An Investment-and-Marriage Model with
Differential Fecundity

Hanzhe Zhang
Michigan State University

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Abstract

I build an equilibrium investment-and-marriage model to provide explanations for an agglomerate of facts on education, income, and marriage that has not been explained in a unified way. Differential fecundity and equilibrium marriage market form the basis of my explanations. Most surprising is an explanation of why in the United States and many other countries women attend college at higher rates and earn lower average incomes than men. The model can also account for gender-specific relationships between age at marriage and personal midlife income as well as the evolving relationship between age at marriage and spousal midlife income for women. Empirical evidence and calibration results support my explanations.

Keywords: college gender gap, relationships between age at marriage and income

JEL: C78, D1

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1 Introduction

Three sets of stylized facts about education, income, and marriage have not been explained in a unified way. First, the college gender gap has reversed but the earnings gender gap persists in the United States and many other developed countries: More women than men are going to college while women continue to earn less than men on average. Second, the relationship between age at marriage and midlife income for American women born in the twentieth century has been positive: The later a woman married, the more she earned on average. In contrast, the same relationship for men has been hump-shaped: Those who married in their mid-twenties had a higher average income than those who married earlier and later. Third, the relationship between age at marriage and spousal income for women has been hump-shaped: The women who married in their mid-twenties had higher-income husbands on average than for those who married earlier and later, suggesting a non-assortative marriage matching in income. Moreover, the average spousal income was lower for the women who married in their thirties than those who married around twenty for the pre-1960 birth cohorts, but was higher for the post-1960 cohorts, especially for those in the states with mandates to cover infertility treatments in insurances passed between 1985 and 1995, suggesting that fertility plays a role in marriage matching.

I propose an equilibrium investment-and-marriage model to explain these facts in a unified way. The key gender difference is fecundity: Women stay fertile for a shorter period of time than men. The most surprising consequence is an explanation of the opposite college and earnings gender gaps. It is natural to think that women’s shorter fertility length deters them from making income-improving investments, but I highlight a general-equilibrium marriage-market effect that results in more women than men going to college. Differential fecundity also generates gender-differential relationships between age at marriage and income, and helps explain the evolving hump-shaped relationship between age at marriage and spousal income for women.

The model proceeds as follows. A new generation of men and women enters the economy at the beginning of each of the infinitely many discrete periods. Individuals differ in their abilities to realize a return from an investment. The least capable individuals forgo college and marry immediately. Those with sufficiently high abilities go to college and realize an uncertain return. The successful ones marry. The unsuccessful ones can take another draw by making what I call a career investment (obtaining additional education, receiving additional training, or searching for a new job). However, this career investment is costlier to women due to it delaying fertility to a less fertile period of life. Once the returns to this investment realize, all men and women marry.

Because the career investment is costlier to women, only highly talented unsuccessful women will invest while the majority of unsuccessful college-educated men will invest. Therefore, (i) the mix of women who marry immediately after college will be negatively selected relative to men,
because some unsuccessful women will marry anyway, and (ii) the mix of women who marry in the third period will be positively selected relative to men, because those who invest will have a high success rate. Since the untalented marry in the first period and earn low incomes, this leads to a positive relationship between age at marriage and income for women and a hump-shaped relationship for men.

Turning to the marriage market, we can produce the hump-shaped relationship between age at marriage and spousal income. Even though women who marry in the third period have a higher average income than those who marry in the second period, they are less fertile. Lower fertility may hurt the marriage prospects of those who marry in the third period. If the effect is sufficiently large, the average spousal income for the women who marry in the third period is the lowest, matching the pattern in the pre-1960 birth cohorts; if the effect is sufficiently small, the average spousal income for them is higher than that for the untalented women who marry in the first period, matching the pattern in the post-1960 birth cohorts in mandated states.

Most importantly, the model can also produce a higher number of women selecting into college investment in the first period. The marriage market will endogenously produce higher returns to high-income women, since there will be fewer high-income women than high-income men given that fewer women make career investments. Thus, more women will want to invest at first, to have a chance of being high income and being rewarded on the marriage market. The reticence to make the second investment also leads to the persistent gender earnings gap, sustaining the two opposite gender gaps at the same time.

Altogether, a very simple model can match a large number of stylized facts. In summary, the paper makes three contributions. First, the paper provides the first explanation of the college and earnings gender gaps using only one gender difference, contributing to the line of research that studies these two gender gaps, especially the effects of the marriage market on them (Ilyigun and Walsh, 2007; Chiappori et al., 2009; Ge, 2011; Lafortune, 2013; Bruze, 2015; Greenwood et al., 2016; Chiappori et al., 2017).1 Second, the paper provides a detailed account and a unified explanation of the relationships between age at marriage and personal midlife income, based on labor-market shocks and differential fecundity, complementing the explanations based on search frictions in the marriage market (Becker, 1974) and information frictions in the marriage market (Bergstrom and Bagnoli, 1993).2 Third, the paper provides a theory consistent with the previ-

1 Other explanations for the college gender gap include gender differences in distributions of noncognitive skills (Buchmann and DiPrete, 2006; Goldin et al., 2006; Becker et al., 2010a,b), in labor-market returns to college (Dougherty, 2005; Mulligan and Rubinstein, 2008; Hubbard, 2011), and in occupational choices (Charles and Luoh, 2003; Olivieri, 2014), and in opportunity costs of college (Chuan, 2018). Existing explanations for the earnings gender gap include gender differences in occupational choices (Bronson, 2015), in social roles (Goldin, 2014), and in career costs of children (Adda et al., 2017).

2 The relationship between age at marriage and personal income has been documented for American men and women in the 1960 census (Keeley, 1974, 1977, 1979) and in the 1980 census (Bergstrom and Schoeni, 1996). It has
ous fertility-based explanation of the relationship between age at marriage and spousal income for women (Low, 2017; Gershoni and Low, 2017), and shows that differential fecundity is able to explain even more gender differences in economic and marital outcomes than the previous literature suggests (Siow, 1998; Greenwood et al., 2003; Coles and Francesconi, 2011; Díaz-Giménez and Giolito, 2013).³

Furthermore, I provide distinguishing empirical evidence to support my explanations. First, I verify the key implication of my explanation of the college gender gap: The marriage market returns to college are higher for women than for men. Second, I show lifecycle income paths that can only be explained by an investment-and-marriage model but cannot be explained by alternative theories based on marriage frictions. Third, I show that mandates to cover infertility treatments in insurances improved the marital outcomes of women who married in their thirties, supporting my explanation of the average spousal income for women based on fertility and income tradeoffs.

Finally, I calibrate my model to (i) validate its quantitative, (ii) quantify the relative importance of marriage-market frictions and labor-market shocks on marriage timing, (iii) quantify the counterfactual effects of infertility treatment insurance mandates on marital and labor outcomes, and (iv) conduct several counterfactual analyses of how gender equality would affect labor-market and marriage-market outcomes.

The rest of the paper is organized as follows. Section 2 discusses the three sets of stylized facts in detail. Section 3, includes the stylized model, characterization of its unique equilibrium, and the theoretical implications of the stylized facts, with omitted proofs in Appendix A and justifications for assumptions in Appendix B. Section 4 summarizes the key empirical evidence and calibration results, with details in Appendices C and D, respectively. Section 5 concludes.

2 Documenting the Stylized Facts

Figure 1 summarizes the three sets of stylized facts to be explained by the model: (a) a reversed college gender gap and a persistent earnings gender gap, (b) a hump-shaped relationship between age at marriage and midlife income for men and a positive relationship for women, and (c) a hump-shaped and evolving relationship between age at marriage and spousal income for women.

I use the decennial censuses of 1960, 1970, and 1980 and five-year American Community Surveys (ACS) 2010 and 2015 in the Integrated Public Use Microdata Series (IPUMS) USA (Ruggles et al., 2017). Age at (first) marriage is either reported directly (as variable AGEMARR) in these three decennial censuses or imputed from the year entering current marriage (variable YRMARR) also been documented for Taiwanese men in their 1989 census (Zhang, 1995) as well as for Canadians in their 1981 census and Brazilians in their 1991 census (Zhang, 2015). Relatedly, Oppenheimer (1988); Todd et al. (2005); Iyigun and Lafontune (2016) study age patterns at marriage.

³See also Siow and Zhu (2002); Schmidt (2005, 2007); Buckles (2007, 2008); Dessy and Djebbari (2010); Coles and Francesconi (2017, 2018); Bitler and Schmidt (2012); Garcia-Moran (2018).
Figure 1: Stylized Facts

(a) Reversed college gender gap and persistent earnings gender gap from 1960 to 2015

[Graph showing the percent with college degrees among 35-39 year-olds for men and women from 1960 to 2010, with a reversed college gender gap and persistent earnings gender gap.]

(b) Relationships between age at marriage and midlife income for men and women

[Graph showing average log personal midlife total income by age at marriage for men and women from the 1900s to the 1970s, with a range of ages 16-39.]

(c) Relationship between age at marriage and spousal income for women

[Graph showing average log spousal midlife total income by age at marriage for women, with categories for overall, mandated states, and nonmandated states.]
in ACS since 2008 for those who have married once and stayed married. The measure of income is reported total pre-tax personal gains or losses from all sources in the previous calendar year, inflation-adjusted to 1999 USD (i.e., INCTOT × CPI99).\(^4\) Midlife income is measured by income between ages 41 and 50 whenever possible.\(^5\)

(a) College and earnings gender gaps

The share of the college-educated among Americans aged 35-39 was higher for men before 2000 but higher for women after 2000 (the left column of Figure 1a), while the average labor income has been consistently higher for men than for women (the right column of Figure 1a). The coexistence of these two opposite gender gaps is not uniquely American but a global phenomenon: in 2010, women went to college at higher rates than men in sixty-seven countries across all inhabited continents, but earned less than men on average in each of these countries (Becker et al., 2010a).\(^6\)

(b) Relationships between age at marriage and midlife income

Men who married in their mid-twenties had a higher average midlife income than men who married earlier and later (Figure 1b). To match the three periods in the model, I compare birth-year by birth-year the average incomes of early, middle, and late grooms, those who first married between ages 16 and 22, between ages 23 and 29, and between ages 30 and 39, respectively. Middle grooms born in almost every year between 1900 and 1979 earned a statistically and economically significantly higher average midlife income than early and late grooms born in the same year; compared to middle grooms, on average, early grooms earned 13.1 percent less and late grooms earned 13.3 percent less (Figure A2a).

In contrast, the later a woman married, the more she earned on average (Figure ??). Early brides had on average 11.8 percent less midlife income than middle brides born in the same year; middle brides had on average 2.1 percent less midlife income than late brides born in the same year, but the differences between middle and late brides were not statistically significant for the majority of the birth years (Figure A2b). The observed gender difference in the relationships between age at marriage and personal income suggests that there is a fundamental gender asymmetry that results in gender-differential marriage timing and labor decisions.

\(^4\) Similar relationships are obtained if inflation-adjusted wage income (i.e., INCWAGE × CPI99) is used instead.

\(^5\) Since spousal income was not reported in the 1950 census, I use the income between ages 51 and 60 in the 1960 census for the 1900s birth cohort. Since age at marriage was not present in IPUMS USA between 1980 and 2008, age at marriage and income between ages 41 and 50 are not simultaneously available for the 1940s and 1950s birth cohorts; I use the income between ages 61 and 70 for the 1940s birth cohort, and the income between ages 51 and 60 for the 1950s birth cohort.

The relationship between age at marriage and spousal income for women

The husbands of the women who married in their mid-twenties earned a higher average midlife income than the husbands of those who married earlier and later (Figure 1c). More precisely, the husbands of early brides and of late brides respectively earned 13.4 percent and 17.5 percent less midlife income on average than the husbands of middle brides (Figure A2c).

Furthermore, the relationship changed over time. Early brides in the pre-1950 birth cohorts had higher-income husbands than late brides, but the pattern was reversed for post-1950 birth cohorts (Figure 1c). This change was more pronounced in the thirteen states that passed mandates between 1985 and 1995 requiring infertility treatments to be covered or offered by insurance. Because infertility treatment was (and still is) quite expensive, the laws reduced the costs for women and effectively extended the biological clock of the women in these states. We should expect that the marital outcome of early brides would have dropped and the marital outcome of late brides would have improved more in these thirteen states after the laws were passed. Indeed, we see that the right shoulder of the hump is raised above the left shoulder in the mandated states but not in the nonmandated states for the 1960s and 1970s birth cohorts (Figure 1c). The average spousal income of late brides statistically significantly surpassed that of early brides born after 1960 in the mandated states but not in the nonmandated states (Figure A2d). This observation suggests that gender-differential fertility length can help explain the observed relationship between age at marriage and spousal income for women.

3 Explaining the Stylized Facts

In this section, I set up the model, characterize the unique equilibrium of the model, and show how the predictions of the model are consistent with the stylized facts documented above.

3.1 Model

The model has an infinite number of discrete periods. At the beginning of each period, a unit mass of men and a unit mass of women enter the economy. Each agent is endowed with a heterogeneous ability \( \theta \in [0, 1] \). Let \( F_m \) and \( F_w \) denote the continuous and strictly increasing cumulative distribution of abilities for men and for women, respectively. Men and women make investment and marriage decisions over the next three periods of their lives to maximize their lifetime utilities. Think of the three periods as ages 16-22, 23-29, and 30-39. Each agent pays investment costs, receives a reservation payoff from working, and receives an additional endogenously determined marriage payoff if married. Each agent is risk-neutral and does not discount.

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3.1.1 Investments

Figure 2 illustrates an agent’s investment and marriage decisions over the three periods. In period 1, each agent decides whether to go to college. Anyone who decides not to go to college earns a low lifetime income and enters the marriage market immediately. Anyone who decides to go to college pays a cost, $c_m$ for a male and $c_w$ for a woman, and is assumed to delay marriage.\(^8\)

In period 2, an ability-$\theta$ agent who went to college gets on the path to a high lifetime income with probability $\theta$. Anyone who does not get on the path to a high lifetime income decides whether to make a career investment, which costs the same as a college investment. A career investment is (i) obtaining additional education beyond college, (ii) obtaining additional training, or (iii) finding a new job.\(^9\) Anyone who does not make a career investment earns a low lifetime income and enters the marriage market in the second period. Anyone who makes a career investment gets another chance to improve his/her lifetime income but delays marriage.

In period 3, an ability-$\theta$ agent who made a career investment enters the marriage market, either with a high lifetime income with probability $\theta$ or with a low lifetime income otherwise.

3.1.2 Differential Fecundity

A man is fertile for all three periods, but a woman is fertile for only the first two periods and is less fertile in the third period. In the marriage market, men are distinguished by income only, but women are distinguished by income and fertility. Let $T_m = \{H, L\}$ and $T_w = \{H, L, h, l\}$ denote the sets of marital characteristics for men and for women, respectively; letters $h$ and $H$ denote high-income types, letters $l$ and $L$ denote low-income types, uppercase letters denote fertile types, and lowercase letters denote less fertile types.

Income and fertility determine each agent’s payoff as follows. Each income-$y$ agent can gen-

\(^8\)The strategy of entering the marriage market while investing is assumed to be infeasible in the basic model. I provide in appendix B.1 theoretical and empirical justifications for this assumption, and relax this assumption in the calibration exercises in appendix D.

\(^9\)I show in appendix B that individuals with a lower income are indeed more likely to choose a career investment.
erate a reservation utility of \( z(y) \) without being married. An income-\( y_m \) man and an income-\( y_w \) fertility-\( \phi_w \) woman would generate a total utility of \( z(y_m, y_w, \phi_w) \) from marriage. Hence, the surplus due to marriage is \( s(y_m, y_w, \phi_w) = z(y_m, y_w, \phi_w) - z(y_m) - z(y_w) \). Assume the marriage surplus is nonnegative, strictly increasing in income, and strictly increasing in fertility. Furthermore, assume the surplus is strictly supermodular in incomes and strictly supermodular in husband’s income and wife’s fitness. Formally, if we let \( s \) the surplus is strictly supermodular in incomes and strictly supermodular in husband’s income and wife’s fitness. Formally, if we let \( s_{\tau_m \tau_w} = s(\tau_m, \tau_w) \) denote the surplus of a type-\( \tau_m \) man and a type-\( \tau_w \) woman and define \( \delta_{\tau_w} = s_{H \tau_w} - s_{L \tau_w} \), then strict supermodularity in incomes means \( \delta_H > \delta_L \) and \( \delta_h > \delta_l \), strict supermodularity in husband’s income and wife’s fitness means \( \delta_{H \tau_w} > \delta_{h \tau_w} \) and \( \delta_{L \tau_w} > \delta_{l \tau_w} \). The two assumptions together imply \( \delta_H \) is the largest and \( \delta_l \) is the smallest, and \( \delta_h \) can be larger, smaller, or equal to \( \delta_l \). These supermodularity assumptions will help us pin down the stable matching patterns.\(^{10}\)

3.1.3 The Marriage Market

Overlapping generations of men and women meet and bargain over the division of their marriage surplus until they reach a stable outcome in which no one can improve his or her payoff. Formally, a marriage market is described by distributions of marriage characteristics, \( G_m = \{G_{m \tau_m}\}_{\tau_m \in T_m} \) and \( G_w = \{G_{w \tau_w}\}_{\tau_w \in T_w} \), where \( G_{m \tau_m} \) is the mass of type-\( \tau_m \) men and \( G_{w \tau_w} \) is the mass of type-\( \tau_w \) women. A stable outcome of the marriage market \( (G_m, G_w) \) consists of stable matching \( G = \{G_{\tau_m \tau_w}\}_{(\tau_m, \tau_w) \in T_m \times T_w} \) and stable marriage payoffs \( v_m = \{v_{m \tau_m}\}_{\tau_m \in T_m} \) and \( v_w = \{v_{w \tau_w}\}_{\tau_w \in T_w} \).

Stable matching \( G \) satisfies feasibility: \( \sum_{\tau_w} G_{\tau_m \tau_w} \leq G_{m \tau_m} \) for any \( \tau_m \in T_m \) and \( \sum_{\tau_m} G_{\tau_m \tau_w} \leq G_{w \tau_w} \) for any \( \tau_w \in T_w \). Stable marriage payoffs \( v_m \) and \( v_w \) satisfy (i) individual rationality: \( v_{m \tau_m} \geq 0 \) for any \( \tau_m \in T_m \) and \( v_{w \tau_w} \geq 0 \) for any \( \tau_w \in T_w \) (every person receives at least as much as they would have if they had remained single); (ii) pairwise efficiency: \( v_{m \tau_m} + v_{w \tau_w} = s_{\tau_m \tau_w} \) if \( G_{\tau_m \tau_w} > 0 \) (every married couple divides the entire marriage surplus); and (iii) Pareto efficiency: \( v_{m \tau_m} + v_{w \tau_w} \geq s_{\tau_m \tau_w} \) for all \( \tau_m \in T_m \) and \( \tau_w \in T_w \) (no man-woman pair not married to each other can simultaneously improve their marriage payoffs by marrying each other). A stable outcome exists for any marriage market (theorem 2 of Gretsky et al. (1992)).

3.2 Unique Equilibrium

Define \( \sigma_m(\theta) \) and \( \sigma_w(\theta) \) as the probability of an ability-\( \theta \) man investing in the first and second period, respectively, and define \( \sigma_m(\theta) \) and \( \sigma_w(\theta) \) for an ability-\( \theta \) woman similarly. Strategies are summarized by functions \( \sigma_m = (\sigma_m^1, \sigma_m^2) \) and \( \sigma_w = (\sigma_w^1, \sigma_w^2) \). We say strategies \( \sigma_m \) and \( \sigma_w \) induce the marriage market \( (G_m, G_w) \) if the distributions of men’s and women’s marriage characteristics in each period are \( G_m \) and \( G_w \), respectively, when men and women of every generation respectively choose strategies \( \sigma_m \) and \( \sigma_w \).

\(^{10}\)I provide in appendix B.3 a microfoundation of the marriage surplus function based on intra-household allocation of private and public goods.
Definition 1. A quadruple \((\sigma_m^v, \sigma_w^v, v_m^v, v_w^v)\) is an equilibrium if (i) \(\sigma_m^v(\theta)\) and \(\sigma_w^v(\theta)\) respectively maximize each ability-\(\theta\) man’s and each ability-\(\theta\) woman’s expected utility when the marriage payoffs are \(v_m^v\) and \(v_w^v\); and (ii) \(v_m^v\) and \(v_w^v\) are stable marriage payoffs of the marriage market \((G_m, G_w)\) induced by \(\sigma_m^v\) and \(\sigma_w^v\).

3.2.1 Equilibrium Investments

Optimal investments differ by gender because of differential fecundity, and the difference in investments is crucial to explain all the stylized facts regarding gender differences. Men’s optimal investments can be solved by backward induction. If an ability-\(\theta\) man who receives a low-income offer after college decides to make a career investment, then he incurs a cost \(c_m\), and expects a lifetime income gain \(\theta(z_mH - z_mL)\) and a lifetime marriage gain \(\theta(v_mH - v_mL)\). An ability-\(\theta\) man makes a career investment if and only if the expected gain outweighs the cost, that is, if and only if his ability is above
\[
\theta_m := \frac{c_m}{z_mH - z_mL + v_mH - v_mL}.
\]
A man goes through the same cost-benefit analysis to decide on optimal college investment. Therefore, in any equilibrium, any man with an ability above \(\theta_m\) makes a college investment, and makes a career investment if he receives a low-income offer after college, while any man with an ability below \(\theta_m\) makes no investment.\(^{11}\)

Women’s optimal investments can be solved by backward induction, too. If an ability-\(\theta\) woman who receives a low-income offer after college makes a career investment, then her expected income gain is \(\theta(z_wH - z_wL)\) and her expected marriage gain is \(\theta(v_wh - v_wl) - (v_wL - v_wl)\), where the term \(v_wL - v_wl\) represents her loss in marriage payoff due to fertility decline. She makes a career investment if and only if her ability \(\theta\) is above
\[
\theta_{w2} := \frac{c_w + v_wL - v_wl}{z_wH - z_wL + v_wh - v_wl}.
\]
In contrast, a woman who makes a college investment does not expect an immediate fertility decline. An ability-\(\theta\) woman makes a college investment if and only if her ability is above
\[
\theta_{w1} := \frac{c_w}{z_wH - z_wL + v_wH - v_wL}.
\]
Note that \(\theta_{w1} < \theta_{w2}\): some women would not make a career investment. In summary, any woman whose ability is above \(\theta_{w2}\) makes a college investment and, in case her college investment fails, makes a career investment; any woman whose ability is between \(\theta_{w1}\) and \(\theta_{w2}\) makes a college investment only; and any woman whose ability is below \(\theta_{w1}\) makes no investment.

The induced distributions of marriage characteristics can be characterized straightforwardly

\(^{11}\)Ability-\(\theta_m\) men are indifferent between investing and not investing. It is without loss of generality to assume that they invest whenever they are indifferent. It is without loss of generality because the distribution of abilities is atomless and there is measure 0 of ability-\(\theta_m\) men, hence the stable outcome of the marriage market is not affected by the investment decisions of ability-\(\theta_m\) men.
from optimal investments. Type-\(H\) men consist of men with an ability above \(\theta_m\) who receive a high-income offer either after a college investment or after a career investment, so \(G_{mh} = \int_{\theta_m}^{1} [\theta + (1 - \theta)\theta] dF_m(\theta)\). Type-\(L\) men consist of (i) all men with an ability below \(\theta_m\) and (ii) men with an ability above \(\theta_m\) who fail to receive a high income after college and career investments. Because there is a unit mass of men in each period’s marriage market, the mass of low-income men is simply \(G_{ml} = 1 - G_{mh}\).\(^{12}\) Type-\(H\) women are those with an ability above \(\theta_{w1}\) who succeed right after college: \(G_{wH} = \int_{\theta_{w1}}^{1} \theta dF_w(\theta)\). Type-\(h\) women are those with an ability above \(\theta_{w2}\) who succeed only after their career investment: \(G_{wh} = \int_{\theta_{w2}}^{1} (1 - \theta)\theta dF_w(\theta)\). Type-\(L\) women consist of (i) all women with an ability below \(\theta_{w1}\) and (ii) women with an ability between \(\theta_{w1}\) and \(\theta_{w2}\) who fail after college and do not make a career investment: \(G_{wL} = F_w(\theta_{w1}) + \int_{\theta_{w1}}^{\theta_{w2}} (1 - \theta) dF_w(\theta)\). Finally, type-\(L\) women are those with an ability above \(\theta_{w2}\) who fail to receive a high income after college and career investments: \(G_{wl} = 1 - G_{wH} - G_{wL} - G_{wh}\).

### 3.2.2 Equilibrium Matching

The tradeoff between income and fertility in the marriage market helps explain the observed marriage matching patterns. First, because the marriage surplus is assumed to be strictly supermodular in incomes, given two equally fertile women, a higher-income woman almost surely marries a higher-income man.\(^{13}\) Second, because the surplus is assumed to be strictly supermodular in husband’s income and wife’s fertility, given two women with the same income, a more fertile woman almost surely marries a higher-income man. The two results together imply that (i) type-\(H\) women almost surely marry higher-income husbands than women of any other type, and (ii) type-\(L\) women almost surely marry lower-income husbands than women of any other type. Whether a type-\(h\) woman or a type-\(L\) woman marries a higher-income husband depends on an additional condition. A type-\(h\) woman almost surely marries a man with a higher income than a type-\(L\) woman does if and only if \(\delta_h > \delta_L\). In summary, stable matching is positive-assortative in men’s income and women’s type, provided that women’s types are ranked according to (i) \(H \succ h \succ L \succ l\) when \(\delta_h > \delta_L\), (ii) \(H \succ L \succ h \succ l\) when \(\delta_L > \delta_h\), or (iii) \(H \succ L \sim h \succ l\) when \(\delta_L = \delta_h\).

### 3.2.3 Equilibrium Marriage Payoffs

Stable marriage payoffs are characterized as follows. Because there is an equal mass of men and women in the marriage market, there is a positive mass of marriages between the bottom-

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\(^{12}\)There is a unit mass of men and women in each period’s marriage market because of the overlapping-generations nature of the model. Period-\(t\) marriage market consists of (i) those who are born in period \(t - 2\) and enter the marriage market in the third period of their lives, (ii) those who are born in period \(t - 1\) and enter the marriage market in the second period of their lives, and (iii) those who are in period \(t\) and enter the marriage market in the first period of their lives. Because the optimal investments are stationary, there is a unit mass of men and women in each period’s marriage market.

\(^{13}\)The modifier “almost surely” is needed because the marriage market consists of a continuum of men and women, rather than a finite number of agents as in Shapley and Shubik (1972) and Becker (1973). It is a standard quantifier in the literature (Chiappori and Oreffice, 2008; Chiappori et al., 2012a,b).
ranked type-$L$ men and type-$l$ women. By pairwise efficiency, $v_{mL} + v_{wl} = s_{ll}$. However, also because there is an equal mass of men and women, neither $v_{mL}$ nor $v_{wl}$ is determinate. Stable marriage payoffs can be determined only up to a constant. They can be characterized by differences between marriage payoffs of adjacently ranked types.

We first derive men’s stable marriage premium $\pi_m \equiv v_{mH} - v_{mL}$. There are two cases. In the first case, the mass of high-income men is between the mass of women strictly higher-ranked than $t'_w$ and the mass of women weakly higher-ranked than $t'_w$, for some female type $t'_w$. Type-$t'_w$ women marry high-income men and low-income men with positive probabilities, so $v_{mH} + v_{w't'_w} = s_H t'_w$ and $v_{mL} + v_{w't'_w} = s_L t'_w$, which together imply $\pi_m = s_H t'_w - s_L t'_w$. In the second case, the mass of high-income men equals the mass of women weakly higher-ranked than $t'_w$. Women weakly higher-ranked than $t'_w$ almost surely marry high-income men, and women strictly lower-ranked than $t'_w$ almost surely marry low-income men. Since there is no type of women who marry both types of men with positive probabilities, $\pi_m$ is indeterminate. It can take any value between $\delta_{t'_w}$ and $\delta_{t'_w}$, where $t'_w$ is the female type ranked just below $t'_w$.\footnote{If $G_{mH} = G_{w\geq t''_w}$, then the lowest type of women marrying high-income men with a positive probability is $t''_w$, and the highest type of women marrying low-income men is $t''_w$, the type ranked right below $t''_w$. By pairwise efficiency, a high-income man marrying a type-$t''_w$ woman gets a payoff of $v_{mH} = s_H t''_w - v_{w't''_w}$. By Pareto efficiency, a low-income man is weakly better off staying in his current match than marrying a type-$t''_w$ woman, so $v_{mL} \geq s_L t''_w - v_{w't''_w}$. The two conditions together imply the upper-bound $\delta_{t''_w}$ for $v_{mH} - v_{mL}$. Because $t''_w$ is the highest-ranked type of women marrying low-income men with a positive probability, it follows that pairwise efficiency condition $v_{mL} \geq s_L t''_w - v_{w't''_w}$ and Pareto efficiency condition $v_{mH} \geq s_H t''_w - v_{w't''_w}$ imply the lower-bound $\delta_{t''_w}$ for $v_{mH} - v_{mL}$.}

This indeterminacy in $\pi_m$ will dissipate in equilibrium, however, when marriage payoffs and investments are jointly determined.

Now we derive the payoff differences between any two adjacently ranked female types $t_w$ and $t'_w$. Either (i) type-$t_w$ and type-$t'_w$ women both marry type-$t_m$ men with positive probabilities so that $v_{w't'_w} - v_{w't'_w} = s_m t_w - s_m t'_w$; or (ii) almost all type-$t_w$ women marry type-$H$ men and almost all type-$t'_w$ women marry type-$L$ men so that $v_{w't_w} - v_{w't'_w} = (s_H t_w - s_L t'_w) - \pi_m$. Hence, the difference between the marriage payoffs of any two types of women is either determinate or depends on $\pi_m$. Consequently, the difference between the marriage payoffs of any two types of women can be expressed as a function of $\pi_m$.

We see from the derivations above that the difference between the marriage payoffs of any two types can be represented by men’s marriage premium $\pi_m$. Furthermore, the three optimal investment thresholds in equations (1)-(3) are uniquely determined by payoff differences, and thus, uniquely determined by $\pi_m$. Therefore, any equilibrium can be simply represented by the one number of $\pi_m$.

### 3.2.4 Equilibrium Existence and Uniqueness

**Theorem 1.** An equilibrium exists. Equilibrium investments are uniquely determined, and equilibrium marriage payoffs are uniquely determined up to a constant.
The proof, presented in Appendix A.2, follows three steps. First, I construct (i) a correspondence that represents the demand for high-income men in the marriage market and (ii) a function that represents the supply. Second, I argue that each intersection of the constructed demand and supply curves corresponds to an equilibrium. Third, I show that (i) the constructed demand and supply curves always intersect, proving equilibrium existence, and (ii) the demand curve is downward-sloping and the supply curve is upward-sloping, proving equilibrium uniqueness.

3.3 Explanations of the Stylized Facts

(a) College and earnings gender gaps

Proposition 1. Suppose women are less fertile in the third period while investment costs \(c_m\) and \(c_w\), labor-market opportunities \(F_m\) and \(F_w\), income premiums \(z_{mH} - z_{mL}\) and \(z_{wH} - z_{wL}\), as well as the marriage surpluses \(s_{HL}\) and \(s_{LH}\) are all gender-symmetric. Strictly more women than men go to college in equilibrium. Strictly fewer women than men earn a high lifetime income in equilibrium if \(G_{mH}(\delta_l) > G_{wH}(\delta_l) + G_{wh}(\delta_l)\).

Existing papers (cited in the introduction) have explained the college gender gap using gender differences in psychic and monetary costs of investments, in labor-market opportunities, in college income premiums, and in marital roles. Proposition 1 states that even in a model that does not include any of these gender differences, it could be the case that more women than men attend college. Adding any of these gender differences into the model would only reinforce the female-dominated college gender gap. Furthermore, the female-dominated college gender gap can be sustained even when gender differences that deter women’s college investments are included. Therefore, the model highlights a new fundamental force rooted in differential fecundity and propagated through the marriage market contributing to the global college gender gap. At the same time, the earnings gender gap is maintained, a result unattainable from previous models explaining the college gender gap without including additional gender differences.

While I will present the formal proof of the proposition by contradiction in appendix A.3, I provide an economic explanation here. Define the difference between the marriage payoffs of a fertile high-income earner and a fertile low-income earner, \(\pi_i \equiv v_{iH} - v_{iL}\), \(i = m, w\), as the marriage premium. The college ability cutoffs are simply determined by the investment cost divided by the income premium and the marriage premium, \(\theta_i = c_i/(z_{iH} - z_{iL} + \pi_i)\), \(i = m, w\). When the

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For example, it has been argued that women’s college income premium is lower than men’s. Dougherty (2005) shows that the college income premium for women (defined as the difference in log wages of non-college-educated female high school graduates and female college graduates) was higher than men’s. Hubbard (2011) shows that the gender difference in the college income premiums was nonexistent after correcting for income top-coding bias which has previously underestimated men’s college income premium. Estimates presented in figure 6 of DiPrete and Buchmann (2006) and figure 9 of Chiappori et al. (2017) provide additional evidence that the gender difference in college income premiums is not enough to explain the college gender gap.

The shares of men and women going to college are \(1 - F_i((c_i/(z_{iH} - z_{iL} + \pi_i^*)))\), \(i = m, w\).
setting is gender-symmetric, more women than men go to college if and only if the endogenous marriage premium $\pi_i$ is higher for women than for men. If the marriage premiums $\pi_m$ and $\pi_w$ were exogenously fixed to be the same, the same number of men and women would go to college, and fewer women than men would make a career investment because of differential fecundity. Consequently, fewer women than men earn a high income. However, the marriage premiums are endogenously determined in the model. Precisely because fewer women than men earn a high income, the women who earn a high income are more scarce and more “valuable” in the marriage market than the men who achieve the same feat. Women’s endogenously higher marriage premium prompts more women than men to make a college investment. Hence, a key implication of the model is a higher marriage premium for women than for men. I will test and confirm this implication empirically in Appendix C.1.

The two key drivers of our main result are differential fecundity and the endogenous division of the marriage surplus. In a model without differential fecundity, the setting is entirely gender-symmetric, so the same number of men and women would go to college, make career investments, and earn a high income. In a model that incorporates differential fecundity but omits the endogenous division of the marriage surplus (i.e., the marriage premiums are exogenously the same for the two genders), the same number of men and women would go to college, but fewer women than men would make a career investment and earn a high income; such a model would be able to explain the earnings gender gap but not the college gender gap.

Hence, the combination of differential fecundity and endogenous surplus division is needed to account for the opposite gender gaps. Differential fecundity directly reduces women’s career investments but does not directly increase their college investments. College and career investments are not directly substitutes to improve income, but endogenous marriage surplus division makes these investments strategic substitutes.\footnote{Thomas (2018) considers the possibility that college and career investments are direct substitutes.} Specifically, the decline in fertility directly discourages intermediate-ability college-investing women (namely, women with an ability close to $q_{w2}^*$) from making career investments, and indirectly encourages lower-ability women (namely, women with an ability close to $\theta_{w1}^*$) to go to college through endogenous marriage surplus division.

Furthermore, surplus supermodularity in incomes is necessary to explain the college gender gap. If the surplus is submodular in incomes, the same number of men and women would go to college. Surplus supermodularity in incomes is theoretically grounded in the intrahousehold allocation model presented in the appendix, and is empirically supported by our subsequent estimates of the marriage surplus function as well as findings on positive assortative matching in incomes and educations in the United States and other countries (Lam, 1988; Blossfeld and Timm, 2003; Schwartz and Mare, 2005; Stevenson and Wolfers, 2007; Greenwood et al., 2014; Siow, 2015; Greenwood et al., 2016; Chiappori et al., 2017).
(b) Relationships between age at marriage and income for men and women

**Proposition 2.** The relationship between age at marriage and income for men is hump-shaped in equilibrium: the average income is the highest for middle grooms. The relationship between age at marriage and income for women can be positive, positive-then-flat, or hump-shaped in equilibrium.

Early grooms in the model are the men with an ability below $\theta_m^{*}$, and they earn a low lifetime income without making any investment. Middle grooms are the men with an ability above $\theta_m^{*}$, who get on the path to a high lifetime income after college. Late grooms are the remaining men with an ability above $\theta_m^{*}$ who fail to realize a high income after college and consequently make a career investment, and some of them receive a high income, and the rest of them receive a low income, so the average income is lower for late grooms than for middle grooms.

For the upward-sloping portion of the relationship, early grooms earn less than middle grooms on average because early grooms invest less than middle grooms. Bergstrom and Bagnoli (1993) also predict a positive relationship between age at marriage and income for men, but there is a difference between their model and the current model. In their model, high-income men wait to marry because they cannot credibly signal their earning abilities when they are young. In contrast, there is no private information in the current model. Even if a man can choose to marry during college, he weakly prefers to wait to marry until after he resolves his post-college income uncertainty (as shown in section B.1). The reason to delay marriage in this model is rooted in the inherent nature of the marriage market. A man who has uncertainty about his future lifetime income may not be able to marry the woman he could marry when he has a high lifetime income for sure, so he chooses to delay marriage. Moreover, the downward-sloping portion of the relationship cannot be explained by Bergstrom and Bagnoli (1993) but can be explained by the current model.

For the downward-sloping portion of the relationship, middle grooms earn more than late grooms on average because middle grooms are the college-educated men who get on the path to a high lifetime income soon after college, and late grooms are the college-educated men who fail to do so and end up with a lower income on average. Becker (1974) and Keeley (1979) also predict a negative relationship, but their explanation is different. Whereas higher-ability men in their models marry earlier because they encounter less marriage-market friction, higher-ability men in the current model do so because they are less likely to encounter an adverse labor-market shock. Lower-income men involuntarily delay marriage due to marriage-market frictions in their models but voluntarily delay marriage due to labor-market shocks in the current model. Section 3.3 will present patterns that can only be explained by the impacts of labor-market shocks on marriage timing. The calibration in appendix D will quantify the respective impacts of marriage-market frictions and labor-market shocks on marriage timing.

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18 The discussion in Oppenheimer (1988) that people would like to be sure of their career prospects before marrying is formalized by this model.
In equilibrium, early brides earn a low income because they are the low-ability women (those with an ability below $\theta^*_{w1}$) who do not go to college. Middle brides consist of all the intermediate-ability women (those with an ability between $\theta^*_{w1}$ and $\theta^*_{w2}$) and the higher-ability women (those with an ability above $\theta^*_{w2}$) who earn a high income right after college. Late brides are the higher-ability women who do not receive a high income after college. The model predicts that early brides earn less than middle brides and late brides, but the model does not make a definitive prediction about whether middle brides or late brides earn less.

In the model, early brides earn less than middle brides, because early brides are those who do not go to college but middle brides are those who go to college with many ending up with a high income. The impact of human capital investment on women’s marriage timing is completely missing in Becker (1974) and Keeley (1979) (which predict a positive relationship between age at marriage and income due to marriage-market frictions) and Bergstrom and Bagnoli (1993) (which predicts no relationship between age at marriage and income for women). The current model naturally incorporates this effect.

In the model, middle brides tend to earn less than late brides, because middle brides mostly consist of intermediate-ability women who fail to receive a high income after college but nonetheless choose to marry, but late brides are the high-ability women who do not receive a high income right out of college but receive a high income with a large probability after career investments. In short, labor-market shocks and the fertility-income tradeoff result in a positive selection in delayed marriage. Becker (1974) predicts a positive relationship between age at marriage and income driven by marriage-market frictions. Section 3.3 will present evidence that can only be explained by labor-market shocks, and the calibration in Appendix D will quantify the respective impacts of marriage-market frictions and labor-market shocks on marriage timing.

(c) Relationship between age at marriage and spousal income for women

Proposition 3. The relationship between age at marriage and spousal income for women is hump-shaped in equilibrium: the average spousal income is the highest for those who marry in the second period. The average spousal income is higher for early brides when $\delta_h > \delta_L$, and is higher for late brides when $\delta_L > \delta_h$.

In the model, early brides are fertile low-income earners, and middle brides consist of both fertile low-income earners and fertile high-income earners; since fertile high-income women’s husbands almost always have a higher income than fertile low-income women’s, middle brides are predicted to have a higher average spousal income than early brides. Late brides consist of both high-income and low-income earners, but they are less fertile than middle brides. Since (i) for any two women with the same income, the more fertile one marries a higher-income husband in equilibrium, and (ii) late brides do not earn significantly more than middle brides on average, the
average spousal income is predicted to be lower for late brides than for middle brides.

The key to explaining whether early brides or late brides marry higher-income husbands is “non-assortative matching” in incomes (Low, 2017) in the marriage market. According to the model, early brides are fertile low-income earners, and late brides consist of less fertile women with a higher average income. If fertility is more important than income in the marriage market (i.e., \( \delta_L > \delta_h \)), type-\( L \) women marry higher-income husbands than type-\( h \) women, and consequently, early brides’ average spousal income would be lower than late brides’. Otherwise (i.e., \( \delta_h > \delta_L \)), it is possible that less fertile high-income women’s husbands have a higher income than fertile low-income women’s, and the average spousal income is higher for less fertile women than for fertile low-income women.

The fact that late brides had higher-income husbands than early brides in the states with infertility mandates suggests that the evolution of the difference in average spousal income between early brides and late brides was at least partially driven by the change in the relative importance of income and fertility in the marriage market. These changes in the marriage market can be thought of as a decrease in the demand for and/or an increase in the supply of “reproductive capital” (Low, 2017) in the marriage market.

On the one hand, the demand for “reproductive capital” has decreased. First, the desired (and actual) family size has decreased; in the United States, the average desired number of children has declined from 3.6 to 2.6 from 1960 to 2010 (Livingston and Cohn, 2010). Many families have shifted from a demand for quantity of children to a demand for quality, as Becker and Lewis (1973) predicted. Women’s fertility has become less of a concern in marriage decisions than women’s income and education. Second, an increase in income gain from college and career investments also contributes to a decrease in the relative importance of fertility; the benefit of the career investment in the labor market outweighs the cost of delayed marriage in the marriage market.

On the other hand, an increase in the supply of “reproductive capital” has been achieved by advances in medical technology such as in-vitro fertilization, egg freezing, and more cost-effective maternal health services, all of which have resulted in a higher probability of staying fertile and conceiving. Older women can have children with less financial burden, more physical ease, and fewer adverse health effects than in the past. Gershoni and Low (2017) present causal evidence that policies that have made assistive reproductive technology less expensive and more accessible directly improved education, labor-market, and marital outcomes of Israeli women who married late. We show similar evidence for American women.

4 Supporting the Explanations

In this section, I summarize the key distinguishing evidence supporting my explanations of the stylized facts as well as the results of calibration that validate the quantitative fit of the model and
counterfactual analyses based on the calibrated models. I leave the details in online appendices.

4.1 Key Empirical Evidence

(a) College and earnings gender gaps

A key implication of the model is a higher marriage premium for women than for men when more women than men go to college. I estimate marriage premiums from 1960 to 2015. Women’s marriage premium was smaller than men’s in 1960, 1970, and 1980, when fewer women than men, ages 35-39, graduated from college; and was greater than men’s in 2010 and 2015, when more women than men, ages 35-39, graduated from college. I adopt the technique developed by Choo and Siow (2006) to exactly identify the marriage surplus function and compute the marriage premiums from the estimated marriage surplus function.

In the model, women’s college investment rate and average income in equilibrium both increase if any or any combination of the following events happens: (i) women face a lower investment cost; (ii) women’s labor-market opportunities improve; (iii) women’s college income premium increases; and (iv) women’s college marriage premium increases. These predictions are consistent with the empirical literature studying the rise of women’s college enrollment and labor force participation, cited in the introduction.

(b) Relationships between age at marriage and income

If labor-market shocks indeed delay marriages of the college-educated men as the model suggests, then we would expect that, compared to college-educated middle grooms, college-educated late grooms should have (i) a lower average income when they have just finished college and (ii) a higher income growth rate following college because they engage in more career investments. In contrast, if marriage-market friction is the sole determinant of marriage timing, then we would expect that college-educated late grooms should have a lower income growth rate following college than college-educated middle grooms. For the three cohorts tracked by National Longitudinal Study of Youth (NLSY), four-year-college-educated late grooms, the men who received exactly four years of postsecondary education and first married between ages 30 and 39, (i) earned a lower average income than four-year-college-educated middle grooms in their twenties, and (ii) caught up and earned almost as much as four-year-college-educated middle grooms in their thirties.

If the labor-market shocks indeed delay marriages of the college-educated women as the model suggests, then we would expect that college-educated late brides should have a lower income in their twenties and a higher income in their thirties than college-educated early brides. The three cohorts of NLSY showed exactly those patterns: four-year-college-educated late brides earned a lower average income in their early twenties and a higher average income in their late twenties and early thirties than four-year-college-educated middle brides.
(c) Relationship between age at marriage and spousal income for women

If fertility and income are two important factors that determine women’s marital outcome, then a potential technological improvement in fertility should improve the relative marital outcome of late brides. The marital outcome of late brides, measured by spousal income and education ranks, indeed improved in the thirteen states that passed mandates to cover or offer infertility treatments in insurances between 1985 and 1995. The same results hold if the marital outcome is measured by absolute spousal income or statewide spousal income z-score.

4.2 Calibration

I calibrate my theoretical model on the U.S. data to show its good quantitative fit. Furthermore, based on the calibrated models, I provide counterfactual analyses of the infertility treatment insurance mandate and gender equality in the marriage market and the labor market. The results of calibration can be summarized in the following four steps.

First, I calibrate the benchmark model to examine the quantitative validity of the model. The calibration matches each targeted marriage-age distribution within 0.7 percent, and except for the average income of late grooms born in the 1930s and that of late brides in the 1960s, the calibration matches each of the targeted average incomes within 5 percent. Non-targeted average spousal incomes were also matched fairly well.

Second, I incorporate marriage-market frictions into the benchmark model to separate investment and marital timing decisions in order to match (i) age distributions at marriage by education level, (ii) average personal midlife income by age at marriage for men and for women, (iii) average spousal income by women’s age at marriage, and (iv) men’s and women’s college enrollment rates, as well as to quantify the importance of labor-market shocks relative to marriage-market frictions in explaining marriage timing. With the calibration of the extended model, I quantify the relative impacts of marriage-market frictions and labor-market shocks on marriage timing decisions.

For the 1930s birth cohort, there was an estimated 17.1 percent chance that college-educated women who decided to enter the marriage market before age 30 involuntarily delayed their marriage until after age 30. For the college-educated men and women who married between ages 30 and 39, essentially all men delayed marriages due to labor-market shocks, and all women delayed marriages due to marriage-market frictions (consistent with the fact that a tiny portion of women in this cohort chose to make a career investment).

For the 1960s birth cohort, the chance for a college-educated man who decided to enter the marriage market between ages 23 and 29 not being able to marry before age 30 was 22.2 percent, and the chance that a college-educated woman who decided to enter the marriage market between ages 23 and 29 not being able to marry before age 30 was 23.8 percent. The calibration shows that 42.7 percent of college-educated men and 24.6 percent of college-educated women delayed their
marriages due to labor-market shocks, and the rest delayed due to marriage-market frictions.

Third, I calibrate the model on four groups – 1930s birth cohort and 1960s birth cohort in the mandated and nonmandated states – to examine the impacts of the mandates on marriage and labor outcomes. The quantitative fit of the model is not affected by the division into two groups of states. The counterfactual analyses based on the calibrated model show that the mandate increases the marriage age of women and the marital outcome of those women who marry after age 30.

Fourth and finally, I use the calibrated parameters of the 1960s birth cohort to provide counterfactual effects of equalizing men and women in the marriage market and the labor market. The first counterfactual analysis shows that removing differential fecundity would increase the fraction of women marrying after age 30 and would increase middle and late brides’ marital outcome, although it would not increase the average personal income of those who marry in the thirties as the relationship between age at marriage and income for women would become more similarly hump-shaped as the relationship for men. The second counterfactual analysis shows that equalizing labor market opportunities for women would slightly decrease the college enrollment of women because the opportunity cost of college increases and would slightly increase the incentives for women to make career investments as the labor premiums of investments increases. The two effects result in the reduction of those who marry between ages 23 and 29 and an increase in those who marry earlier and later. The marital outcome of middle and late brides increases because of their increased income. The third counterfactual analysis shows that if the ability distributions are the same for men and women, then the college enrollment for women would decrease and the fraction of those who marry after age 30 would also decrease.

5 Conclusion

I built an equilibrium investment-and-marriage model with one gender asymmetry – differential fecundity – to account for a set of phenomena that has not been explained under a unified framework. Most notably, I (i) provided a new explanation for the college gender gap and the earnings gender gap, based on differential fecundity and the equilibrium marriage market, (ii) explained the relationships between age at marriage and personal income for both men and women, and (iii) encapsulated in my model previous explanation of the relationship between age at marriage and spousal income for women. The model accounted for these phenomena both qualitatively and quantitatively. Labor-market shocks, marriage-market frictions, and differential fecundity were shown to play important roles. I believe the theoretical and empirical analyses helped to achieve a better understanding of complex labor-market and marriage-market phenomena, and could serve as stepping stones to achieve an even better understanding of these intertwined phenomena.
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Appendix

A Additional Figures and Omitted Proofs

A.1 Additional Figures

Figure A1 shows the relationships between age at marriage and labor income (rather than total income). Figure A2 shows the income differences between different age-at-marriage groups birth-year by birth-year.

Figure A1: Stylized Facts with Labor Income
(a) Relationships between age at marriage and labor income for men and women

(b) Relationship between age at marriage and spousal labor income for women
Figure A2: Evolution of the average income differences between age-at-marriage groups
(a) Difference in average log personal midlife total income from middle grooms

(b) Difference in average log personal midlife total income from middle brides

(c) Difference in average log spousal midlife total income from middle brides

(d) Difference in average spousal incomes between early and late brides
A.2 Proof of Theorem 1

Let \( \theta_{m}(\pi_{m}), \theta_{w1}(\pi_{m}), \) and \( \theta_{w2}(\pi_{m}) \) denote the ability cutoffs characterizing optimal human capital investments when men’s stable marriage premium is \( \pi_{m} \) (and women’s stable marriage-payoff differences are pinned down by \( \pi_{m} \)). Let \( G_{m}(\pi_{m}) \) and \( G_{w}(\pi_{m}) \) denote the induced distributions of men’s and women’s marriage characteristics, respectively, when the investment strategies are the ones characterized by the ability cutoffs \( \theta_{m}(\pi_{m}), \theta_{w1}(\pi_{m}), \) and \( \theta_{w2}(\pi_{m}) \). Let \( \Pi_{m}(G_{m},G_{w}) \) denote the set of men’s stable marriage premiums (and associated stable marriage payoffs of women) in the marriage market \( (G_{m},G_{w}) \). Construct the correspondence

\[
D_{mH}(\pi_{m}) := \{G_{mH} \in [0,1] : \pi_{m} \in \Pi_{m}((G_{mH},1-G_{mH}),G_{w}(\pi_{m}))\}.
\]

For any \( \pi_{m} \in [\delta_{i},\delta_{H}] \), each element in the set \( D_{mH}(\pi_{m}) \) is a mass \( G_{mH} \) of high-income men such that \( \pi_{m} \) is men’s stable marriage premium in the marriage market \( ((G_{mH},1-G_{mH}),G_{w}(\pi_{m})) \). Explicitly, (i) if \( \pi_{m} = \delta_{t}^{*} \) for a certain type \( \tau_{t}^{*} \in T_{w} \), then \( D_{mH}(\pi_{m}) = [G_{w};\delta_{t}^{*};\delta_{t'}^{*}(\pi_{m})] \); and (ii) if \( \pi_{m} \in (\delta_{t'}^{*},\delta_{t''}^{*}) \) for a certain pair of adjacent types \( \tau_{t}^{*} \in T_{w} \) and \( \tau'_{t} \in T_{w} \), then \( D_{mH}(\pi_{m}) = G_{w};\delta_{t}^{*};\delta_{t''}^{*}(\pi_{m}) \).

I prove the claim that there exists an equilibrium in which men’s stable marriage premium is \( \pi_{m}^{*} \) if and only if \( G_{mH}(\pi_{m}^{*}) \in D_{mH}(\pi_{m}^{*}) \). First, the only if part. Suppose men’s equilibrium marriage premium is \( \pi_{m}^{*} \). The induced mass of high-income men is \( G_{mH}(\pi_{m}^{*}) \), and the induced distribution of women’s marriage characteristics is \( G_{w}(\pi_{m}^{*}) \). Since \( \pi_{m}^{*} \in \Pi_{m}((G_{mH}(\pi_{m}^{*}),1-G_{mH}(\pi_{m}^{*})),G_{w}(\pi_{m}^{*})) \), by definition of \( D_{mH}(\pi_{m}^{*}) \), we have \( G_{mH}(\pi_{m}^{*}) \in D_{mH}(\pi_{m}^{*}) \). Reversely, the if only part. If \( G_{mH}(\pi_{m}^{*}) \in D_{mH}(\pi_{m}^{*}) \), then by definition of \( D_{mH}(\pi_{m}^{*}) \), \( \pi_{m}^{*} \in \Pi_{m}((G_{mH}(\pi_{m}^{*}),1-G_{mH}(\pi_{m}^{*})),G_{w}(\pi_{m}^{*})) \), so \( \pi_{m}^{*} \) is men’s equilibrium marriage premium.

It follows from the claim above that an equilibrium exists if and only if the graph of function \( G_{mH}(\cdot) \) and the graph of correspondence \( D_{mH}(\cdot) \) intersect at least once. Equilibrium marriage-payoff differences and equilibrium investments are uniquely determined if and only if the graph of function \( G_{mH}(\cdot) \) and the graph of correspondence \( D_{mH}(\cdot) \) intersect once and only once. The existence of an equilibrium is guaranteed because \( G_{mH}(\cdot) \) has a range \([0,1]\) and is continuous, and \( D_{mH}(\cdot) \) has a range \([0,1]\) and is upperhemicontinuous.

It remains for us to prove equilibrium uniqueness. \( G_{mH}(\pi_{m}) = \int_{\theta_{m}(\pi_{m})}^{1} \theta(2-\theta)dF_{m}(\theta) \) is strictly increasing in \( \pi_{m} \) because \( \theta_{m}(\pi_{m}) = \frac{c_{m}}{\epsilon_{mH} - \epsilon_{mL} + \pi_{m}} \) is strictly decreasing in \( \pi_{m} \). It suffices to show \( D_{mH}(\pi_{m}) \) is weakly decreasing in the following sense: for any \( \pi_{m} \) and \( \pi'_{m} > \pi_{m} \), \( \max D_{mH}(\pi'_{m}) \leq \min D_{mH}(\pi_{m}) \). For the remainder of the proof, we mechanically show that \( D_{mH}(\pi_{m}) \) is decreasing. Depending on \( \delta_{H} > \delta_{L} \), \( \delta_{H} < \delta_{L} \), or \( \delta_{H} = \delta_{L} \), \( D_{mH}(\pi_{m}) \) is characterized differently. I discuss the three cases separately.
Case 1. Suppose $\delta_L > \delta_h$. Explicitly,

$$D_{mH}(\pi_m) = \begin{cases} 
    [G_{w:\geq h}(\pi_m), 1] & \text{if } \pi_m = \delta_l \\
    G_{w:\geq h}(\pi_m) & \text{if } \pi_m \in (\delta_l, \delta_h) \\
    [G_{w:\geq L}(\pi_m), G_{w:\geq h}(\pi_m)] & \text{if } \pi_m = \delta_h \\
    G_{w:\geq L}(\pi_m) & \text{if } \pi_m \in (\delta_h, \delta_L) \\
    [G_{wH}(\pi_m), G_{w:\geq L}(\pi_m)] & \text{if } \pi_m = \delta_L \\
    G_{wH}(\pi_m) & \text{if } \pi_m \in (\delta_L, \delta_H) \\
    [0, G_{wH}(\pi_m)] & \text{if } \pi_m = \delta_H 
\end{cases}$$

It remains to show that (i) $G_{w:\geq h}(\pi_m)$ is strictly decreasing when $\pi_m \in (\delta_l, \delta_h)$, (ii) $G_{w:\geq L}(\pi_m)$ is strictly decreasing when $\pi_m \in (\delta_h, \delta_L)$, and (iii) $G_{wH}(\pi_m)$ is strictly decreasing when $\pi_m \in (\delta_L, \delta_H)$.

(i) To show $G_{w:\geq h}(\pi_m) = 1 - \int_{\theta_{w2}(\pi_m)}^1 (1 - \theta)^2 dF_w(\theta)$ is strictly decreasing when $\pi_m \in (\delta_l, \delta_h)$, it suffices to show $\theta_{w2}(\pi_m)$ is strictly decreasing when $\pi_m \in (\delta_l, \delta_h)$. Men’s stable marriage premium can be $\pi_m \in (\delta_l, \delta_h)$ only when $G_{mH} = G_{w:\geq h}$. When $G_{mH} = G_{w:\geq h}$, given men’s stable marriage premium $\pi_m$, women’s stable marriage-payoff differences are $v_{wl} - v_{wl} = s_{HL} - s_{LI} - \pi_m$, $v_{wh} - v_{wl} = s_{HH} - s_{HL}$, and $v_{wh} - v_{wl} = s_{Lh} - s_{LI} - \pi_m$, so

$$\theta_{w2}(\pi_m) = \frac{c_w + (v_{wl} - v_{wl})}{z_{wh} - z_{wl} + (v_{wh} - v_{wl})} = \frac{c_w + (s_{HL} - s_{LI} - \pi_m)}{z_{wh} - z_{wl} + (s_{HH} - s_{LI} - \pi_m)}$$

Since $\theta_{w2}(\pi_m) < 1$, $\theta_{w2}(\pi_m) < 0$ when $\pi_m \in (\delta_l, \delta_h)$.

(ii) To show $G_{w:\geq L}(\pi_m) = 1 - \int_{\theta_{w2}(\pi_m)}^1 (1 - \theta)dF_w(\theta)$ is strictly decreasing when $\pi_m \in (\delta_h, \delta_L)$, it suffices to show $\theta_{w2}(\pi_m)$ is strictly decreasing when $\pi_m \in (\delta_h, \delta_L)$. Men’s stable marriage premium can be $\pi_m \in (\delta_h, \delta_L)$ only when $G_{mH} = G_{w:\geq L}$. When $G_{mH} = G_{w:\geq L}$, given men’s stable marriage premium $\pi_m$, women’s stable marriage-payoff differences are $v_{wl} - v_{wl} = s_{HL} - s_{LI} - \pi_m$, $v_{wh} - v_{wl} = s_{HH} - s_{HL}$, and $v_{wh} - v_{wl} = s_{Lh} - s_{LI} - \pi_m$, so

$$\theta_{w2}(\pi_m) = \frac{c_w + (s_{HL} - s_{LI} - \pi_m)}{z_{wh} - z_{wl} + (s_{Lh} - s_{LI})}$$

Therefore, $\theta_{w2}(\pi_m)$ is strictly decreasing when $\pi_m \in (\delta_h, \delta_L)$.

(iii) To show $G_{wh}(\pi_m) = \int_{\theta_{w1}(\pi_m)}^1 \theta dF_w(\theta) + \int_{\theta_{w2}(\pi_m)}^1 (1 - \theta)dF_w(\theta)$ is strictly decreasing when $\pi_m \in (\delta_L, \delta_H)$, it suffices to show $\theta_{w1}(\pi_m)$ and $\theta_{w2}(\pi_m)$ are strictly increasing when $\pi_m \in (\delta_L, \delta_H)$. Men’s stable marriage premium is $\pi_m \in (\delta_L, \delta_H)$ only when $G_{mH} = G_{wh}(\pi_m)$. When $G_{mH} = G_{wh}$, given men’s stable marriage premium $\pi_m$, women’s stable marriage-payoff differences are $v_{wl} - v_{wl} = s_{LL} - s_{LI}$, $v_{wh} - v_{wl} = s_{HH} - s_{LI} - \pi_m$, and $v_{wh} - v_{wl} =$
\[ s_{Lh} - s_{Ll}, \text{ so} \]

\[ \theta_{w1}(\pi_m) = \frac{c_w}{z_{wH} - z_{wL} + s_{HH} - s_{LL} - \pi_m}, \]

and

\[ \theta_{w2}(\pi_m) = \frac{c_w + (s_{LL} - s_{Ll})}{z_{wH} - z_{wL} + (s_{Lh} - s_{Ll})}. \]

Therefore, both \( \theta_{w1}(\pi_m) \) and \( \theta_{w2}(\pi_m) \) are increasing when \( \pi_m \in (\delta_{L}, \delta_{H}) \).

**Case 2.** Suppose \( \delta_{h} \geq \delta_{L} \). Explicitly,

\[
D_{mH}(\pi_m) = \begin{cases} 
[G_{w;L}(\pi_m), 1] & \text{if } \pi_m = \delta_{l} \\
G_{w;L}(\pi_m) & \text{if } \pi_m \in (\delta_{l}, \delta_{L}) \\
[G_{w;h}(\pi_m), G_{w;L}(\pi_m)] & \text{if } \pi_m = \delta_{L} \\
G_{w;h}(\pi_m) & \text{if } \pi_m \in (\delta_{L}, \delta_{h}) \\
[G_{wH}(\pi_m), G_{w;h}(\pi_m)] & \text{if } \pi_m = \delta_{h} \\
G_{wH}(\pi_m) & \text{if } \pi_m \in (\delta_{h}, \delta_{H}) \\
[0, G_{wH}(\pi_m)] & \text{if } \pi_m = \delta_{H} 
\end{cases}
\]

It suffices to show that (i) \( G_{w;L}(\pi_m) \) is strictly decreasing when \( \pi_m \in (\delta_{l}, \delta_{L}) \), (ii) \( G_{w;h}(\pi_m) \) is strictly decreasing when \( \pi_m \in (\delta_{L}, \delta_{h}) \), and (iii) \( G_{wH}(\pi_m) \) is strictly decreasing when \( \pi_m \in (\delta_{h}, \delta_{H}) \).

(i) To show \( G_{w;L}(\pi_m) = 1 - \int_{0}^{1} \left[ \theta_{w2}(\pi_m) \right] (1 - \theta)^2 \, dF_w(\theta) \) is strictly decreasing when \( \pi_m \in (\delta_{l}, \delta_{L}) \), it suffices to show \( \theta_{w2}(\pi_m) \) is strictly decreasing when \( \pi_m \in (\delta_{l}, \delta_{L}) \). Men’s stable marriage premium can be \( \pi_m \in (\delta_{l}, \delta_{L}) \) only when \( G_{mH} = G_{w;L} \). When \( G_{mH} = G_{w;L} \), given men’s stable marriage premium \( \pi_m \), women’s stable marriage-payoff differences are \( v_{WL} - v_{wL} = s_{HL} - s_{LL} - \pi_m \), \( v_{WH} - v_{WL} = s_{HH} - s_{HL} \), and \( v_{WH} - v_{wL} = s_{HH} - s_{HL} - \pi_m \), so

\[ \theta_{w2}(\pi_m) = \frac{c_w}{z_{wH} - z_{wL} + s_{HH} - s_{HL} - \pi_m}. \]

Since \( \theta_{w2}(\pi_m) < 1 \), \( \theta'_{w2}(\pi_m) < 0 \) when \( \pi_m \in (\delta_{l}, \delta_{L}) \).

(ii) To show \( G_{w;h}(\pi_m) = \int_{0}^{1} \left[ \theta_{w1}(\pi_m) \right] \theta \, dF_w(\theta) + \int_{0}^{1} \left[ \theta_{w2}(\pi_m) \right] (1 - \theta)^2 \, dF_w(\theta) \) is strictly decreasing when \( \pi_m \in (\delta_{h}, \delta_{L}) \), it suffices to show both \( \theta_{w1}(\pi_m) \) and \( \theta_{w2}(\pi_m) \) are strictly increasing when \( \pi_m \in (\delta_{h}, \delta_{L}) \). Men’s stable marriage payoff can be \( \pi_m \in (\delta_{h}, \delta_{L}) \) only when \( G_{mH} = G_{w;h} \). When \( G_{mH} = G_{w;h} \), given men’s stable marriage premium \( \pi_m \), women’s stable marriage-payoff differences are \( v_{WH} - v_{wL} = s_{HH} - s_{LL} - \pi_m \), \( v_{WL} - v_{wL} = s_{LL} - s_{LL} \), and \( v_{WH} - v_{wL} = s_{HH} - s_{LL} - \pi_m \), so

\[ \theta_{w1}(\pi_m) = \frac{c_w}{z_{wH} - z_{wL} + s_{HH} - s_{LL} - \pi_m} \]

and

\[ \theta_{w2}(\pi_m) = \frac{c_w + (s_{LL} - s_{Ll})}{z_{wH} - z_{wL} + (s_{Lh} - s_{Ll} - \pi_m)}. \]
Therefore, both $\theta_{w1}(\pi_m)$ and $\theta_{w2}(\pi_m)$ are strictly increasing when $\pi_m \in (\delta_l, \delta_h)$.

(iii) To show $G_{wH}(\pi_m) = \int_{\theta_w(\pi_m)}^{1} \theta dF_w(\theta)$ is strictly decreasing when $\pi_m \in (\delta_h, \delta_H)$, it suffices to show $\theta_{w1}(\pi_m)$ is strictly increasing when $\pi_m \in (\delta_h, \delta_L)$. Men’s stable marriage premium can be $\pi_m \in (\delta_h, \delta_L)$ only when $G_{mH} = G_{wH}$. When $G_{mH} = G_{wH}$, given men’s stable marriage premium $\pi_m$, women’s stable marriage-payoff difference $v_{wH} - v_{wL} = s_{HH} - s_{LL} - \pi_m$, so

$$\theta_{w1}(\pi_m) = \frac{c_w}{z_{wH} - z_{wL} + s_{HH} - s_{LL} - \pi_m}.$$

Therefore, $\theta_{w1}(\pi_m)$ is strictly decreasing when $\pi_m \in (\delta_h, \delta_L)$.

Case 3. Suppose $\delta_h = \delta_L$. Types are ranked as $H \succ L \sim h \succ l$. Let $\tau_2 := L \sim h$. Explicitly,

$$D_{mH}(\pi_m) = \begin{cases} 
[ G_{w; \geq \tau_2}(\pi_m), 1 ] & \text{if } \pi_m = \delta_l \\
G_{w; \geq \tau_2}(\pi_m) & \text{if } \pi_m \in (\delta_l, \delta_{\tau_2}) \\
[ G_{wH}(\pi_m), G_{w; \geq \tau_2}(\pi_m) ] & \text{if } \pi_m = \delta_{\tau_2} \\
G_{wH}(\pi_m) & \text{if } \pi_m \in (\delta_{\tau_2}, \delta_H) \\
[0, G_{wH}(\pi_m)] & \text{if } \pi_m = \delta_H
\end{cases}.$$

It remains to show that (i) $G_{w; \geq \tau_2}(\pi_m)$ is strictly decreasing when $\pi_m \in (\delta_l, \delta_{\tau_2})$, and (ii) $G_{wH}(\pi_m)$ is strictly decreasing when $\pi_m \in (\delta_{\tau_2}, \delta_H)$.

(i) To show $G_{w; \geq \tau_2}(\pi_m) = 1 - \int_{\theta_w(\pi_m)}^{1} (1 - \theta)^2 dF_w(\theta)$ is strictly decreasing when $\pi_m \in (\delta_l, \delta_{\tau_2})$, it suffices to show $\theta_{w2}(\pi_m)$ is strictly decreasing when $\pi_m \in (\delta_l, \delta_{\tau_2})$. Men’s stable marriage premium can be $\pi_m \in (\delta_l, \delta_L)$ only when $G_{mH} = G_{w; \geq \tau_2}$. When $G_{mH} = G_{w; \geq \tau_2}$, given men’s stable marriage premium $\pi_m$, women’s stable marriage-payoff differences are $v_{wL} - v_{wl} = s_{HL} - s_{LL} - \pi_m$, $v_{wH} - v_{wL} = s_{HH} - s_{HL}$, and $v_{wH} - v_{wl} = s_{HH} - s_{HL} - \pi_m$, so

$$\theta_{w2}(\pi_m) = \frac{c_w + s_{HL} - s_{LL} - \pi_m}{z_{wH} - z_{wL} + s_{HH} - s_{HL} - \pi_m}.$$

Since $\theta_{w2}(\pi_m) < 1$, $\theta_{w2}(\pi_m)$ is strictly decreasing when $\pi_m \in (\delta_l, \delta_{\tau_2})$.

(ii) To show $G_{wH}(\pi_m) = \int_{\theta_w(\pi_m)}^{1} \theta dF_w(\theta)$ is strictly decreasing when $\pi_m \in (\delta_{\tau_2}, \delta_H)$, it suffices to show $\theta_{w1}(\pi_m)$ is strictly increasing when $\pi_m \in (\delta_{\tau_2}, \delta_H)$. Men’s stable marriage premium can be $\pi_m$ only when $G_{mH} = G_{wH}$. When $G_{mH} = G_{wH}$, given men’s stable marriage premium $\pi_m$, women’s stable marriage-payoff difference $v_{wH} - v_{wL} = s_{HH} - s_{LL} - \pi_m$, so

$$\theta_{w1}(\pi_m) = \frac{c_w}{z_{wH} - z_{wL} + s_{HH} - s_{LL} - \pi_m}.$$

Therefore, $\theta_{w1}(\pi_m)$ is strictly increasing when $\pi_m \in (\delta_{\tau_2}, \delta_H)$. $QED$

A.3 Proof of Proposition 1

I first prove the college gender gap. Suppose by way of contradiction that weakly fewer women than men go to college in equilibrium: $1 - F_w(\theta_w^*) \leq 1 - F_m(\theta_m^*)$. First, since $F_m = F_w$ by assump-
tion, $F_w(\theta^*_w) \geq F_m(\theta^*_m)$ implies $\theta^*_w = c_m/(z_{wH} - z_{wL} + v^*_w - v^*_m) \geq \theta^*_m = c_m/(z_{mH} - z_{mL} + v^*_m - v^*_L)$. Since $z_{wH} - z_{wL} = z_{mH} - z_{mL}$ by assumption, $v^*_w - v^*_m \leq v^*_m - v^*_L$.

Second, $\theta^*_w > \theta^*_w$, so strictly fewer women than men make a career investment in equilibrium. Since weakly fewer women go to college by our premise and strictly fewer women make a career investment, strictly fewer women than men earn a high income, i.e., $G^*_w + G^*_wh < G^*_mH$. As a result, there is a positive mass of type-$L$ women marrying high-income men. By pairwise efficiency, $v^*_L = s_{HL} - v^*_mH$. Since there is always a positive mass of $(H,H)$ couples, by pairwise efficiency, $v^*_L = s_{HH} - v^*_mH$. The two pairwise efficiency conditions together imply $v^*_w - v^*_L = s_{HH} - s_{HL}$. By $s_{HL} = s_{H}L$, $v^*_w - v^*_L = s_{HH} - s_{HL} - s_{H}L = \delta_H$. Because a positive mass of type-$H$ men marries type-$L$ women in equilibrium, $v^*_mH = s_{HL} - v^*_wL$. Furthermore, by Pareto efficiency, $v^*_m \geq s_{LL} - v^*_wL$. The two conditions together imply $v^*_w - v^*_m \leq s_{HL} - s_{LL}$. Since the surplus is strictly super-modular in incomes, $v^*_w - v^*_m = \delta_H > \delta_L = v^*_mH - v^*_mL$.

The two conclusions, $v^*_w - v^*_wL \leq v^*_mH - v^*_mL$ and $v^*_w - v^*_wL > v^*_mH - v^*_mL$, contradict each other. Therefore, there must be strictly more women than men going to college.

I now prove the earnings gender gap. Consider the assumption $G_{mH}(\delta_l) > G_{wH}(\delta_l) + G_{wh}(\delta_l)$. It states that when men’s stable marriage premium $\pi_m$ is $\delta_l$ the lowest value possible, mass $G_{mH}(\delta_l)$ of high-income men is strictly greater than the mass $G_{wH}(\delta_l) + G_{wh}(\delta_l)$ of high-income women. That is, even when men have the smallest possible marriage premium $\pi_m = \delta_l = s_{H}L - s_{H}H$ and women have the largest possible marriage premium $\pi_w = s_{HH} - s_{HL}$, fewer women will end up with a high income than men. Therefore, the earnings gender gap always holds.

Without the assumption, I can show that there are weakly fewer fertile high-income women than high-income men in equilibrium. Suppose by way of contradiction that there are strictly fewer high-income men than fertile high-income women in equilibrium: $G^*_mH < G^*_wH$. As a result, low-income men marry type $H$ women with a positive probability: $v^*_mL = s_{HL} - v^*_wL$. In addition, almost all high-income men marry type $H$ women, so $v^*_mH = s_{HH} - v^*_wL$. The two conditions together imply $v^*_mH - v^*_mL = s_{HH} - s_{HL} = \delta_H$. Since low-income men marry both high-income men and low-income men with positive probabilities, $v^*_wH - v^*_wL = s_{HL} - s_{LL} = \delta_L$, where the second equality follows $s_{HL} = s_{LL}$. Since $v^*_mH - v^*_mL > v^*_wH - v^*_wL$, $\theta^*_m > \theta^*_w1 > \theta^*_w2$. Since more men make college investments as well as career investments, there cannot be strictly fewer high-income men than high-income fertile women:

$$G^*_mH = \int_{\theta^*_m}^{1} pdF_m(p) + \int_{\theta^*_m}^{1} p(1 - p)dF_m(p) > \int_{\theta^*_w1}^{1} pdF_w(p) + \int_{\theta^*_w2}^{1} p(1 - p)dF_w(p) = G^*_wH,$$

contradicting the premise.

\[QED\]
Online Appendix

B Justifications for Assumptions

B.1 Entering the Marriage Market After Investing

I claim in footnote 8 that the strategy of entering the marriage market while investing is weakly dominated by the strategy of entering the marriage market after investing. I extend the basic model to allow this strategy, and show that the strategy is weakly dominated in the extended model. Since lifetime income can only be high or low in the model, a man’s marriage type can be simply represented by the probability of obtaining a high lifetime income when he enters the marriage market. The marriage surplus of a man who obtains a high income with probability \( \theta_m \) and a fertile woman who obtains a high income with probability \( \theta_w \) is

\[
\tilde{s}(\theta_m, \theta_w, +) = \theta_m \theta_w s_{HH} + \theta_m (1 - \theta_w) s_{HL} + (1 - \theta_m) \theta_w s_{LH} + (1 - \theta_m)(1 - \theta_w) s_{LL}.
\]

The marriage surplus of a type-\( \theta_m \) man and a less fertile type-\( \theta_w \) woman is

\[
\tilde{s}(\theta_m, \theta_w, -) = \theta_m \theta_w s_{HH} + \theta_m (1 - \theta_w) s_{HL} + (1 - \theta_m) \theta_w s_{LH} + (1 - \theta_m)(1 - \theta_w) s_{LL}.
\]

The marriage market in the extended model is organized in the same way as in the basic model. Namely, let \( \tilde{T}_m = [0, 1] \) and \( \tilde{T}_w = [0, 1] \times \Phi \) represent the expanded sets of marriage types, \( \tilde{G}_m \) and \( \tilde{G}_w \) distributions of marriage types, \( \tilde{G} \) a matching, and \( \tilde{\nu}_m \) and \( \tilde{\nu}_w \) marriage payoffs. The outcome \((\tilde{G}, \tilde{\nu}_m, \tilde{\nu}_w)\) is stable in the marriage market \((\tilde{G}_m, \tilde{G}_w)\) if (1) (individual rationality) \( \tilde{\nu}_m \tau_m \geq 0 \) for all \( \tau_m \in \tilde{T}_m \) and \( \tilde{\nu}_w \tau_w \geq 0 \) for all \( \tau_w \in \tilde{T}_w \), (2) (pairwise efficiency) \( \tilde{\nu}_m \tau_m + \tilde{\nu}_w \tau_w = \tilde{s}_m \tau_m \tau_w \) when \( \tilde{G}(\tau_m, \tau_w) > 0 \), and (3) (Pareto efficiency) \( \tilde{\nu}_m \tau_m + \tilde{\nu}_w \tau_w \geq \tilde{s}_m \tau_m \tau_w \) for any pair of \( \tau_m \in \tilde{T}_m \) and \( \tau_w \in \tilde{T}_w \).

I now show that the strategy of simultaneously investing and marrying is weakly dominated by investing and then marrying after income is realized for a college man. Let \( \tau_w \) denote the type of woman an ability-\( \theta \) man who has made the college investment and will not make the corner investment marries in the stable matching. His stable marriage payoff is

\[
\tilde{\nu}_m \theta = \tilde{s}_0 \tau_w - \tilde{\nu}_w \tau_w = p\tilde{s}_1 \tau_w + (1 - \theta)\tilde{s}_0 \tau_w - \tilde{\nu}_w \tau_w.
\]

By Pareto efficiency, the marriage payoff of each man weakly exceeds what he would get if he marries a type-\( \tau_w \) woman: for type-1 (i.e., high-income) and probability-0 (i.e., low-income) men, \( \tilde{\nu}_m \tau_m \geq \tilde{s}_1 \tau_w - \tilde{\nu}_w \tau_w \) and \( \tilde{\nu}_m \tau_0 \geq \tilde{s}_0 \tau_w - \tilde{\nu}_w \tau_w \). If the same ability-\( \theta \) man makes a college investment and marries after income is realized, his expected marriage payoff is \( p\tilde{\nu}_m + (1 - \theta)\tilde{\nu}_0 \), which, by the two inequalities above, is greater than \( p\tilde{s}_1 \tau_w + (1 - \theta)\tilde{s}_0 \tau_w - \tilde{\nu}_w \tau_w \), which is the expected payoff the man gets from simultaneously investing and marrying. The same argument applies to any man or

\[19\text{Because income is uncertain when agents enter the marriage market while investing, their marriage characteristics are represented by distributions of incomes rather than realized incomes (Borch, 1962; Wilson, 1968; Chiappori and Reny, 2016; Zhang, 2017).} \]
any woman who chooses an investment strategy that results in a high income with probability \( q \), for any \( \theta \).

Empirically, most people chose not to marry when they were making human capital investments. First, 86 to 96 percents of college-educated men and 80 to 90 percents of college-educated women did not marry between ages 18 and 21, their college years (table B1), besides the two outliers, 1930s and 1940s birth cohorts, many of whom were rushed into marriage to avoid being drafted to the Vietnam War. Second, 78 to 92 percents of men with advanced degrees and 79 to 92 percents of women with advanced degrees did not marry between ages 22 and 23 (table B2).

| Table B1: Proportion of college degrees marrying between ages 18 and 21 |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
|                 | 1900s | 1910s | 1920s | 1930s | 1940s | 1950s | 1960s | 1970s |
| Men             | 0.06  | 0.06  | 0.14  | 0.18  | 0.19  | 0.12  | 0.06  | 0.04  |
| Women           | 0.10  | 0.12  | 0.23  | 0.29  | 0.29  | 0.20  | 0.12  | 0.10  |

| Table B2: Proportion of advanced degrees marrying between ages 22 and 23 |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
|                 | 1900s | 1910s | 1920s | 1930s | 1940s | 1950s | 1960s | 1970s |
| Men             | 0.09  | 0.13  | 0.20  | 0.21  | 0.22  | 0.15  | 0.09  | 0.08  |
| Women           | 0.08  | 0.13  | 0.20  | 0.21  | 0.20  | 0.15  | 0.12  | 0.11  |

B.2 More Career Investments By Low-Income Individuals

I define a career investment as obtaining additional education beyond college, obtaining additional training, or switching to a new job. I use NLSY79 and NLSY97 to verify the assumption that those who earned a lower income were more likely to make a career investment between ages 23 and 29.

I run the simple OLS regression, probit regression, as well as logistic regression to test the following relation,

\[ \text{investment}_{i, \text{age}} = \alpha_{i} + \beta \text{logincome}_{i, \text{age} - 1} + \epsilon_{i, \text{age}}. \]

| Table B3: Relation between career and logincome, men |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
|                 | (1) | (2) | (3) | (4) | (5) | (6) |
| ols79           |  -0.0969***    |  -0.447***     |  -0.261***     |  -0.0947***    |  -0.406***     |  -0.250***     |
| logit79         | (0.0142) | (0.0647) | (0.0370) | (0.0161) | (0.0723) | (0.0439) |
| probit79        |  -0.000539     |  0.00561       |  -0.000519     |  -0.0244***    |  -0.108***     |  -0.0664***    |
| ols97           | (0.00741) | (0.0308) | (0.0188) | (0.00719) | (0.0324) | (0.0199) |
| logit97         |  -0.000539     |  0.00561       |  -0.000519     |  -0.0244***    |  -0.108***     |  -0.0664***    |
| probit97        | (0.00741) | (0.0308) | (0.0188) | (0.00719) | (0.0324) | (0.0199) |

| \( N \) | 1659 | 1659 | 1659 | 1638 | 1638 | 1638 |

Marginal effects; Standard errors in parentheses
(d) for discrete change of dummy variable from 0 to 1
* \( p < 0.05 \), ** \( p < 0.01 \), *** \( p < 0.001 \)
College-educated men with a lower income in the previous year were more likely to make a career investment between ages 23 and 29. Table B3 below shows that college-educated men with a percent lower income in the previous year were nine percentage points more likely to make a career investment for both the NLSY79 and NLSY97 cohorts.

### Table B4: Relation between career and income, women

<table>
<thead>
<tr>
<th></th>
<th>ols79</th>
<th>logit79</th>
<th>probit79</th>
<th>ols97</th>
<th>logit97</th>
<th>probit97</th>
</tr>
</thead>
<tbody>
<tr>
<td>logincome</td>
<td>-0.0231</td>
<td>-0.0991</td>
<td>-0.0605</td>
<td>-0.0538***</td>
<td>-0.230***</td>
<td>-0.141***</td>
</tr>
<tr>
<td></td>
<td>(0.0133)</td>
<td>(0.0540)</td>
<td>(0.0329)</td>
<td>(0.0124)</td>
<td>(0.0523)</td>
<td>(0.0317)</td>
</tr>
<tr>
<td>age</td>
<td>-0.0320***</td>
<td>-0.131***</td>
<td>-0.0822***</td>
<td>-0.0503***</td>
<td>-0.213***</td>
<td>-0.138***</td>
</tr>
<tr>
<td></td>
<td>(0.00602)</td>
<td>(0.0249)</td>
<td>(0.0154)</td>
<td>(0.00563)</td>
<td>(0.0249)</td>
<td>(0.0153)</td>
</tr>
<tr>
<td>N</td>
<td>2155</td>
<td>2155</td>
<td>2155</td>
<td>2284</td>
<td>2284</td>
<td>2284</td>
</tr>
</tbody>
</table>

Marginal effects; Standard errors in parentheses
(d) for discrete change of dummy variable from 0 to 1

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

College-educated women with a lower income in the previous year were also more likely to make a career investment between ages 23 and 29 (and to a lesser extent than men). Table B4 below shows that a one percent lower income in the previous year was associated with two percentage points higher chance of making a career investment for the NLSY79 cohort and five percentage points higher chance for the NLSY97 cohort.

### B.3 Surplus Supermodularity and Transferable Utility

The following intra-household public-good consumption problem justifies the surplus monotonicity assumptions, the surplus supermodularity assumptions, and the assumption of transferable utilities in the marriage market. Low (2017) presents a similar microfoundation. Bergstrom and Cornes (1983); Chiappori and Gugl (2015); Chiappori et al. (2017) provide more general discussions of the microfoundation.

An unmarried income-$y_m$ man who derives utility $q_m$ from consuming $q_m$ units of a composite private good derives a utility of $z(y_m) = y_m$, and an unmarried income-$y_w$ woman who derives utility $q_w$ from consuming $q_w$ units of the same composite private good derives a utility of $z(y_w) = y_w$. A couple with children spends their income $y_m + y_w$ on $q_m$ and $q_w$ units of the private good as well as on $Q$ units of a public good to derive a utility of $q_m(1 + Q)$ for the husband and a utility of $q_w(1 + Q)$ for the wife. To maximize joint utility $(q_m + q_w)(1 + Q)$ subject to the budget constraint $q_m + q_w + Q \leq y_m + y_w$, the couple consumes $q_m + q_w = (y_m + y_w + 1)/2$ units of the private good and $Q = (y_m + y_w - 1)/2$ units of the public good for a joint utility $(y_m + y_w + 1)^2/4 = (y_m + y_w - 1)^2/4 + y_m + y_w$. A couple without children spends income on the composite private good only to derive a joint utility of $y_m + y_w$. With probability $\phi_w$ a woman with fitness level $\phi_w$.
can have children. Therefore, the marriage surplus an income-$y_m$ man and an income-$y_w$ woman with fitness level $\phi_w$ generate is

$$s(y_m, y_w, \phi_w) = z(y_m, y_w, \phi_w) - z(y_m) - z(y_w)$$

$$= \phi_w[y_m + y_w - 1] + (1 - \phi_w)(y_m + y_w - y_m - y_w)$$

$$= \phi_w(y_m + y_w - 1)^2/4.$$

The surplus is strictly increasing in $y_m$, $y_w$, and $\phi_w$, when $y_m + y_w - 1 > 0$. The surplus is strictly supermodular in $y_m$ and $y_w$ as well as in $y_m$ and $\phi_w$. Moreover, any division of the marriage surplus between a couple can be achieved through the allocation of the private good. When there are children, $q_m$ units of the private good are allocated to the husband and $q_w$ units of the private good are allocated to the wife, where $q_m + q_w = (y_m + y_w + 1)/2$.

References in Appendix B


Online Appendix

C Supporting the Explanations: Key Empirical Evidence

C.1 College and Earnings Gender Gaps

The model predicts that women’s marriage premium is higher than men’s when more women than men go to college, but is lower than men’s when fewer women than men go to college.

Claim 1. Suppose the marriage surpluses $s_{HL}$ and $s_{LH}$ are gender-symmetric and more men than women earn a high income in equilibrium. Women’s marriage premium $\pi^w = s_{HH} - s_{HL}$ is larger than men’s marriage premium $\pi^m = s_{HL} - s_{LL}$ in equilibrium.

Previous literature has presented evidence consistent with this prediction (Chiappori et al., 2009, 2017). However, in the previous papers, each individual’s marriage type is education or age rather than income or fertility. Although income is positively correlated with education and fertility is negatively correlated with age, these papers do not provide direct evidence for our predictions. I directly test this key implication with data.

Figure C1 shows estimated marriage premiums from 1960 to 2015. The estimation is consistent with our prediction: women’s marriage premium (i) was smaller than men’s in 1960, 1970, and 1980, when fewer women than men, ages 35-39, graduated from college; and (ii) was greater than men’s in 2010 and 2015, when more women than men, ages 35-39, graduated from college. We adopt the technique developed by Choo and Siow (2006) to exactly identify the marriage surplus function, and compute the marriage premiums from the estimated marriage surplus function. We detail the estimation procedure below.

We only need to estimate $s_{HH}$, $s_{HL}$, and $s_{LL}$ to compute the marriage premiums, because, according to claim 1, men’s marriage premium is $\pi^m = s_{HL} - s_{LL}$, and women’s marriage premium is $\pi^w = s_{HH} - s_{HL}$. We modify our matching model to adopt the technique of Choo and Siow (2006). The marriage payoff of a type-$\tau_m$ man $i$ married to a type-$\tau_w$ woman is

$$v^i_{m\tau_m \tau_w} = z^m_{\tau_m \tau_w} - t_{\tau_m \tau_w} + \epsilon^i_{\tau_m \tau_w},$$

where $z^m_{\tau_m \tau_w}$ is the systematic gross return to a type-$\tau_m$ man married to a type-$\tau_w$ woman, $t_{\tau_m \tau_w}$ is the transfer from a type-$\tau_m$ man to a type-$\tau_w$ woman, and $\epsilon^i_{\tau_m \tau_w}$ is an independently and identically distributed random variable with a type I extreme-value distribution, i.e., $F(\epsilon) = \exp[-\exp(-\epsilon)]$. The marriage payoff of a type-$\tau_w$ woman $j$ married to a type-$\tau_m$ man is

$$v^j_{w\tau_w \tau_m} = z^w_{\tau_w \tau_m} + t_{\tau_m \tau_w} + \epsilon^j_{\tau_m \tau_w},$$

$^{20}$We are not able to produce estimates for 1990 and 2000, because age at marriage, the information needed to construct the marriage market, was neither reported nor inferable in the censuses in these years. We are also not able to produce estimates for years prior to 1960, because spousal income – the information needed to construct marriage types and to compute the number of marriages between different marriage types – was not reported in the censuses.
where $z_{\tau_m \tau_w}$ is the systematic return to a type-\(\tau_w\) woman married to a type-\(\tau_m\) man, and $\varepsilon_{\tau_m \tau_w}^i$ is an i.i.d. random variable with a T1EV distribution. The payoff to man $i$ who remains unmarried is

$$\gamma_{\tau_m}^i = z_{\tau_m \tau_w}^m + \varepsilon_{\tau_m}^i,$$

where $\varepsilon_{\tau_m}^i$ is also an i.i.d. random variable with a T1EV distribution. The systematic marriage surplus for a type-\(\tau_m\) man married to a type-\(\tau_w\) woman is $s_{\tau_m \tau_w}^m = z_{\tau_m \tau_w}^m - z_{\tau_m \tau_w}^m$. Similarly, the systematic marriage surplus for a type-\(\tau_w\) woman married to a type-\(\tau_m\) man is $s_{\tau_m \tau_w}^w = z_{\tau_m \tau_w}^w - z_{\tau_m \tau_w}^w$.

Therefore, the total systematic marriage surplus of a type-\(\tau_m\) man and a type-\(\tau_w\) woman is $s_{\tau_m \tau_w} = s_{\tau_m \tau_w}^m + s_{\tau_m \tau_w}^w$. Following Choo and Siow (2006),

$$\hat{s}_{\tau_m \tau_w} = 2\ln \left[ \frac{\widehat{G}_{\tau_m \tau_w}}{\sqrt{\widehat{G}_{\tau_m \theta} \widehat{G}_{\theta \tau_w}}} \right],$$

where $\widehat{G}_{\tau_m \tau_w}$ is the estimated measure of marriages between type-\(\tau_m\) men and type-\(\tau_w\) women, $\widehat{G}_{\tau_m \theta}$ is the estimated measure of unmarried type-\(\tau_m\) men, and $\widehat{G}_{\theta \tau_w}$ is the estimated measure of unmarried type-\(\tau_w\) women. By claim 1, point estimates of the marriage premiums are $\widehat{\pi}_{mH}^* = \hat{s}_{HL} - \hat{s}_{LL}$ and $\widehat{\pi}_{wH}^* = \hat{s}_{HH} - \hat{s}_{HL}$. Standard errors of the marriage premiums are obtained from simulated measures of marriage characteristics.

What remains is to specify the marriage market and to assign each individual fertility and income marriage characteristics. We include in the marriage market all never-married individuals between ages 16-39 who were not in school, and all heterosexual couples who were both between ages 16-39 and who had both married for the first time within the two years under consideration. We categorize an agent as a high-income type if he or she earns more than the median personal labor income of the college graduates of the same age, and as a low-income agent otherwise. We treat men between ages 16-39 and women between ages 16-29 as fertile, and we treat women between ages 30-39 as less fertile.

The model also shows the factors that contribute to the rise of women’s college enrollment and earnings over time.

**Claim 2.** Suppose more men than women earn a high income in equilibrium before and after the changes in the primitives of the model. Women’s college investment rate and average income in equilibrium both increase if any or any combination of the following events happens: (i) women’s investment cost $c_w$ decreases; (ii) women’s labor-market opportunities $F_w$ (first-order stochastically) increase; (iii) women’s income premium $z_{wH} - z_{wL}$ increases; and (iv) the surplus difference $s_{HH} - s_{HL}$ (women’s equilibrium marriage premium $\pi_{wH}^*$) increases.

Existing literature (cited in the introduction) has thoroughly studied how monetary and psychic college investment costs, labor-market opportunities, and income premium for women (as well as for men) have evolved over the past decades, and how these changes have contributed to...
the changes in the college and earnings gender gaps. Predictions (i)-(iii) are consistent with previous findings and do not add any new theoretical insights. The change in the marriage premium is relatively less studied. In equilibrium, women’s marriage premium equals the difference between (i) the marriage surplus when a high-income man marries a high-income woman and (ii) the marriage surplus when he marries a low-income woman. Its increase is associated with technological and social changes that affect intrahousehold consumption and time-allocation decisions. Technological progress freed women from some household activities and made women with high earning abilities more valuable in the labor market as well as in the marriage market (Greenwood et al., 2014, 2016). In addition, an increasing focus on human capital of children also made highly educated and highly skilled women more valuable in the marriage market (Chiappori et al., 2009, 2017). We will empirically confirm that the marriage premium for women has indeed increased and has gradually surpassed the marriage premium for men over the last several decades.

C.2 Relationship between Age at Marriage and Income for Men

An alternative explanation for the relationship between age at marriage and income for men is the marriage market frictions. It is possible that the marriage timing differs by income and education, and the marriage frictions alone could explain the observed relationship. I present a piece of evidence that can only be explained by the labor-market shocks, however. If labor-market shocks indeed affect marriage timing as the model suggests, then we would expect that, compared to college-educated middle grooms, college-educated late grooms should have (i) a lower average income when they have just finished college and (ii) a higher income growth rate following college. Figure C2 shows exactly such patterns for the three cohorts tracked by National Longitudinal Surveys of Youth (NLSY). Four-year-college-educated late grooms, the men who received exactly four years of postsecondary education and first married between ages 30 and 39 (i) earned a lower average income than four-year-college-educated middle grooms in their twenties, and (ii) caught up and earned almost as much as four-year-college-educated middle grooms in their thirties.

C.3 Relationship between Age at Marriage and Income for Women

Marriage frictions can potentially fully explain the observed relationship between age at marriage and income for women. I present a piece of evidence that cannot be explained by marriage market frictions but can be explained by labor market shocks, however. The model predicts that, because they receive an adverse labor-market shock and consequently make a career investment, compared to the college-educated women who marry before age thirty, the college-educated women who marry after age thirty have a lower post-college income initially and a steeper income gain afterwards, on average. Data matches this prediction: four-year-college-educated late brides had a lower average income right out of college but quickly caught up with and later on surpassed four-year-college-educated middle brides (figure C3).
Figure C1: Marriage premiums from 1960 to 2015

Marriage premium (utils)

Census year

Men's marriage premium $\pi_m = s_{HL} - s_{LL}$
Women's marriage premium $\pi_w = s_{HH} - s_{HL}$

Figure C2: Income ratio of college late grooms to college middle grooms, NLSY

Income ratio of 4-year-college late grooms to 4-year-college middle grooms

Age

1941-54 birth cohort 1957-64 birth cohort 1980-84 birth cohort

Figure C3: Income ratio of college late brides to college middle brides, NLSY

Income ratio of 4-year-college late brides to 4-year-college middle brides

Age

1941-54 birth cohort 1957-64 birth cohort 1980-84 birth cohort
Figure C4: Income and education in mandated versus nonmandated states, 1930-1979

(a) Spousal total income percentile rank by birth year

(b) Spousal education percentile rank by birth year

(c) Personal total income percentile rank by birth year

(d) Personal education percentile rank by birth year
Figure C5: Robustness checks

(a) Spousal total income z-score by birth year

(b) Spousal total income by birth year

(c) Spousal total income z-score difference by birth year

(d) Spousal total income difference by birth year

C6
C.4 Relationship between Age at Marriage and Spousal Income for Women

Figures C4a and C4b show that early brides’ marital outcome deteriorated and late brides’ marital outcome improved in mandated states, relative to nonmandated states, where the marital outcome is measured by spousal income rank and spousal education rank. Meanwhile, the marital outcome of the middle brides did not evolve differently for the two sets of states.

The same pattern emerges even when we use alternative measures of the marital outcome, such as spousal total income statewide z-score (Figures C5a) or the raw spousal total income (Figure C5b). The difference in the marital outcome in the two sets of states was not statistically significant, but became statistically significant after the mandates were passed. Figures C5c and C5d show that the marital outcome, measured by spousal total income z-score and spousal income, was not significantly different for early brides in the mandated and nonmandated states before the mandates were passed, but became statistically significantly worse for early brides in mandated states. Quantitatively, the average spousal income of early brides born in mandated states in 1980 were about 0.15 standard deviation worse than that of early brides born in nonmandated states. In contrast, the marital outcome was not significantly different for late brides in the mandated and nonmandated states before the mandates were passed, but became significantly better for late brides in mandated states. The average spousal income of late brides born in mandated states in 1980 was about 0.15 standard deviation better than that of late brides in nonmandated in 1980. Meanwhile, the average marital outcome of middle brides did not differ by much.

In addition, it is worth mentioning that the mandates did not significantly improve the average education and total income rank of late brides in those states (figures C4c and C4d), consistent with previous studies (Buckles, 2007). The result is also consistent with our theory: more women may make a career investment following the relaxation of the fertility constraint, but since these women have intermediate abilities and may have a relatively low chance of receiving a high income, late brides’ average income may not increase. In contrast, the improvements in Israeli women’s education and earnings were more pronounced (Gershoni and Low, 2017), partially because Israelis go to college later due to mandatory military services.

References in Appendix C


Online Appendix

D  Supporting the Explanations: Calibration

D.1  Benchmark Model

Suppose the ability distributions for men and for women are beta distributions with parameters $(\alpha_m, \beta_w)$ and $(\alpha_w, \beta_w)$, respectively. Since the model predicts that those who marry in the first period are low-income earners, we use the average labor income of men and women who first married between ages 16 and 22 to estimate $y_{mL}$ and $y_{wL}$, respectively. Since the model predicts that men who marry in the second period are high-income earners, we use the average labor income of men who married between ages 23 and 29 to estimate $y_{mH}$. We use the average labor income of the unmarried women to estimate $y_{wH}$. Total investment costs $c_m$ and $c_w$ are two years of low incomes. We use two years because the college-educated on average marry two years later than the non-college-educated (Coles and Francesconi (2017) use four years of minimum wages). Annual investment cost is total investment cost divided by 40. The marriage surplus in monetary terms is $k$ times the marriage surplus in utils estimated in section 3.3.

Twelve moments are targeted: observed percentage and average midlife labor income of early, middle, and late grooms (denoted by $\hat{G}_{ma}$ and $\hat{y}_{ma}$, $a \in \{1,2,3\}$), and of early, middle, and late brides ($\hat{G}_{wa}$ and $\hat{y}_{wa}$, $a \in \{1,2,3\}$). Define the penalty function with five arguments, $\alpha_m$, $\alpha_w$, $\beta_m$, $\beta_w$, and $k$ and find the parameters to minimize it:

$$D_1(\alpha_m, \alpha_w, \beta_m, \beta_w, k) = \sqrt{\sum_{i \in \{m,w\}} \left[ \sum_{a=1}^3 \frac{G_{ia} - \hat{G}_{ia}}{\hat{G}_{ia}} \right]^2 + \sum_{a=1}^3 \left| \frac{y_{ia} - \hat{y}_{ia}}{\hat{y}_{ia}} \right|^2}.$$

We find the parameters for the 1930s and the 1960s birth cohorts, respectively. Table D1 shows the fit of the model. The calibration matches each targeted marriage-age distribution within 0.7 percent, and except for the average income of late grooms born in the 1930s and that of late brides in the 1960s, the calibration matches each of the targeted average incomes within 5 percent. The non-targeted average spousal incomes are also matched fairly well. Table D2 shows the estimated parameters of the model. Labor-market opportunities are estimated to be much greater for women born in the 1960s and slightly lower for men born in the 1960s (figure D1), consistent with the results in Coles and Francesconi (2018).

D.2  Extended Model

Not everyone who decides to marry can get married right away, and not everyone who goes to college waits to marry after college. To separate college attendance and marriage timing, I extend the model to allow college men and women to marry in the first period and noncollege men and
Table D1: Fit of the benchmark model

<table>
<thead>
<tr>
<th>moments</th>
<th>30s target</th>
<th>30s model</th>
<th>difference</th>
<th>60s target</th>
<th>60s model</th>
<th>difference</th>
</tr>
</thead>
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<tr>
<td>$G_{m1}$</td>
<td>0.48476</td>
<td>0.48927</td>
<td>0.0346%</td>
<td>0.30756</td>
<td>0.307632</td>
<td>0.0233%</td>
</tr>
<tr>
<td>$G_{m2}$</td>
<td>0.411344</td>
<td>0.411096</td>
<td>-0.005%</td>
<td>0.451633</td>
<td>0.451472</td>
<td>-0.008%</td>
</tr>
<tr>
<td>$G_{m3}$</td>
<td>0.103896</td>
<td>0.103977</td>
<td>0.0782%</td>
<td>0.240807</td>
<td>0.240896</td>
<td>0.037%</td>
</tr>
<tr>
<td>$G_{w1}$</td>
<td>0.740591</td>
<td>0.740641</td>
<td>0.0063%</td>
<td>0.4494</td>
<td>0.450621</td>
<td>0.272%</td>
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<tr>
<td>$G_{w2}$</td>
<td>0.206928</td>
<td>0.206863</td>
<td>-0.005%</td>
<td>0.381204</td>
<td>0.378867</td>
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</tr>
<tr>
<td>$G_{w3}$</td>
<td>0.0524809</td>
<td>0.0524961</td>
<td>0.0007%</td>
<td>0.169396</td>
<td>0.170512</td>
<td>0.011%</td>
</tr>
</tbody>
</table>

Table D2: Estimated parameters of the benchmark model

<table>
<thead>
<tr>
<th></th>
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<th>1960s</th>
<th>1930s</th>
<th>1960s</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_m$</td>
<td>Beta(1.02,0.837)</td>
<td>Beta(0.462,0.523)</td>
<td>$s_{HH}$</td>
<td>$s_{HL}$</td>
</tr>
<tr>
<td>$F_w$</td>
<td>Beta(0.0535,0.196)</td>
<td>Beta(0.165,0.373)</td>
<td>$s_{HL}$</td>
<td>$s_{HH}$</td>
</tr>
<tr>
<td>$c_m$</td>
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<td>$21.0646$</td>
<td>$2055.66$</td>
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<td>$c_w$</td>
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<td>$1004.55$</td>
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<td>$y_mH$</td>
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<td>$42783.4$</td>
<td>$48376.5$</td>
<td>$49895.6$</td>
</tr>
<tr>
<td>$y_mL$</td>
<td>$40209.7$</td>
<td>$43806.4$</td>
<td>$61434.7$</td>
<td>$54923.9$</td>
</tr>
<tr>
<td>$y_wH$</td>
<td>$14902.3$</td>
<td>$13445.7$</td>
<td>$26080.1$</td>
<td>$28028.3$</td>
</tr>
<tr>
<td>$y_wL$</td>
<td>$12049.7$</td>
<td>$14145.5$</td>
<td>$47016.2$</td>
<td>$48873.3$</td>
</tr>
</tbody>
</table>

Figure D1: Estimated CDF and PDF of abilities in the benchmark model
women to marry in the second and third periods.

The ability distributions are beta distributions as in the benchmark model. We use the average incomes of noncollege men and women as low incomes \( y_{mL} \) and \( y_{wL} \), respectively, and the average incomes of college men and women as high incomes \( y_{mH} \) and \( y_{wH} \), respectively. The total investment cost is the opportunity cost in the form of two years of low incomes. The annual investment cost is the total investment cost divided by 40. The surplus in monetary terms is again \( k \) times the surplus in utils estimated in section 3.3. Now, frictions. First, not all noncollege men and women marry between ages 16 and 22. The actual probabilities that they married after age 22 are taken from the data. Let \( h_{iNa}, i \in \{m, w\}, a \in \{1, 2\} \), denote the hazard rate of a noncollege man or a noncollege woman marrying in period \( a \). Let \( h_{iC2} \) denote the hazard rate of a college man or a college woman marrying between ages 23 and 29. Second, not all college men and college women delay marriage until after college: let \( h_{iC1}, i \in \{m, w\} \), denote the probability that a college man or a college woman marries between ages 16 and 22.

We target seventeen moments: the college enrollment rates of men and women (denoted by \( G_{mC} \) and \( G_{wC} \), respectively), the average incomes of men who married early, middle, and late brides (denoted by \( x_{w1}, x_{w2}, \) and \( x_{w3} \), respectively), as well as the twelve moments targeted in the benchmark model.

To estimate the seven parameters \((\alpha_m, \alpha_w, \beta_m, \beta_w, k, \mu_m, \mu_w)\), we define the penalty function

\[
D_2 = \left( \sum_{i \in \{m, w\}} \left[ \frac{G_{iC} - \hat{G}_{iC}}{\hat{G}_{i, col}} \right]^2 + \sum_{a=1}^{3} \left[ \frac{G_{ia} - \hat{G}_{ia}}{\hat{G}_{ia}} \right]^2 + \sum_{a=1}^{3} \left[ \frac{y_{ia} - \hat{y}_{ia}}{\hat{y}_{ia}} \right]^2 \right)^{3/2} + \sum_{a=1}^{3} \left[ \frac{x_{wa} - \hat{x}_{wa}}{\hat{x}_{wa}} \right]^2.
\]

We find the parameters to minimize the penalty.

We test the performance of the extended model for the 1930s and 1960s birth cohorts, too. Table D3 shows how well the model matches the data. The average error between targeted and calibrated moments is 1.71 percent for the 1930s birth cohort, and is 1.51 percent for the 1960s birth cohort. Table D4 shows the model’s calibrated parameters.

We can examine the relative importance of labor-market shocks to marriage-market frictions in influencing marriage timing.

For the 1930s birth cohort, there was an estimated 17.1 percent chance that college-educated women who decided to enter the marriage market before age 30 involuntarily delayed their marriage until after age 30, slightly higher than the 21.2 percent chance that noncollege women involuntarily delayed their marriage until after age 30. Among the men and women who married between ages 30 and 39, among the college-educated, essentially all men delayed marriages due to labor-market shocks, and all women delayed marriages due to marriage-market frictions (consistent with the fact that a tiny portion of women in this cohort chose to make a career investment).

For the 1960s birth cohort, the chance for a college-educated man who decided to enter the
Table D3: Fit of the extended model

<table>
<thead>
<tr>
<th>moments</th>
<th>30s target</th>
<th>30s model</th>
<th>difference</th>
<th>60s target</th>
<th>60s model</th>
<th>difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_{m1}$</td>
<td>0.48476</td>
<td>0.484451</td>
<td>-0.0637%</td>
<td>0.30756</td>
<td>0.307372</td>
<td>-0.0613%</td>
</tr>
<tr>
<td>$G_{m2}$</td>
<td>0.411344</td>
<td>0.412559</td>
<td>0.295%</td>
<td>0.451633</td>
<td>0.452309</td>
<td>0.15%</td>
</tr>
<tr>
<td>$G_{m3}$</td>
<td>0.103896</td>
<td>0.102989</td>
<td>-0.872%</td>
<td>0.240807</td>
<td>0.24032</td>
<td>-0.202%</td>
</tr>
<tr>
<td>$G_{w1}$</td>
<td>0.740591</td>
<td>0.740591</td>
<td>0.000051%</td>
<td>0.4494</td>
<td>0.449534</td>
<td>0.0299%</td>
</tr>
<tr>
<td>$G_{w2}$</td>
<td>0.206928</td>
<td>0.206847</td>
<td>-0.0393%</td>
<td>0.381204</td>
<td>0.380081</td>
<td>-0.295%</td>
</tr>
<tr>
<td>$G_{w3}$</td>
<td>0.0524809</td>
<td>0.0525618</td>
<td>0.154%</td>
<td>0.169396</td>
<td>0.170385</td>
<td>0.584%</td>
</tr>
<tr>
<td>$G_{w,\text{col}}$</td>
<td>0.218733</td>
<td>0.220363</td>
<td>0.745%</td>
<td>0.379722</td>
<td>0.380819</td>
<td>0.289%</td>
</tr>
<tr>
<td>$y_{m1}$</td>
<td>0.119257</td>
<td>0.119255</td>
<td>-0.00131%</td>
<td>0.390058</td>
<td>0.389479</td>
<td>-0.148%</td>
</tr>
<tr>
<td>$y_{m2}$</td>
<td>40209.7</td>
<td>39603.7</td>
<td>-1.51%</td>
<td>44571.6</td>
<td>44730.5</td>
<td>0.357%</td>
</tr>
<tr>
<td>$y_{m3}$</td>
<td>43820.8</td>
<td>43915.8</td>
<td>0.217%</td>
<td>56434.2</td>
<td>56524.6</td>
<td>0.16%</td>
</tr>
<tr>
<td>$y_{w1}$</td>
<td>37442.</td>
<td>38350.9</td>
<td>2.43%</td>
<td>48376.5</td>
<td>48589.3</td>
<td>0.44%</td>
</tr>
<tr>
<td>$y_{w2}$</td>
<td>12409.</td>
<td>11696.3</td>
<td>-2.93%</td>
<td>20091.</td>
<td>20510.</td>
<td>2.09%</td>
</tr>
<tr>
<td>$y_{w3}$</td>
<td>12457.2</td>
<td>12739.2</td>
<td>2.26%</td>
<td>24627.8</td>
<td>25169.9</td>
<td>2.2%</td>
</tr>
<tr>
<td>$x_{m1}$</td>
<td>12886.1</td>
<td>12421.</td>
<td>-3.61%</td>
<td>26080.1</td>
<td>24207.1</td>
<td>-7.18%</td>
</tr>
<tr>
<td>$x_{w1}$</td>
<td>41269.2</td>
<td>41155.8</td>
<td>-0.275%</td>
<td>46138.3</td>
<td>47051.6</td>
<td>1.98%</td>
</tr>
<tr>
<td>$x_{w2}$</td>
<td>45269.5</td>
<td>42290.6</td>
<td>-6.58%</td>
<td>58701.2</td>
<td>55594.8</td>
<td>-5.29%</td>
</tr>
<tr>
<td>$x_{w3}$</td>
<td>35537.5</td>
<td>38066.9</td>
<td>7.12%</td>
<td>48666.8</td>
<td>50699.8</td>
<td>4.18%</td>
</tr>
<tr>
<td>average</td>
<td>$\rightarrow$</td>
<td>$\rightarrow$</td>
<td>1.71%</td>
<td>$\rightarrow$</td>
<td>$\rightarrow$</td>
<td>1.51%</td>
</tr>
</tbody>
</table>

Table D4: Estimated parameters of the extended model

| $F_m$ | Beta(0.0165,0.0757) | Beta(0.0244,0.0501) | $y_{mH}$ | Beta(0.0507,0.138) | $y_{mL}$ | $y_{wH}$ | Beta(0.0757,0.138) | $y_{wL}$ | $h_{mC1}$ | Beta(0.009999,0.7783) | $h_{mC2}$ | $h_{mN1}$ | Beta(0.052621,0.372798) | $h_{mN2}$ | $h_{wC1}$ | Beta(0.052741,0.307963) | $h_{wC2}$ | $h_{wN1}$ | Beta(0.0769455,0.539849) | $h_{wN2}$ | Beta(0.0788497,0.675267) |
|-------|-------------------|-------------------|----------|-------------------|----------|----------|-------------------|----------|--------------|-------------------|----------|--------------|-------------------|----------|--------------|-------------------|----------|--------------|-------------------|----------|
| $F_w$ | Beta(0.0236,0.519) | Beta(0.0244,0.0501) | $y_{mH}$ | Beta(0.0507,0.138) | $y_{mL}$ | $y_{wH}$ | Beta(0.0757,0.138) | $y_{wL}$ | $h_{mC1}$ | Beta(0.036696,0.200993) | $h_{mC2}$ | $h_{mN1}$ | Beta(0.052621,0.372798) | $h_{mN2}$ | $h_{wC1}$ | Beta(0.052741,0.307963) | $h_{wC2}$ | $h_{wN1}$ | Beta(0.0769455,0.539849) | $h_{wN2}$ | Beta(0.0788497,0.675267) |

Figure D2: Estimated CDF and PDF of abilities in the extended model
marriage market between ages 23 and 29 not being able to marry before age 30 was 22.2 percent, and the chance that a college-educated woman who decided to enter the marriage market between ages 23 and 29 not being able to marry before age 30 was 23.8 percent. We find that 42.7 percent of college-educated men and 24.6 percent of college-educated women delayed their marriages due to labor-market shocks (and the rest delayed due to marriage-market frictions).

We can also calculate men’s and women’s marriage premiums. Comparing these two birth cohorts, men’s marriage premium decreased from $26410.4 to $6911.67, and women’s marriage premium increased from $10331.8 to $14177.6. Furthermore, we find that the marriage premium is 43.6 percent of the total premium for men and 40.9 percent of the total premium for women born in the 1930s, and is 12.6 percent for men and 29.4 percent for women born in the 1960s. In comparison, Bruze (2015) estimates that the marriage premium is about 40 to 70 percent of the total premium for men and women.

D.3  Mandate Analyses

In this section, I first calibrate the extended model on the 1930s and 1960s birth cohorts in the mandated states and the nonmandated states, and then provide counterfactual analyses that show the impacts of the mandate on men and women’s investment and marriage decisions.

The quantitative fit of the model on the 1930s and 1960s cohorts in the mandated states is shown in Tables D5 and D6. The quantitative fit of the model on the two cohorts in the nonmandated states is shown in Tables D7 and D8. The fits are similar to those on the entire birth cohorts shown in Tables D3 and ?? in the previous section. The moments are calibrated within 4 percent of the targets on average. Figures D3 and D4 show the estimated ability distributions. They all take the same shape as those in Figures ??, further suggesting the stability of the calibration results.

Tables D9 and D10 show the results of two counterfactual analyses for the 1960s cohort – the outcome when mandated states did not pass the mandates and the outcome when nonmandated states become mandated. Namely, I examine the marriage, college, and income patterns when the mandated states have a marriage market that operates as that in the nonmandated states and when the nonmandated states have a marriage market that operates as that in the mandated states.

Table D9 shows that without the mandate, the fraction of late brides would decrease from 19.4 percent to 17 percent. The fraction of women attending college decreases from 37.1 percent to 35.9 percent. The average spousal income of late brides would decrease, and the average spousal income of early brides would increase by 2.9 percent. The average spousal income does not decrease by much because of a countervailing effect that removing the mandate would deter intermediate ability women from making career investments and increase the average income of those who choose to make career investments and delay marriages. These results are qualitatively and quantitatively sensible.
### Table D5: Fit of the extended model, mandated states

<table>
<thead>
<tr>
<th>moments</th>
<th>30s target</th>
<th>30s model</th>
<th>difference</th>
<th>60s target</th>
<th>60s model</th>
<th>difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_{m_1}$</td>
<td>0.451869</td>
<td>0.451556</td>
<td>-0.0693%</td>
<td>0.271852</td>
<td>0.271602</td>
<td>-0.092%</td>
</tr>
<tr>
<td>$G_{m_2}$</td>
<td>0.430358</td>
<td>0.431748</td>
<td>0.323%</td>
<td>0.462758</td>
<td>0.463643</td>
<td>0.191%</td>
</tr>
<tr>
<td>$G_{m_3}$</td>
<td>0.117773</td>
<td>0.116697</td>
<td>-0.914%</td>
<td>0.26539</td>
<td>0.264754</td>
<td>-0.239%</td>
</tr>
<tr>
<td>$G_{w_1}$</td>
<td>0.712169</td>
<td>0.714571</td>
<td>0.337%</td>
<td>0.40867</td>
<td>0.415509</td>
<td>1.67%</td>
</tr>
<tr>
<td>$G_{w_2}$</td>
<td>0.227668</td>
<td>0.221022</td>
<td>-2.92%</td>
<td>0.403811</td>
<td>0.390709</td>
<td>-3.24%</td>
</tr>
<tr>
<td>$G_{w_3}$</td>
<td>0.0601629</td>
<td>0.0644064</td>
<td>7.05%</td>
<td>0.187518</td>
<td>0.193783</td>
<td>3.34%</td>
</tr>
<tr>
<td>$G_{m, col}$</td>
<td>0.240621</td>
<td>0.242344</td>
<td>0.716%</td>
<td>0.392051</td>
<td>0.393502</td>
<td>0.37%</td>
</tr>
<tr>
<td>$G_{w, col}$</td>
<td>0.131002</td>
<td>0.12084</td>
<td>-7.76%</td>
<td>0.400299</td>
<td>0.370931</td>
<td>-7.34%</td>
</tr>
<tr>
<td>$y_{m_1}$</td>
<td>42549.9</td>
<td>41471.4</td>
<td>-2.53%</td>
<td>45833.3</td>
<td>46347.3</td>
<td>1.12%</td>
</tr>
<tr>
<td>$y_{m_2}$</td>
<td>46013.6</td>
<td>46116.1</td>
<td>0.223%</td>
<td>59531.3</td>
<td>59658.5</td>
<td>0.214%</td>
</tr>
<tr>
<td>$y_{m_3}$</td>
<td>38934.8</td>
<td>40058.4</td>
<td>2.89%</td>
<td>52070.5</td>
<td>52371.7</td>
<td>0.579%</td>
</tr>
<tr>
<td>$y_{w_1}$</td>
<td>12664.9</td>
<td>12918.8</td>
<td>2.01%</td>
<td>20453.6</td>
<td>21866.4</td>
<td>6.91%</td>
</tr>
<tr>
<td>$y_{w_2}$</td>
<td>13050.4</td>
<td>15802.5</td>
<td>21.1%</td>
<td>25514.7</td>
<td>28767.5</td>
<td>12.7%</td>
</tr>
<tr>
<td>$y_{w_3}$</td>
<td>13429.7</td>
<td>12946.1</td>
<td>-3.6%</td>
<td>27373.5</td>
<td>25741.2</td>
<td>-5.96%</td>
</tr>
<tr>
<td>$x_{w_1}$</td>
<td>43941.9</td>
<td>42819.1</td>
<td>-2.56%</td>
<td>48004.4</td>
<td>47777.3</td>
<td>-0.473%</td>
</tr>
<tr>
<td>$x_{w_2}$</td>
<td>47304.5</td>
<td>45972.1</td>
<td>-2.82%</td>
<td>62317.6</td>
<td>60849.6</td>
<td>-2.36%</td>
</tr>
<tr>
<td>$x_{w_3}$</td>
<td>37059.8</td>
<td>39648.9</td>
<td>6.99%</td>
<td>52485.</td>
<td>54120.2</td>
<td>3.12%</td>
</tr>
<tr>
<td>average</td>
<td>-&gt;</td>
<td>-&gt;</td>
<td>3.81%</td>
<td>-&gt;</td>
<td>-&gt;</td>
<td>2.94%</td>
</tr>
</tbody>
</table>

### Table D6: Estimated parameters of the extended model, mandated states

<table>
<thead>
<tr>
<th></th>
<th>1930s,M</th>
<th>1960s,M</th>
<th>1930s,M</th>
<th>1960s,M</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_m$</td>
<td>Beta(0.0198,0.0821)</td>
<td>Beta(0.0233,0.0448)</td>
<td>$y_{mH}$</td>
<td>$$64700.7$</td>
</tr>
<tr>
<td>$F_w$</td>
<td>Beta(0.00759,0.0748)</td>
<td>Beta(0.0161,0.0325)</td>
<td>$y_{mL}$</td>
<td>$$37863.7$</td>
</tr>
<tr>
<td>$c_m$</td>
<td>$$1893.18$</td>
<td>$$1747.65$</td>
<td>$y_{wH}$</td>
<td>$$32931.3$</td>
</tr>
<tr>
<td>$c_w$</td>
<td>$$576.1$</td>
<td>$$795.52$</td>
<td>$y_{wL}$</td>
<td>$$1152.$</td>
</tr>
<tr>
<td>$S_{HH}$</td>
<td>$$68279.$</td>
<td>$$20532.$</td>
<td>$h_{mC1}$</td>
<td>0.313768</td>
</tr>
<tr>
<td>$S_{HL}$</td>
<td>$$44160.9$</td>
<td>$$9067.66$</td>
<td>$h_{mC2}$</td>
<td>0.999997</td>
</tr>
<tr>
<td>$S_{HL}$</td>
<td>$$41450.1$</td>
<td>$$19328.8$</td>
<td>$h_{mN1}$</td>
<td>0.495628</td>
</tr>
<tr>
<td>$S_{HH}$</td>
<td>$$45384.4$</td>
<td>$$10326.6$</td>
<td>$h_{mN2}$</td>
<td>0.7824</td>
</tr>
<tr>
<td>$S_{LL}$</td>
<td>$$22639.5$</td>
<td>$$2409.42$</td>
<td>$h_{wC1}$</td>
<td>0.506763</td>
</tr>
<tr>
<td>$S_{LH}$</td>
<td>$$20218.5$</td>
<td>$$399.895$</td>
<td>$h_{wC2}$</td>
<td>0.973855</td>
</tr>
<tr>
<td>$S_{LH}$</td>
<td>$$20815.7$</td>
<td>$$3742.54$</td>
<td>$h_{wN1}$</td>
<td>0.743135</td>
</tr>
<tr>
<td>$S_{LL}$</td>
<td>$$21866$</td>
<td>$$43174.8$</td>
<td>$h_{wN2}$</td>
<td>0.783044</td>
</tr>
</tbody>
</table>

### Figure D3: Estimated CDF and PDF of abilities in the extended model, mandated states

![Estimated CDF and PDF of abilities](image-url)
Figure D4: Estimated CDF and PDF of abilities in the extended model, nonmandated
### Table D9: Counterfactual M→N. Mandated States Stay Nonmandated

<table>
<thead>
<tr>
<th>moment</th>
<th>note</th>
<th>calibrated</th>
<th>counterfactual</th>
<th>percent change</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_{m1}$</td>
<td>fraction of early grooms</td>
<td>0.271602</td>
<td>0.271591</td>
<td>-0.00423343</td>
</tr>
<tr>
<td>$G_{m2}$</td>
<td>fraction of middle grooms</td>
<td>0.463643</td>
<td>0.463617</td>
<td>-0.00573727</td>
</tr>
<tr>
<td>$G_{m3}$</td>
<td>fraction of late grooms</td>
<td>0.264754</td>
<td>0.264792</td>
<td>0.0143901</td>
</tr>
<tr>
<td>$G_{w1}$</td>
<td>fraction of early brides</td>
<td>0.415509</td>
<td>0.486025</td>
<td>16.971</td>
</tr>
<tr>
<td>$G_{w2}$</td>
<td>fraction of middle brides</td>
<td>0.390709</td>
<td>0.344268</td>
<td>-11.8863</td>
</tr>
<tr>
<td>$G_{w3}$</td>
<td>fraction of late brides</td>
<td>0.193783</td>
<td>0.169708</td>
<td>-12.4238</td>
</tr>
<tr>
<td>$G_{m,\text{col}}$</td>
<td>fraction of college men</td>
<td>0.393502</td>
<td>0.393569</td>
<td>0.0169436</td>
</tr>
<tr>
<td>$G_{w,\text{col}}$</td>
<td>fraction of college women</td>
<td>0.370931</td>
<td>0.358943</td>
<td>-3.23186</td>
</tr>
<tr>
<td>$y_{m1}$</td>
<td>avg inc of early grooms</td>
<td>46347.3</td>
<td>46347.8</td>
<td>0.0011738</td>
</tr>
<tr>
<td>$y_{m2}$</td>
<td>avg inc of middle grooms</td>
<td>59658.5</td>
<td>59660.0</td>
<td>0.00260003</td>
</tr>
<tr>
<td>$y_{m3}$</td>
<td>avg inc of late grooms</td>
<td>52371.7</td>
<td>52369.6</td>
<td>-0.0040448</td>
</tr>
<tr>
<td>$y_{w1}$</td>
<td>avg inc of early brides</td>
<td>21866.4</td>
<td>22052.5</td>
<td>0.850921</td>
</tr>
<tr>
<td>$y_{w2}$</td>
<td>avg inc of middle brides</td>
<td>28767.5</td>
<td>28524.5</td>
<td>-0.844894</td>
</tr>
<tr>
<td>$y_{w3}$</td>
<td>avg inc of late brides</td>
<td>25741.2</td>
<td>25709.2</td>
<td>-0.124429</td>
</tr>
<tr>
<td>$x_{w1}$</td>
<td>avg spousal inc of early brides</td>
<td>47777.3</td>
<td>49171.5</td>
<td>2.918</td>
</tr>
<tr>
<td>$x_{w2}$</td>
<td>avg spousal inc of middle brides</td>
<td>60849.6</td>
<td>61119.7</td>
<td>0.443882</td>
</tr>
<tr>
<td>$x_{w3}$</td>
<td>avg spousal inc of late brides</td>
<td>54120.2</td>
<td>54057.8</td>
<td>-0.115388</td>
</tr>
</tbody>
</table>

### Table D10: Counterfactual N→M. Nonmandated States Become Mandated

<table>
<thead>
<tr>
<th>moment</th>
<th>description</th>
<th>calibrated</th>
<th>counterfactual</th>
<th>percent change</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_{m1}$</td>
<td>fraction of early grooms</td>
<td>0.334418</td>
<td>0.33459</td>
<td>0.0514688</td>
</tr>
<tr>
<td>$G_{m2}$</td>
<td>fraction of middle grooms</td>
<td>0.444872</td>
<td>0.445255</td>
<td>0.0861058</td>
</tr>
<tr>
<td>$G_{m3}$</td>
<td>fraction of late grooms</td>
<td>0.220711</td>
<td>0.220155</td>
<td>-0.251542</td>
</tr>
<tr>
<td>$G_{w1}$</td>
<td>fraction of early brides</td>
<td>0.485707</td>
<td>0.415194</td>
<td>-14.5176</td>
</tr>
<tr>
<td>$G_{w2}$</td>
<td>fraction of middle brides</td>
<td>0.354892</td>
<td>0.403018</td>
<td>13.5607</td>
</tr>
<tr>
<td>$G_{w3}$</td>
<td>fraction of late brides</td>
<td>0.159401</td>
<td>0.181788</td>
<td>14.0447</td>
</tr>
<tr>
<td>$G_{m,\text{col}}$</td>
<td>fraction of college men</td>
<td>0.373063</td>
<td>0.372044</td>
<td>-0.273165</td>
</tr>
<tr>
<td>$G_{w,\text{col}}$</td>
<td>fraction of college women</td>
<td>0.36033</td>
<td>0.372282</td>
<td>3.31698</td>
</tr>
<tr>
<td>$y_{m1}$</td>
<td>avg inc of early grooms</td>
<td>43444.2</td>
<td>43438.1</td>
<td>-0.0140087</td>
</tr>
<tr>
<td>$y_{m2}$</td>
<td>avg inc of middle grooms</td>
<td>54176.3</td>
<td>54156.6</td>
<td>-0.0364412</td>
</tr>
<tr>
<td>$y_{m3}$</td>
<td>avg inc of late grooms</td>
<td>45506.</td>
<td>45531.6</td>
<td>0.0563763</td>
</tr>
<tr>
<td>$y_{w1}$</td>
<td>avg inc of early brides</td>
<td>20950.</td>
<td>20800.6</td>
<td>-0.713539</td>
</tr>
<tr>
<td>$y_{w2}$</td>
<td>avg inc of middle brides</td>
<td>26551.5</td>
<td>26748.1</td>
<td>0.740465</td>
</tr>
<tr>
<td>$y_{w3}$</td>
<td>avg inc of late brides</td>
<td>22856.7</td>
<td>22872.5</td>
<td>0.0691212</td>
</tr>
<tr>
<td>$x_{w1}$</td>
<td>avg spousal inc of early brides</td>
<td>43993.5</td>
<td>42686.1</td>
<td>-2.97182</td>
</tr>
<tr>
<td>$x_{w2}$</td>
<td>avg spousal inc of middle brides</td>
<td>55561.4</td>
<td>55306.3</td>
<td>-0.459177</td>
</tr>
<tr>
<td>$x_{w3}$</td>
<td>avg spousal inc of late brides</td>
<td>47599.5</td>
<td>47632.5</td>
<td>0.0692529</td>
</tr>
</tbody>
</table>

D8
Table D10 shows the counterfactual effects of mandates in the nonmandated states. The mandate would (1) increase the fraction of late brides from 15.9 percent to 18.2 percent and decrease the fraction of early brides from 48.6 percent to 41.5 percent, (2) increase the fraction of women attending college from 36 percent to 37.2 percent, and (3) decrease the average spousal income of early brides by 2.97 percent and slightly increase the average spousal income of late brides.

D.4 Gender Equality Analyses

In this section, I conduct three counterfactual analyses on the 1960s birth cohort to investigate the effects of gender equality of the marriage market and the labor market on women’s marriage timing, college, income, and marital outcome.

First, I consider the possibility that differential fecundity is completely eradicated. Namely, women are distinguished by income just like men are, regardless of age. Table D11 shows the results. In equilibrium, 4.95 percent of women would switch from marrying between ages 23 and 29 to marrying between ages 30 and 39. The college investment is not affected directly. The average income of middle brides increases by 5.43 percent and that of late brides decreases by 3.61 percent, making the relationship between age at marriage and income more hump-shaped like the relationship for men. The average spousal income of middle brides increases by 3.27 percent because the average income of middle brides increases. The average spousal income of late brides increases despite of a lower average personal income, because fertility is not a concern in the marriage market in this counterfactual environment.

<table>
<thead>
<tr>
<th>moment</th>
<th>note</th>
<th>calibrated</th>
<th>counterfactual</th>
<th>percent change</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_{w1}$</td>
<td>fraction of early brides</td>
<td>0.449534</td>
<td>0.449534</td>
<td>0</td>
</tr>
<tr>
<td>$G_{w2}$</td>
<td>fraction of middle brides</td>
<td>0.380081</td>
<td>0.330948</td>
<td>−12.9269</td>
</tr>
<tr>
<td>$G_{w3}$</td>
<td>fraction of late brides</td>
<td>0.170385</td>
<td>0.219518</td>
<td>28.8361</td>
</tr>
<tr>
<td>$G_{w,col}$</td>
<td>fraction of college women</td>
<td>0.389479</td>
<td>0.389479</td>
<td>0</td>
</tr>
<tr>
<td>$y_{w1}$</td>
<td>avg inc of early brides</td>
<td>20510</td>
<td>20510</td>
<td>0</td>
</tr>
<tr>
<td>$y_{w2}$</td>
<td>avg inc of middle brides</td>
<td>25169.9</td>
<td>26537</td>
<td>5.43158</td>
</tr>
<tr>
<td>$y_{w3}$</td>
<td>avg inc of late brides</td>
<td>24207.1</td>
<td>23332.3</td>
<td>−3.61388</td>
</tr>
<tr>
<td>$x_{w1}$</td>
<td>avg spousal inc of early brides</td>
<td>47051.6</td>
<td>46073.3</td>
<td>−2.07922</td>
</tr>
<tr>
<td>$x_{w2}$</td>
<td>avg spousal inc of middle brides</td>
<td>55594.8</td>
<td>57414.3</td>
<td>3.27287</td>
</tr>
<tr>
<td>$x_{w3}$</td>
<td>avg spousal inc of late brides</td>
<td>50699.8</td>
<td>51384.1</td>
<td>1.3497</td>
</tr>
</tbody>
</table>

Second, I consider the equalization of labor-market opportunities. Namely, suppose women earn just as much as men and pay the same investment costs as men. Table D12 shows the results. The fraction of women marrying between ages 23 and 29 would decrease from 38 percent to 36.89 percent. Of this 1.11 percent of the population, 0.2 percent would choose to marry earlier and 0.91 percent would choose to marry later. The fraction of early brides slightly increases because the fraction of women attending college would decrease from 38.9 percent to 38.3 percent and some of them would choose to marry early. The average spousal income of early brides would decrease,
and the average spousal income of middle brides would increase by 0.68 percent and that of late brides would increase by 0.37 percent.

### Table D12: Counterfactual 2. Equalized Labor-Market Opportunities

<table>
<thead>
<tr>
<th>Moment</th>
<th>Note</th>
<th>Calibrated</th>
<th>Counterfactual</th>
<th>Percent Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_{w1}$</td>
<td>Fraction of early brides</td>
<td>0.449534</td>
<td>0.451106</td>
<td>0.349743</td>
</tr>
<tr>
<td>$G_{w2}$</td>
<td>Fraction of middle brides</td>
<td>0.380081</td>
<td>0.368892</td>
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</tr>
<tr>
<td>$G_{w3}$</td>
<td>Fraction of late brides</td>
<td>0.170385</td>
<td>0.180002</td>
<td>5.6441</td>
</tr>
<tr>
<td>$G_{w;col}$</td>
<td>Fraction of college women</td>
<td>0.389479</td>
<td>0.382699</td>
<td>-1.74082</td>
</tr>
<tr>
<td>$y_{w1}$</td>
<td>Avg inc of early brides</td>
<td>20510.</td>
<td>43183.8</td>
<td>110.55</td>
</tr>
<tr>
<td>$y_{w2}$</td>
<td>Avg inc of middle brides</td>
<td>25169.9</td>
<td>53199.7</td>
<td>111.363</td>
</tr>
<tr>
<td>$y_{w3}$</td>
<td>Avg inc of late brides</td>
<td>24207.1</td>
<td>50888.1</td>
<td>110.219</td>
</tr>
<tr>
<td>$x_{w1}$</td>
<td>Avg spousal inc of early brides</td>
<td>47051.6</td>
<td>46851.4</td>
<td>-0.425445</td>
</tr>
<tr>
<td>$x_{w2}$</td>
<td>Avg spousal inc of middle brides</td>
<td>55594.8</td>
<td>55790.4</td>
<td>0.675739</td>
</tr>
<tr>
<td>$x_{w3}$</td>
<td>Avg spousal inc of late brides</td>
<td>50699.8</td>
<td>50888.1</td>
<td>0.371288</td>
</tr>
</tbody>
</table>

Third, I consider the equalization of investment opportunities. Namely, suppose the ability distribution of women would be just like that of men in the model. Table D13 shows the results. In equilibrium, the fraction of women going to college would decrease and the fraction of women choosing to marry later would also decrease. The average incomes would however increase. The average spousal income of middle and late brides would increase by 1.85 and 3.66 percent, respectively. The fact that women’s college enrollment would decrease is in line with the empirical finding that women’s higher average non-cognitive skills result in a higher college enrollment for them. If women’s ability distribution is fixed to be the same as men’s, college enrollment would decrease.

### Table D13: Counterfactual 3. Equalized Investment Opportunities

<table>
<thead>
<tr>
<th>Moment</th>
<th>Note</th>
<th>Calibrated</th>
<th>Counterfactual</th>
<th>Percent Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_{w1}$</td>
<td>Fraction of early brides</td>
<td>0.449534</td>
<td>0.450571</td>
<td>0.230603</td>
</tr>
<tr>
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<td>Fraction of middle brides</td>
<td>0.380081</td>
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</tr>
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<td>$G_{w3}$</td>
<td>Fraction of late brides</td>
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</tr>
<tr>
<td>$G_{w;col}$</td>
<td>Fraction of college women</td>
<td>0.389479</td>
<td>0.385009</td>
<td>-1.14781</td>
</tr>
<tr>
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<td>20510.</td>
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<td>$y_{w2}$</td>
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<td>Avg spousal inc of late brides</td>
<td>50699.8</td>
<td>52556.2</td>
<td>3.66142</td>
</tr>
</tbody>
</table>

References in Appendix D


**Coles, Melvyn G. and Marco Francesconi**, “Equilibrium Search and the Impact of Equal Oppor-
opportunities for Women,” June 2017. IZA DP No. 10827.