An Investment-and-Marriage Model with Differential Fecundity

Hanzhe Zhang

Michigan State University

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http://www.msu.edu/~hanzhe/
Four Stylized Facts
1. College and Earnings Gender Gaps

![Graph 1: Percent with college degrees among 35-39 year-olds](chart1)

- **Percent with college degrees among 35-39 year-olds**
- **Male and Female Comparison**

![Graph 2: Avg labor inc (thousands $) among 35-39 year-olds](chart2)

- **Avg labor inc (thousands $) among 35-39 year-olds**
- **Male and Female Comparison**
2. Men’s Midlife Income by Age at Marriage

**1930-39**

- Men's personal midlife total income
- 95% CI

**1960-69**

- Men's personal midlife total income
- 95% CI
3. Women’s Midlife Income by Age at Marriage

- **1930-39**
  - Midlife total income (thousand $)
  - Age at marriage
  - Women's personal midlife total income
  - 95% CI

- **1960-69**
  - Midlife total income (thousand $)
  - Age at marriage
  - Women's personal midlife total income
  - 95% CI
4(i). Women’s Spousal Income by Age at Marriage

![Graph showing women's spousal midlife total income by age at marriage for two periods: 1930-39 and 1960-69. The graphs display data with 95% confidence intervals.]
Mandates to Cover/Offer Infertility Treatments in Insurances

- Between 1985 and 1995, 13 U.S. states passed mandates to cover/offer infertility treatments in insurances.
4(ii). Women’s Spousal Income by Age at Marriage

- **1930-1939**
  - Spousal log income
  - Mandate states
  - Non-mandate states

- **1960-1969**
  - Spousal log income
  - Mandate states
  - Non-mandate states

**Spousal Midlife Total Income (thousand $)**

- **1930-1939**
  - Mandate states
  - Non-mandate states

- **1960-1969**
  - Mandate states
  - Non-mandate states
Four Stylized Facts

1. More women than men go to college, but fewer women than earn a high income.

2. Relationship between age at marriage and personal midlife income for men has been persistently hump-shaped.

3. Relationship between age at marriage and personal midlife income for women has been persistently positive.

4. Relationship between age at marriage and spousal income for women has been persistently hump-shaped with a changing marital outcome for early brides versus late brides.
Model
Human Capital Investments
Human Capital Investments

An agent enters the marriage market immediately after income is determined.
Human Capital Investments

ages 16-22

ages 23-29

ages 30-39
Human Capital Investments

A crew enters the marriage market immediately after income is determined.

- ages 16-22
- ages 23-29
- ages 30-39
Human Capital Investments

An agent enters the marriage market immediately after income is determined.

- Offer
  - College
    - Ages 16-22
  - No college
    - Ages 23-29
    - Ages 30-39
Human Capital Investments

Offer

College

No college

$\theta$

L

Ages 16-22

Ages 23-29

Ages 30-39
Human Capital Investments

- colleges
- no college

Ages 16-22

Ages 23-29

Ages 30-39
Human Capital Investments

- ages 16-22 with college
- ages 16-22 without college
- ages 23-29
- ages 30-39

An agent enters the marriage market immediately after income is determined.
Human Capital Investments

- College offer
- No college

- Ages 16-22
- Ages 23-29
- Ages 30-39

An agent enters the marriage market immediately after income is determined.
Human Capital Investments

An agent enters the marriage market immediately after income is determined.

- College: \( \theta \)
  - Offer: \([\theta] \)
  - No college: \([1 - \theta] \)

- H
- L

Ages 16-22
Ages 23-29
Ages 30-39
Human Capital Investments

An agent enters the marriage market immediately after income is determined.

- College: \(\theta\)
- No college: \(1 - \theta\)

- Offer
  - [\(\theta\)]
  - [\(1 - \theta\)]

- Ages 16-22
- Ages 23-29
- Ages 30-39
Human Capital Investments

- Ages 16-22
  - College
  - No college
  - Ages 16-22

- Ages 23-29
  - [\theta]
  - Career
  - Ages 23-29

- Ages 30-39
  - [1 - \theta]
  - Offer
  - Ages 30-39
Human Capital Investments

An agent enters the marriage market immediately after income is determined.
Human Capital Investments

Ages 16-22
- College
  - Offer \[\theta\]
  - No college \[1 - \theta\]

Ages 23-29
- Career
  - Offer
  - No career

Ages 30-39
Human Capital Investments

An agent enters the marriage market immediately after income is determined.

- Ages 16-22
  - College: [1 - \(\theta\)]
  - No college: \(\theta\)

- Ages 23-29
  - Career: \(\theta\)
  - No career: [1 - \(\theta\)]

- Ages 30-39
  - Offer: \(H\)
  - No offer: \(L\)
Human Capital Investments

ages 16-22

no college

college

[1 - \theta]

offer

[\theta]

career

no career

L

L

H

H

\theta

\theta

ages 23-29

ages 30-39
Human Capital Investments

An agent enters the marriage market immediately after income is determined.

- Ages 16-22: College offer
- Ages 23-29: Career offer
- Ages 30-39: No career offer

- College: 1 - θ
- No college: θ
- No career: θ
- Career: 1 - θ

L: Low
H: High
An agent enters the marriage market immediately after income is determined.
Differential Fecundity

\[ y + v - c \]

income + marital payoff (income, fertility) – investment costs

- Men who marry in any of the three periods have the same fertility level.
- Women who marry in the third period may have a lower fertility level than those who marry in the first two periods.
- Husband’s income and wife’s income and fertility determine marriage surplus: \( s(y_m, y_w, \phi_w) \equiv s(\tau_m, \tau_w) \).
  - Surplus is increasing in each argument, supermodular in incomes, and supermodular in income and fertility.
Division of the marriage surplus is endogenously determined:
\[ u_{m\tau_m} + u_{w\tau_w} = s(\tau_m, \tau_w) \] for any married couple \( \tau_m \) and \( \tau_w \).

Marriages are stable: \( u_{m\tau_m} + u_{w\tau_w} \geq s(\tau_m, \tau_w) \) for any pair.
Explanations
1. Men’s Midlife Income by Age at Marriage

**1930-39**

- Age at marriage: 16-22, 23-29, 30-39
- Midlife total income (thousand $): 44-52
- 95% CI

**1960-69**

- Age at marriage: 16-22, 23-29, 30-39
- Midlife total income (thousand $): 50-65
- 95% CI
\[
\theta_m = \frac{c_m}{(y_{mH} - y_{mL}) + (v_{mH} - v_{mL})} \equiv \frac{c_m}{\Delta y_m + \Delta v_m}
\]
2. Women’s Midlife Income by Age at Marriage

- Women's personal midlife total income
- 95% CI
\[ \theta_{w1} = \frac{c_w}{\Delta y_w + v_{wH} - v_{wL}} < \theta_{w2} = \frac{c_w + v_{wL} - v_{wl}}{\Delta y_w + v_{wh} - v_{wl}} \]
Suppose the setting is gender-symmetric except for fertility length. More women than men go to college in equilibrium. 

\[
\begin{align*}
&\text{no college, no career} \\
&\text{college, career}
\end{align*}
\]

\[
\begin{align*}
&\text{no fertility difference} \\
&\text{fertility difference}
\end{align*}
\]

Only college-educated men make a career investment. Only some college-educated women make a career investment. Fewer women than men earn a high income. High-income women are more scarce than high-income men in MM. College generates higher MM returns for women than for men.
Suppose the setting is gender-symmetric except for fertility length. *More* women than men go to college in equilibrium.
Suppose the setting is gender-symmetric except for fertility length. *More* women than men go to college in equilibrium.

| 0 | no college, no career | $\theta_m^*$ | college, career | 1 |

- All college-educated men make a career investment.
Suppose the setting is gender-symmetric except for fertility length. More women than men go to college in equilibrium.

\[
\begin{array}{ccc}
0 & \text{no college, no career} & \theta^*_m & \text{college, career} & 1 \\
\hline
\text{no fertility difference} & & \theta^*_w = \theta^*_w \\
0 & \text{no college, no career} & \text{college, career} & 1 \\
\end{array}
\]

- All college-educated men make a career investment.
College and Earnings Gender Gaps

Suppose the setting is gender-symmetric except for fertility length. More women than men go to college in equilibrium.

\[
\begin{array}{ccc}
0 & \text{no college, no career} & \theta_m^* & \text{college, career} & 1 \\
\hline
\text{fertility difference} & \theta_{w1}^* = \theta_{w2}^* \\
0 & \text{no college, no career} & \text{college, career} & 1
\end{array}
\]

- All college-educated men make a career investment.
Suppose the setting is gender-symmetric except for fertility length. More women than men go to college in equilibrium.

\[
\begin{array}{cccc}
0 & \text{no college, no career} & \theta_m^* & \text{college, career} & 1 \\
\hline
\text{fertility difference} & \theta_{w1}^* < \theta_{w2}^* \\
0 & \text{no college, no career} & \text{college, career} & 1 \\
\hline
\text{only college}
\end{array}
\]

- All college-educated men make a career investment.
- Only some college-educated women make a career investment.
**College and Earnings Gender Gaps**

Suppose the setting is gender-symmetric except for fertility length. *More* women than men go to college in equilibrium.

<table>
<thead>
<tr>
<th></th>
<th>no college, no career</th>
<th>college, career</th>
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<tr>
<td>0</td>
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<td>$\theta^*_m$</td>
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<tr>
<td></td>
<td>fertility difference</td>
<td>$\theta^*_w$</td>
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<td></td>
<td>$\theta^<em>_w_1 &lt; \theta^</em>_w_2$</td>
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<td></td>
</tr>
<tr>
<td>0</td>
<td>no college, no career</td>
<td>college, career</td>
<td>1</td>
</tr>
</tbody>
</table>

- All college-educated men make a career investment.
- Only some college-educated women make a career investment.
- Fewer women than men earn a high income.
Suppose the setting is gender-symmetric except for fertility length. *More* women than men go to college in equilibrium.

\[ \begin{align*}
0 & \quad \text{no college, no career} & \theta^*_m & \quad \text{college, career} & 1 \\
\hline
\text{fertility difference} & \text{(blue line)} & \theta^*_w_1 < \theta^*_w_2 \\
0 & \quad \text{no college, no career} & \quad & \text{college, career} & 1 \\
\hline
\text{only college} & \text{(red line)}
\end{align*} \]

- All college-educated men make a career investment.
- Only some college-educated women make a career investment.
- Fewer women than men earn a high income.
- High-income women are more scarce than high-income men in MM.
Suppose the setting is gender-symmetric except for fertility length. More women than men go to college in equilibrium.

\[
\begin{array}{cccc}
0 & \text{no college, no career} & \theta^*_m & \text{college, career} & 1 \\
\hline
\text{fertility difference} & \theta^*_w_1 < \theta^*_w_2 \\
0 & \text{no college, no career} & \text{college, career} & 1 \\
\text{only college} & \\
\end{array}
\]

- All college-educated men make a career investment.
- Only some college-educated women make a career investment.
- Fewer women than men earn a high income.
- High-income women are more scarce than high-income men in MM.
- College generates higher MM returns for women than for men.
4. Women’s Spousal Income by Age at Marriage

- Women's spousal midlife total income
- 95% CI

**1930-39**
- Women's spousal midlife total income: $40,000 - $30,000

**1960-69**
- Women's spousal midlife total income: $60,000 - $50,000
Fertility-Income Tradeoff

Fraction of ability-\( \theta \) women

- no college or career
  - marry in period 1
    - \( L \)
- college only
  - marry in period 2
    - \( L \)
- college and career
  - marry in period 3
    - \( l \)
    - \( h \)

\( \theta_w \) college only
\( \theta_{w1} \) college
\( \theta_{w2} \) college and career

Ability \( \theta \)

No college or career
Marry in period 1
College only
Marry in period 2
College and career
Marry in period 3

College and, if necessary, career

20
Spousal Total Income Percentile Rank

Ninety-five-percent confidence interval in mandate and non-mandate states

Women age marriage 16-22
Women age marriage 23-29
Women age marriage 30-39

Birth year

Ninety-five-percent confidence interval in mandate and non-mandate states

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<table>
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<th>Women age marriage 16-22</th>
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<td>no one affected</td>
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<td>Women age marriage 23-29</td>
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<td>--------------------------</td>
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<tr>
<td>no one affected</td>
</tr>
<tr>
<td>35</td>
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<tr>
<td></td>
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<tr>
<td>Women age marriage 30-39</td>
</tr>
<tr>
<td>--------------------------</td>
</tr>
<tr>
<td>no one affected</td>
</tr>
<tr>
<td>35</td>
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<td>men</td>
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<td>$H$</td>
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<td>$L$</td>
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Fertility more important

<table>
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<th>men</th>
<th>match</th>
<th>women</th>
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<td>$H$</td>
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<td>$H$</td>
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<tr>
<td>$L$</td>
<td>$Ll$</td>
<td>$l$</td>
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Income more important
Supporting Evidence and Calibration
Age-Income Profiles for Men

Income ratio of 4-year-college late groom to 4-year-college middle groom

1941-54 birth cohort

1957-64 birth cohort

1980-84 birth cohort

Age

22 26 30 34 22 26 30 34 22 26 30 34

6 8 10 12 6 8 10 12 6 8 10 12
Age-Income Profiles for Women

Income ratio of 4-year-college late brides to 4-year-college middle brides

1941-54 birth cohort
1957-64 birth cohort
1980-84 birth cohort

Age

22 26 30 34 22 26 30 34 22 26 30 34
Evolution of the Marriage Premium

Men's marriage premium: $\pi_m = s_{HL} - s_{LL}$

Women's marriage premium: $\pi_w = s_{HH} - s_{HL}$

Marriage premium (utils)

Census year


95% CI
Calibration

- Ability distributions are $\text{Beta}(\alpha_m, \beta_m)$ and $\text{Beta}(\alpha_w, \beta_w)$.
- Low income is average income of the non-college-educated. High income is average income of the college-educated.
- Total investment cost is two years of low incomes; annual cost is total cost/40.
- Marriage surplus in monetary terms is $k$ times surplus in utils estimated.
- Add marriage frictions (possibility of not marrying upon entering MM).
- 19 targeted moments.
  - Percentages of early, middle, late grooms/brides (6).
  - Average personal income of early, middle, late grooms (3).
  - Average personal income of early, middle, late brides (3).
  - Average spousal income of early brides (3).
  - College enrollment rates of men and women (2).
## Fit of the Model

<table>
<thead>
<tr>
<th>moments</th>
<th>30s target</th>
<th>30s model</th>
<th>difference</th>
<th>60s target</th>
<th>60s model</th>
<th>difference</th>
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<td>$G_{m1}$</td>
<td>0.48476</td>
<td>0.484451</td>
<td>-0.0637%</td>
<td>0.30756</td>
<td>0.307372</td>
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<td>$G_{m2}$</td>
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<td>0.412559</td>
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<td>0.452309</td>
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<td>$G_{m3}$</td>
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<td>0.740591</td>
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<td>0.449534</td>
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<tr>
<td>$G_{w2}$</td>
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<td>0.206847</td>
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<td>0.381204</td>
<td>0.380081</td>
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<td>$G_{w3}$</td>
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<td>0.0525618</td>
<td>0.154%</td>
<td>0.169396</td>
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<td>$G_{m,\text{col}}$</td>
<td>0.218733</td>
<td>0.220363</td>
<td>0.745%</td>
<td>0.379722</td>
<td>0.380819</td>
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<td>0.390058</td>
<td>0.389479</td>
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<td>$y_{m1}$</td>
<td>40209.7</td>
<td>39603.7</td>
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<td>44571.6</td>
<td>44730.5</td>
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<td>$y_{m2}$</td>
<td>43820.8</td>
<td>43915.8</td>
<td>0.217%</td>
<td>56434.2</td>
<td>56524.6</td>
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<td>$y_{m3}$</td>
<td>37442</td>
<td>38350.9</td>
<td>2.43%</td>
<td>48376.5</td>
<td>48589.3</td>
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<tr>
<td>$y_{w1}$</td>
<td>12049.0</td>
<td>11696.3</td>
<td>-2.93%</td>
<td>20091.0</td>
<td>20510.0</td>
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<tr>
<td>$y_{w2}$</td>
<td>12457.2</td>
<td>12739.2</td>
<td>2.26%</td>
<td>24627.8</td>
<td>25169.9</td>
<td>2.2%</td>
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<tr>
<td>$y_{w3}$</td>
<td>12886.1</td>
<td>12421.0</td>
<td>-3.61%</td>
<td>26080.1</td>
<td>24207.1</td>
<td>2.09%</td>
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<td>$x_{w1}$</td>
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<td>46138.3</td>
<td>47051.6</td>
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<td>$x_{w2}$</td>
<td>45269.5</td>
<td>42290.6</td>
<td>-6.58%</td>
<td>58701.2</td>
<td>55594.8</td>
<td>1.98%</td>
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<tr>
<td>$x_{w3}$</td>
<td>35537.5</td>
<td>38066.9</td>
<td>7.12%</td>
<td>48666.8</td>
<td>50699.8</td>
<td>-5.29%</td>
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<tr>
<td>average</td>
<td>1.71%</td>
<td>1.51%</td>
<td>1.51%</td>
<td>4.18%</td>
<td>1.51%</td>
<td>4.18%</td>
</tr>
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</table>
Quantitative Results

- Estimated ability distributions (labor-market opportunities).

- Labor-market shocks (due to the possibility that one does not receive a high-income offer after college) contribute to 42.7% of college-educated men and 24% of college-educated women born in the 1960s delaying marriage (the rest are explained by marriage-market frictions).
Conclusion

- College and earnings gender gaps.
- Relationships between age at marriage and personal income for men and women.
- Relationship between age at marriage and spousal income for women.
- Differential fecundity, due to the equilibrium marriage market, leads to many observed economic and social gender differences.
THANK YOU!