policy $\{R_{t+1}, G_t, M^s_{d,t+1}, Y_{t+1}\}_{t \geq 0}$ is prices $\{e_t, p_t\}_{t \geq 0}$, individual and aggregate
quantities \( \{x_{d,t}, x_{f,t}, c_{d,t}, c_{f,t}, m_{d,t+1}, m_{f,t+1}\}_{t \geq 0} \) and \( \{X_{d,t}, X_{f,t}, C_{d,t}, C_{f,t}, M_{d,t+1}, M_{f,t+1}\}_{t \geq 0} \) such that:

1. **Individual quantities** \( \{x_{d,t}, x_{f,t}, c_{d,t}, c_{f,t}, m_{d,t+1}, m_{f,t+1}\}_{t \geq 0} \) solve (10).

2. **Individual and aggregate quantities coincide**

\[
\{x_{d,t}, x_{f,t}, c_{d,t}, c_{f,t}, m_{d,t+1}, m_{f,t+1}\}_{t \geq 0} = \{X_{d,t}, X_{f,t}, C_{d,t}, C_{f,t}, M_{d,t+1}, M_{f,t+1}\}_{t \geq 0}.
\]

3. **Markets clear:**

\[
M_{d,t+1} = M_{d,t+1}^s \quad \forall t \geq 0
\]

\[
R_{t+1} = \frac{R}{\beta} + \eta + \frac{X_{d,t}}{e_t} \quad \forall t \geq 0
\]

\[
Y_t = C_{d,t} + \frac{\eta \zeta}{p_t} + G_t \quad \forall t \geq 0.
\]

(Since this is a small open economy, there is no market clearing for the foreign good.)

To characterize a PSE, we first solve the household’s problem taking as given policy,

\[
\{R_{t+1}, G_t, M_{d,t+1}^s, Y_{t+1}\}_{t \geq 0}, \text{ initial cash holdings } m_{d,0} \text{ and } m_{f,0}, \text{ and price sequences } \{e_t, p_t\}_{t \geq 0}.
\]

Then, given household behavior, we will use feasibility to obtain prices.

The key to the household’s problem is finding \( q_t \), total expenditure (in domestic currency units) at \( t \), defined by \( q_t = p_t c_{d,t} + e_t c_{f,t} \).\(^{11}\) If both CIAs bind, consumers spend all their money \( q_t = m_{d,t} + e_t m_{f,t} \), devoting fractions \( \omega \) and \( 1 - \omega \) to the domestic good \( p_t c_{d,t} \), and the foreign good \( e_t c_{f,t} \), respectively. In some periods, however, at least one CIA does not

\(^{11}\) Of course, \( q_t \) and \( Q_t \) denote, respectively, individual, and aggregate variables, and are equal in equilibrium.
bind. For example, in the crisis period, households save foreign cash to profit from its appreciation.

For a procedure to find \( q_t \), consider first the case in which the peg is maintained forever, i.e. \( e_t = \bar{e} \) for all \( t \geq 0 \). Optimality requires that agents cannot do better by saving cash at \( t \) and spending it at \( t+1 \).\(^{12}\) In other words, the marginal utility of spending a unit of currency spent at time \( t \) (given by \( \omega / p_t c_{d,t} \)) must be at least as large as at \( t+1 \):

\[
\omega / p_t c_{d,t} \geq \omega / p_{t+1} c_{d,t+1},
\]

Using \( p_t c_{d,t} = \omega q_t \), we find that this condition is equivalent to

\[
q_{t+1} \geq \beta q_t, \tag{13}
\]

Note that, if \( q_t < m_{d,t} + e_t m_{j,t} \), (13) holds with equality, and this allows us to determine when CIAs bind. We first guess that they always do, i.e. that agents always want to spend all their cash, and set \( q_t = m_{d,t} + e_t m_{j,t} \) for all \( t \). If this never violates (13), we are done. If it violates (13) at some time \( t \), we find the first period in the future \( z > t \) in which both CIAs bind and use (13), holding with equality, iterating \( z-t \) periods, to obtain \( q_t \), that is, we set

\[
q_t = \beta^{z-t} M_{d,z}^s. \]

Period \( z \) is always well defined, because, as we will show later, if the peg is never abandoned, equilibrium paths always converge to a steady state with binding CIAs within a few periods.

\(^{12}\) When the domestic currency is depreciating, like in the crisis period, consumers save in the foreign currency. If CIAs do not bind while the exchange rate is fixed, the household is indifferent between saving in either currency, in which case I assume, for simplicity, that the household saves in domestic currency. This could be rationalized by an infinitesimal transaction cost of acquiring foreign currency.
For paths that end up in the abandonment of the peg, there exists a first period of floating $s > t$, and the above procedure needs to be adjusted to take this into account. If $e_s > e$ (which, of course, is the natural case in the context of currency crises), then, at time $s - 1$ the foreign CIA is slack, and the domestic binding. In other words, at time $s - 1$, agents save foreign currency, let it appreciate, and exchange it back for domestic currency at time $s$. How much they save is determined by a modified version of (13), namely $q_s = e_s / e \beta q_{s-1}$. Other than this, the procedure for finding $q_t$ is the same as in the case with $e_t = e$ for all $t \geq 0$.

Having solved the consumer’s problem, it is straightforward to find prices: When the exchange rate floats, we can solve for $e_t$ in the market-clearing condition in the assets market, using the facts that $R_t$ and $R_{t+1}$ are given and that $X_{d,t}$ equals $\omega Q_t - M_{d,t}$. Given $e_t$, $p_t$ can be found using (12) and the fact that $C_{d,t}$ equals $\omega Q_t / p_t$.

4. Optimal Policy under Commitment

Having defined a PSE, and given fundamentals $\left(M_{d,0}, M_{f,0}, R_0, Y_0\right)$, we define optimal policy under commitment as the sequence $\left\{R_{t+1}, G_t, M_{d,t+1}, Y_{t+1}\right\}_{t \geq 0}$ that solves

$$\max_{\left\{R_{t+1}, G_t, M_{d,t+1}, Y_{t+1}\right\}_{t \geq 0}} \sum_{t=0}^{\infty} \beta^t \left[ \omega \log C_{d,t} + (1 - \omega) \log C_{f,t} + \gamma \log G_t \right]$$

subject to

$$e_t = \overline{e} \quad \text{if } Y_t = 1$$

(14)
\[ p_t(G_t - \tau Y_t) = M_{d,t+1} - M_{d,t} + \epsilon_t \left( \frac{R_t}{\beta} - R_{t+1} \right) \]

\[ Y_{t+1} = \alpha \quad \text{if} \quad Y_t = \alpha \]

\[ R_t \geq A \]

\( \{e_t, X_{d,t}, X_{f,t}, p_t, C_{d,t}, C_{f,t}, M_{d,t+1}, M_{f,t+1}\}_{t \geq 0} \) are elements of a PSE for fundamentals \((M_{d,0}, M_{f,0}, R_0, Y_0)\) and policy \(\{R_{t+1}, G_t, M_{d,t+1}, Y_{t+1}\}_{t \geq 0}\).

The solution to this problem is summarized by Figure 1, in which the \((M_{d,t}, R_t)\) plane\(^{13}\) is divided into 3 areas, the “Float Immediately Zone”, the “Delay the Penalty Zone”, and the “Fix Forever Zone”. In the “Float Immediately Zone”, if there was a speculative attack, demand of foreign currency by consumers would drive reserves below the threshold \(A\). In those cases, we assume that the exchange rate is already floating, i.e. \(Y_0 = \alpha\).\(^{14}\) In the “Delay the Penalty Zone”, reserves can endure an attack, but it would be too painful to reduce expenditures in order to make the money supply compatible with \(\overline{\sigma}\) in the long run. Thus, the government runs fiscal deficits and finances them via seignorage, making floating ultimately inevitable. However, in order to postpone the penalty for floating, deficits are smaller than under a free float. In this zone, policies similar to those assumed in first-generation models are optimal. Furthermore, as in Lahiri and Végh (2003), the government

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\(^{13}\) We can restrict attention to these two dimensions, since having nonzero initial foreign cash in the hands of consumers is equivalent to having zero initial foreign cash, a greater domestic money supply and greater reserves. That is, the situation of an economy with fundamentals \((M_{d,t}, M_{f,t}, R_t)\) is exactly the same as with \((M_{d,t} + \epsilon M_{f,t}, 0, R_t + \beta M_{f,t})\).

\(^{14}\) This could be rationalized by assuming that, if the government cannot satisfy demand for foreign currency, only a randomly chosen fraction of households can buy the desired quantity of the foreign currency and the foreign good, while the rest can buy none. Since this would imply infinitely negative utility, the government is sure to abandon the peg before entering that zone.
tries, with some success, to delay the crisis for a few periods. Finally, in the “Fix Forever Zone” the cuts needed to keep the peg are not too large and it is optimal to move fundamentals towards a steady state, and never float. In all steady states the current account is balanced, i.e. exports and interest on reserves pay for imports:

\[
\frac{(1 - \omega)M_{d.x}}{\overline{e}} = \eta + R_i \left( \frac{1}{\beta} - 1 \right).
\]  

(15)

In these states, the fixed parity \( \overline{e} \) would clear the market under floating rates if the government neither accumulated, nor lost reserves over time. Within the locus of states defined by (15), the lower reserves, the lower the levels of imports and public spending that can be provided. For this reason, not all states on (15) are steady states, since there is a threshold for reserves below which it is best to increase deficits and eventually float. Within the “Fix Forever Zone”, but to the right (left) of the steady states, the optimal policy involves cuts (increases) in public spending that reduce (expand) the money supply until one of the steady states on (15) is reached.\(^\text{15}\) In choosing the speed of adjustment, the government trades off utility during the transition against steady-state utility—more dramatic spending cuts are more painful, but lead to better steady states, and vice versa. Also, the farther a state is to the right of the steady states, the more painful the transition, and thus, as we keep going to the right, we eventually exit the “Fix Forever Zone” and enter the “Delay the Penalty Zone”.

The size of the “Fix Forever Zone” grows with the size of the penalty for floating, \( 1 - \alpha \). The “Delay the Penalty Zone” is bigger the greater the difference \( \gamma / (\omega + \gamma) - \tau \), where

\(^{15}\) Unlike in most crises models, including Krugman (1979) and Flood and Garber (1984), in our model, even if a peg is expected to last forever, the equilibrium quantity of money need not be constant. Our two-moneys cash-in-advance structure also implies that, as a country approaches a crisis in the “Delay the Penalty” zone, the money supply grows, an implication that is in line with the empirical work by Kaminsky and Reinhardt (1999), as they show that “Excess-M1 balances” are significantly higher before crises than in tranquil times.
\(\gamma/(\omega+\gamma)\) is the optimal \(G_t/Y_t\) ratio if the government can use seignorage freely, as it can under floating exchange rates. Thus, this difference captures how painful it is to renounce to seignorage and keep fiscal policy consistent with the fixed exchange rate.

This completes the qualitative description of optimal policy under commitment. Next, in subsections 4.1 and 4.2, we explain the calculations that generated these results.

4.1 Optimal Policy under Floating Exchange Rates

Suppose that at \(t=0\), the currency is floating and fundamentals are \((M_{d,0}, M_{f,0}, R_0, \alpha)\).\(^{16}\) We guess that, since the exchange rate is no longer a concern, the government will use seignorage to finance whichever level of the public good is optimal given preferences and feasibility. We also guess that, with constant output and concave utility, optimal consumption streams are constant for all three goods. We propose a government policy and an associated PSE that satisfy these properties we suspect are optimal. We verify optimality in the proofs of lemmas 1 and 2, which are available from the author upon request. The policy we propose is simply given by:

\[
R_{t+1} = R_t + \beta M_{f,t}, \quad \text{for all } t \geq 0
\]  

\[
G_t = G^* = \frac{\gamma}{\omega+\gamma} \alpha, \quad \text{for all } t \geq 0
\]

and

\[
M_{d,t+1}^* = M_{d,t} - \epsilon_t \left( \frac{R_t}{\beta} - R_{t+1} \right) + p_t \left( G_t - \tau Y_t \right) \quad \text{for all } t \geq 0.
\]

\(^{16}\) While, as mentioned in footnote (13), in terms of welfare, fundamentals \((M_{d,0}, M_{f,0}, R_0, \alpha)\) and \((M_{d,0}, +e, M_{f,0}, 0, R_t + \beta M_{f,t})\) are equivalent, for this discussion we will keep \(M_{f,0} > 0\), since in the important first period after the crisis, households inevitably hold positive holdings of foreign currency.
Since, as we show in the proof of Lemma 1, if \( e_{t+1} \geq e_t \), CIA constraints always bind under floating exchange rates, we set \( M_{f,t} = 0 \) for all \( t \geq 1 \). Thus, reserves may increase at time 0, but remain constant after that, and hence \( R_0 + \beta M_{f,0} = R_t = R_t \) for all \( t \geq 1 \). In the goods market, the government sets the level of public spending at a constant level \( G^* \) amounting to a fraction \( \gamma/(\omega + \gamma) \) of output, and the money supply is residually given by the budget constraint.

Since all CIAs bind, it is easy find prices and consumption quantities. Set \( Q_t \) equal to \( M_{d,t} + e_t M_{f,t} \) and \( X_{d,t} \) equal to \( \omega Q_t - M_{d,t} \), use (16) (17), and (7) and (12), to obtain \( e_t \) and \( p_t \). Substituting back into CIAs, we obtain consumption of private goods:

\[
C^*_d = \frac{\eta + R_t \left( \frac{1}{\beta} - 1 \right)}{\eta + \omega R_t \left( \frac{1}{\beta} - 1 \right)} \frac{\omega}{\omega + \gamma} \alpha
\]

and

\[
C^*_f = \eta + R_t \left( \frac{1}{\beta} - 1 \right)
\]

The government’s value function captures maximum utility attainable under floating rates, given \( M_{f,0} \) and \( R_0 \),

\[
V^*_e(M_{d,0}, M_{f,0}, R_0, \alpha, \xi_0) = \frac{\omega \log C^*_d + (1 - \omega) \log C^*_f + \gamma \log G^*}{1 - \beta}.
\]

Except for period 0, in which households may start with foreign currency and sell it to the central bank, in all other periods, nominal variables grow at the constant rate:
\[
\frac{M_{d,t+1}}{M_{d,t}} = \frac{P_{t+1}}{p_t} = \frac{e_{t+1}}{e_t} = \frac{\eta + \omega R_t \left(\frac{1}{\beta} - 1\right)}{\eta + R_t \left(\frac{1}{\beta} - 1\right)}(1 - \tau) \frac{\omega + \gamma}{\omega} \quad \text{for } t \geq 1.
\]

To verify that this policy is optimal, we prove the following two lemmas:

**Lemma 1** Given state \( S_0 = (M_{d,0}, M_{f,0}, R_0, \alpha, \xi_0) \) and policy as given by (16)-(18), there is a private sector equilibrium in which private consumption is given by (19) and (20).

**Lemma 2** Policy (16)-(18) is optimal: There is no other policy with an associated private sector equilibrium in which welfare is larger than \( V^*(S_0) \).

The proof of lemma 1 reduces to showing that all CIAs bind in any PSE. We show this by assuming that it is optimal for a household to let one CIA loose, and derive a contradiction.

To prove lemma 2, we solve a pseudo-planner’s problem, and show that, given feasibility, and some implementability constraints, it is impossible to attain higher utility than \( V^*(S_0) \).

4.2 Finding the Optimal Time to Devalue

Given \( V^*_g \), we can find the function \( V^*_g(1) \), which gives maximum welfare attainable when starting period 0 with a fixed exchange rate and abandoning it at the end of that same period, i.e. \( Y_0 = 1 \), and \( Y_1 = \alpha \). To obtain \( V^*_g(1) \) we solve the following problem for arbitrary values of domestic money supply and reserves:

\[
V^*_g(M_{d,0}, 0, R_0, 1, \xi_0) = \max_{R_0, G_0, M_{d,1}} \omega \log C_{d,0} + (1 - \omega) \log C_{f,0} + \gamma \log G_0 \\
+ \beta V^*_g(M_{d,1}, M_{f,1}, R_1, \alpha, \xi_1)
\]

(23)
subject to \( C_{d,0}, C_{f,0}, M_{d,1}, \) and \( M_{f,1} \) being elements of a PSE for a policy consisting of \( \{R_i, G_0, M^*_i\} \) in period 0 and (16)-(18) afterwards. For future use, we label the policy that solves this problem \( \Phi_g^1(M_{d,0}, 0, R_0, 1, \xi_0) \). Note that, since \( M_{d,1} \) does not affect \( V^*_g \), the government can, for all \( t \geq 0 \), freely set \( G_i / Y_i \) at \( \gamma / (\omega + \gamma) \). (For details on why this ratio is optimal, see the proof of lemma 2.) Also note that \( R_i \) is not really chosen by the government since in period 0 the exchange rate is fixed. To characterize consumer behavior, first find expenditure \( Q_0 \) using (13) and the fact that \( Q_1 \) is \( M_{d,1} + e_i M_{f,1} \). Expressing \( M_{d,1}, M_{f,1}, \) and \( e_i \) as a function of \( Q_0 \) and doing a bit of algebra yields

\[
Q_0 = \frac{\bar{e} \left[ \eta + R_0 \left( \frac{1}{\beta - 1} \right) \right] - M_{d,0} (1 - \beta)^2}{\beta + \omega (\beta - \beta^2 - 1)}.
\]  

(24)

Given \( Q_0 \), it is easy to find \( C_{d,0} \) and \( C_{f,0} \) and hence, \( V^1_g \). However, we need to point out two special cases. First, if the RHS of (24) exceeds \( M_{d,0} \), then \( Q_0 \) is \( M_{d,0} \) and \( V^1_g \) must be calculated accordingly. The abandonment of the peg in this case is not a crisis, as there is no run on reserves, and no depreciation when floating. The second special case is for fundamentals in the “Float Immediately Zone”, where \( M_{d,0} \) is so large relative to \( R_0 \) that the drop in reserves prior to the devaluation, given by \( (M_{d,0} - \omega Q_0) / \bar{e} \), would push \( R_i \) below \( A \). Since we assume that the exchange rate must be floating if the economy is in this zone, \( V^1_g \) is not defined at these points.

Next, we iteratively apply the same formula used to obtain \( V^1_g \) to obtain value and policy functions \( V^t_g \) and \( \Phi^t_g \) for any \( t \geq 2 \):
\[
V'_g(M_{d,t}, 0, R_t, 1, \xi) = \max_{R_t+1, G_t, M_{d,t+1}} \omega \log C_{d,t} + (1 - \omega) \log C_{f,t} + \gamma \log G_t + \beta V'_{g-1}(M_{d,t+1}, 0, R_t+1, 1, \xi_t+1)
\]  

subject to \(C_{d,t}, C_{f,t}, M_{d,t+1}\), and \(M_{f,t+1}\) being elements of a PSE for a policy consisting of \(\{R_{t+1}, G_t, M_{d,t+1}\}\) in period \(t\) and \(\Phi_{g-1}\) afterwards. Unlike \(V'_g\), all value functions under fixed exchange rates, i.e. all \(V'_g\) for \(t \geq 1\), depend nontrivially on \(M_{d,t+1}\). This complicates the choice of \(G_t\), as it is no longer static. Thus, we solve (25) by means of an algorithm, which we implement in C/C++.\(^\text{17}\) The algorithm finds the maximum utility attainable if abandoning the fixed exchange rate after \(t\) periods, with \(t \in \{1, \ldots t^{\text{max}}\}\), and \(t^{\text{max}}\) so large that \(V'_{g}^{\text{max}}\) and \(V'_{g}^{\text{max}-1}\) are arbitrarily close (defining distance as maximum over \((M_d, R)\) pairs of \(|V'_g(M_d, R) - V'_{g-1}(M_d, R)|\)).\(^\text{18}\) Given, for every pair \((M_d, R)\), the utility associated with devaluing at any \(t\), we pick the best time to devalue (under commitment) \(T^C(M_d, 0, R, 1, \xi)\) and record the maximal utility as \(V^C_g(M_d, 0, R, 1, \xi)\), that is:

\[
V^C_g(M_d, 0, R, 1, \xi) = \max_{t^{\text{max}}} V'_g(M_d, 0, R, 1, \xi).
\]  

The optimal policy found through this algorithm \(\Phi^C(M_d, 0, R, 1, \xi)\) is the solution to problem (14), and yields the zones of Figure 1. The “Fix Forever Zone” consists of all fundamentals

\(^{17}\) Given the formula for \(V'_t\), we compute \(V^2_e\) as follows: For every pair \((M_e, R)\), and for each value of \(G\) in a grid, we compute the optimal response of consumers, and the associated welfare. Choosing the best \(G\) for every pair \((M_e, R)\) yields numerical values for \(V^2_e\). We do the same to get \(V^3_e, \ldots, V^{t^{\text{max}}}e\).\(^{18}\) The algorithm stops when distance falls below 0.0001. Of course, the resulting \(t^{\text{max}}\) depends on the grid used for computation, and on parameters, especially \(\beta\). For most parameter combinations, including the ones generating figures 1-5, \(t^{\text{max}}\) is between 110 and 150.
with $T^C(M_d, 0, R, 1, \xi) = t^\text{max}$. Fundamentals with $T^C(M_d, 0, R, 1, \xi) < t^\text{max}$ constitute the “Delay the Penalty Zone”.19

5. Lack of Commitment and Self-Fulfilling Crises

In the previous section, given initial fundamentals $(M_{d,t}, 0, R, 1)$, the government commits to an optimal plan $\{R_{t+1}, G_t, M_{d, t+1}^{s}, Y_{t+1}\}_{t=0}$. For initial fundamentals in the “Fix Forever Zone”, the exchange rate survives indefinitely, and thus, there is no speculative attack. For initial fundamentals in the “Delay the Penalty Zone”, the optimal plan schedules floating the currency at a concrete time, and this is foreseen by households, who launch an attack a period before. In this section, we study under what conditions an a priori unexpected sunspot generates a self-fulfilling crisis by triggering a speculative attack that makes it optimal for the government to deviate from its optimal plan and float the currency at that point. In terms of the model’s notation, we assume that if $\xi = 0$, households expect the commitment policy to be carried out, whereas if $\xi = 1$, and fundamentals are such that a speculative attack can succeed, they expect the attack to take place. Following Cole and Kehoe (2000), we call “The Crisis Zone” the area in the space of fundamentals where immediate floating happens if and only if there is speculative attack in the period. To find the crisis zone in the case where the sunspot is a zero probability event ($\pi = 0$), we calculate the government’s optimal response if, given fundamentals $(M_{d,t}, 0, R, 1)$ there is a speculative attack. Letting superscript $A$ denote the value of variables in the period of the attack, we have $Q_t^A$ given by (24) and $X_{f,t}^A$.

19 Again, for most parameter constellations, the longest optimal devaluation time in the “Delay the Penalty Zone” is between 8 and 15 periods.
the amount of foreign currency purchased, given by \( \left( M_{d,t} - \omega Q^t \right) / \bar{e} \), which is higher than under commitment. This changes the government's situation in two ways. First, the government loses interest on reserves for one period. Second, the portfolio \( (M_{d,t} + X_{d,t}^A, M_{f,t} + X_{f,t}^A) \) held by households in the goods market is optimal if there is a crisis but suboptimal if not. If these effects are large enough, the government will change its plan about \( G_t, M_{d,t+1}^s, \) and \( Y_{t+1} \) and abandon the fixed exchange rate. The welfare associated with not floating when attacked, \( V^P_g \), is given by

\[
V^P_g(S, R_{t+1}^A, X_{d,t}^A, X_{f,t}^A) = \max_{G_t, M_{d,t+1}} \omega \log C_{d,t} + (1-\omega) \log C_{f,t} + \gamma \log G_t + \beta V^C(M_{d,t+1}, M_{f,t+1}, R_{t+1}, 1, 0)
\]

subject to the condition that \( C_{d,t}, C_{f,t}, M_{d,t+1}, \) and \( M_{f,t+1} \) are elements of a private sector equilibrium for policy \( (G_t, M_{d,t+1}^s) \) in this period’s goods market and \( \Phi^C \) afterwards. Since \( \xi = 1 \) is a zero probability event, the continuation value is simply the value function under commitment. (Also, to calculate \( V^C_g(M_{d,t+1}, M_{f,t+1}, R_{t+1}, 1, 0) \), we use the fact that fundamentals \( (M_{d,t+1}, M_{f,t+1}, R_{t+1}) \) and \( (M_{d,t+1} + \bar{e} M_{f,t+1}, 0, R_{t+1} + \beta M_{f,t+1}) \) are equivalent.)

The crisis zone consists of all fundamentals with \( T^C(M_{d,t}, 0, R_t) > 1 \) and \( V^P_g(S_t, R_{t+1}^A, X_{d,t}^A, X_{f,t}^A) < V^I_g(S_t) \), that is, fundamentals such that the government, under commitment, was planning not to abandon the peg at time \( t \) but, if there is an attack, the government prefers abandoning the peg at \( t \) than proving consumers wrong.

Keeping parameters as in Figure 1, we add the “Crisis Zone” in Figure 2. The “Crisis Zone” is close to the “Float Immediately Zone”, as the distortions associated with speculation
can make the government devalue immediately only if, even under commitment, the
government could only delay the crisis for a few periods. As we move away from the “Float
Immediately Zone”, and further into the “Delay the Penalty Zone”, optimal devaluation times
under commitment increase, and speculative attacks cannot force an immediate devaluation.
For the parameters in Figure 2, the “Fix Forever Zone” is far enough from the “Float
Immediately Zone” not to intersect with the “Crisis Zone”. Thus, for these parameters, self-
fulfilling crises can only affect the timing of inevitable crises, but not dismantle pegs that are
sustainable in the long run. However, as we can see in Figure 3, a change in parameters that
makes it less painful to maintain the peg, such as an increase in $\tau$, expands the “Fix Forever
Zone” making it intersect with the “Crisis Zone”. For points in this area of intersection, the
possibility arises for “purely” self-fulfilling crisis, in which a peg that, in the absence of
speculation, would have survived in the long run, collapses if attacked.\(^{20}\)

Another parameter that determines whether the model generates purely self-fulfilling
crises or not is the lower limit for reserves $A$. Figures 4 and 5 shows how a tightening of this
limit, keeping all other parameters constant, creates the possibility that a sustainable peg may
be dismantled by a speculative attack, while this possibility did not exist when the
government’s ability to borrow foreign currency was greater. This feature of the model is
reminiscent of the evidence that, in many emerging markets crises, e.g. Mexico (1994-95),

\(^{20}\) Conceptually, the problem defined by (27) may miss some ‘purely’ self-fulfilling crises, because it only takes
into account the possibility of attacks in the present period, but not in the future. However, it is conceivable that
even if $(M_{\rho}, R_{g})$ is in the fix forever zone but not in the crisis zone, at some point the optimal path of
correction of fundamentals may intersect the crisis zone. However, my computations suggest that this does not
happen. The reason for this is that the correction of fundamentals tends to be strongest in the first few periods.
Even though reserves fall along adjustment paths, they do so slowly compared with the domestic money supply,
which is reduced vigorously in the first periods of adjustment, placing fundamentals in the safe zone.
Asia (1997-98) and Argentina (2001-2002), difficulties to maintain a fixed exchange rate come simultaneously with difficulties to roll over debt.

While in figures 3 and 5 the sets of fundamentals for which ‘purely’ self-fulfilling crises are possible are nonempty, these sets are also quite small. In general, the size of the crisis zone increases as $\beta$ falls, since this increases the cost of defending against attacks. The widening of the crisis zone results both in wider ranges of possible floating times for crises that are bound to happen, and in an increase in the size of the set of fundamentals for which ‘purely’ self-fulfilling crises can happen. In figure 6, we show an economy where the cost of fending off attacks is very high, since $\beta = 0.8$. This figure may be plausible for countries in which there is substantial risk of sovereign default.

Since our computational results depend on the parameter values we assume, a natural question is whether those values are reasonably realistic. For some parameters, there is a fairly direct way to determine the ranges of plausibility. The tax revenue/GDP ratio could be a good proxy for $\tau$, and thus may range between, say, 15 percent as a rough average for Latin American countries, 20 percent for south-east Asian countries, and above 30 percent for European countries. $\eta$ would correspond to the exports/GDP ratio, with typical values for Brazil and Argentina below 20 percent (higher for Mexico), in the mid-twenties in Western Europe and 30 percent or more in East Asia. The imports/GDP ratio would be given by $1 - \omega$, and in general, it would take similar values to those of $\eta$, low for Latin America, with the exception of Mexico, and higher for Europe and Asia. Assigning a numerical value to $\gamma$ requires a little bit more imagination. Interpreting the world through the lens of our model, in periods of floating countries should set the $G/Y$ ratio equal to $\gamma/(\omega + \gamma)$. Using this, and
data for public spending to GDP ratios, we could obtain $\gamma$. Values for $A$, $\alpha$ and $\beta$ are even more difficult to pin down. From the point of view of our model, a country’s post-crisis level of reserves (net of foreign debt) need not be equal to the minimum $A$, since multiple equilibria imply several possible crisis times and several possible post-crisis reserves levels. Furthermore, some developed countries have debt/GDP ratios above 100 percent, while in some recent emerging market crises, investors have been unwilling to lend to governments whose debts amounted to only 50 or 60 percent of GDP. Thus, I have experimented with a wide range of values for $A$, from −20% to −100% of GDP. Similarly, $\alpha$ presents difficulties, since it is a penalty that is supposed to be perpetual, but we only observe a few years after the crisis, and we do not observe what would have happened to each country if there had been no crisis. Furthermore, in cases like Britain (1992), GDP grew after floating, which illustrates the fact that, in some cases, the penalty for floating may be best interpreted as representing political embarrassment, or loss of credibility. Given these considerations, I have experimented with small values, between 0.5% and at most 5% of GDP. Finally, the interest rate $1/\beta−1$ at which a country can borrow reserves to fend off attacks depends on factors such as creditworthiness, monetary policy in the anchor country, and loan (or bond) maturity. Yields on foreign-currency denominated bonds can may be a good proxy for yearly interest rates (and these may range from very low, like 5%, to 60% or more in cases with severe sovereign risk). However, it is more difficult to determine what should be the length of a period in our model. Even for very high annual interest rates, if periods are short $1/\beta−1$ will be small. On the other hand, if we think of speculative pressure as lasting for a substantial period of time, like for instance a quarter, we can justify large values for $1/\beta−1$. Hence, I experiment with values $\beta$ between 0.8 and 0.99.
6. Conclusion

We extend second-generation models of currency crises by endogenizing the time paths of fundamentals. We find that, if the optimal policy under commitment is to run expansionary policies and float at a certain period, the possibility of self-fulfilling crises generates multiple possible devaluation times. For some parameters, self-fulfilling crises can only do this, accelerate crises that will happen anyway. For other parameter combinations, self-fulfilling crises can dismantle sustainable pegs. Thus, for these latter parameter configurations, the model generates “purely” self-fulfilling crises and rationalizes the policies assumed in Obstfeld (1986), i.e. run restrictive policies if not attacked, and expansionary policies if attacked.

For future work, it would be interesting to expand the model by making the lower limit for reserves vary endogenously, as in many emerging markets crises, the collapse of currencies come simultaneously with difficulties to roll over debt, or, as in Argentina in 2001-2002, default. A second direction in which the ideas in this paper could be furthered is by studying the possibility of multiple equilibria in other models that derive optimal government policy in times of crises, such as Lahiri and Végh (2003). This extension would answer to what extent optimal interest rate policy is robust, or can be substantially changed by sunspots and self-fulfilling expectations.
References


Figure 1  Equilibrium paths under commitment.

Figure 2  For these parameters, self-fulfilling expectations affect only timing of crises.
Figure 3  An increase in $\tau$ : “Fix Forever Zone” expands and intersects with “Crisis Zone”.

Figures 4 and 5  A reduction in the borrowing limit $A$ creates intersection between “Fix Forever” and “Crisis Zone”, making even sustainable pegs vulnerable to attacks. In this case, the “Delay the Penalty” Zone practically disappears.
Figure 6. As $\beta$ falls, the intersection of the ‘Fix Forever Zone’ and the ‘Crisis Zone’ grows. Parameters are $A = -0.2$, $\alpha = 0.99$, $\beta = 0.8$, $\bar{e} = 1$, $\eta = 0.2$, $\gamma = 0.35$, $\omega = 0.8$ and $\tau = 0.25$. 