Irreversible abatement investment under cost uncertainties: tradable emission permits and emissions charges

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Abstract

A major concern with tradable emission permits is that stochastic permit prices may reduce a firm’s incentive to invest in abatement capital or technologies relative to other policies such as a fixed emissions charge. However, under efficient permit trading, the permit price uncertainty is caused by abatement cost uncertainties which affect investment under both permit and charge policies. We develop a rational expectations general equilibrium model of permit trading and irreversible abatement investment to show how cost uncertainties affect investment under permits. We compare the resulting investment incentive with that under charges. After controlling for the assumption that random shocks affect the abatement cost linearly, we find that firms’ investment incentive decreases in cost uncertainties, but more so under emissions charges than under permits. Therefore, tradable permits in fact may help maintain firms’ investment incentive under uncertainty. © 2002 Elsevier B.V. All rights reserved.

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1. Introduction

Tradable emission permits (TEPs) are gaining popularity in environmental regulation as manifested by the successful sulfur trading in the U.S. and the global carbon trading proposed in the Kyoto Protocol. However, there is a concern that in the long run, a system of grandfathered permits may provide less incentive for firms to invest in abatement...
technologies or capital than a constant emissions tax, because the marginal abatement costs go down as firms invest, reducing the permit price and subsequently the benefits of investment (Milliman and Prince, 1989; Jung et al., 1996). Moreover, under TEPs, the incentive to invest may be further reduced because permit prices are typically random and the investment is to a great extent irreversible (Xepapadeas, 1999; Chao and Wilson, 1993).¹ In contrast, other policies such as standards or taxes do not introduce this additional uncertainty. The performance of TEPs under uncertainty is extremely important given the pervasive existence of uncertainties in permit trading. The literature typically assumes *exogenous* and random permit price processes (Xepapadeas, 1999) or *exogenous* and random demand functions for permits (Chao and Wilson, 1993). In Baldursson and von der Fehr (1999), uncertainty is due to the entry and exit of polluting firms.

These studies point out an important possibility. However, since permit price is directly determined by firms’ abatement costs through (efficient) permit trading, a major force behind price randomness is firms’ abatement cost shocks.² Such shocks will affect the investment decisions under other policies as well. TEPs do not create uncertainties in their own right, but rather ‘transmit’ cost uncertainties into permit price uncertainties. Thus the relevant question is, compared with other policies, whether cost uncertainties reduce the investment incentive by a larger amount under TEPs when the permit price is endogenously determined by abatement costs through permit trading.

The focus of this paper is thus on the effects of cost uncertainties on firms’ incentive to invest in abatement technologies or capital under TEPs vis-à-vis emissions charges. We introduce a general equilibrium model of permit trading by price taking firms with stochastic abatement costs and rational expectations about permit prices. In each period, the government grandfathers a fixed and constant number of emission permits. Firms’ abatement costs are subject to both industry wide and firm specific shocks. Given the (marginal) costs, efficient permit trading endogenously determines the equilibrium permit price. A firm can irreversibly invest in capital or technology to reduce its abatement cost. The aggregate investment behavior of the firms (together with the cost shocks) determines the time path of the permit price. Abatement cost shocks change the price *instantaneously* through permit trading and *overtime* through the firms’ abatement investment.

This model captures several salient features of a TEP system. First, (arguably) the most important determinant of a permit price is the firms’ abatement costs. Firms’ input, output and entry/exit decisions do affect the permit price, but primarily indirectly through altering the abatement costs. For example, railway deregulation in the U.S. raised the use of low sulfur coal by the utility companies, contributing to the lower-than-expected SO₂ permit price (Burtraw, 1996). Here the regulatory change reduced the permit price through lowering the (marginal) abatement costs. We model cost shocks without restricting them to be from a particular source. Second, a TEP system is in essence similar to a pure exchange economy with fixed endowments of permits. There are no *exogenous* permit demand or supply functions. Rather, firms *choose* to be permit suppliers or buyers through

¹ That irreversibility and uncertainty (and future learning) reduce investment is a standard conclusion of real option theory (Arrow and Fisher, 1974; Dixit and Pindyck, 1994).

² Throughout this paper, we will maintain the efficient permit trading hypothesis. This hypothesis is confirmed in one of the best known TEP systems, the SO₂ trading in the U.S. (Joskow et al., 1998).
investment. Finally, abatement investments are difficult to reverse. For example, a utility company will find it costly to get rid of a scrubber it has installed.

Under TEPs, we have to solve for firms’ optimal investment strategies in a rational expectations general equilibrium. Compared with the typical (partial equilibrium) real options model, such as Arrow and Fisher (1974) and Kolstad (1996), the general equilibrium framework complicates both the analysis and the intuition. Partial equilibrium models argue that if a firm has an investment opportunity, it may wish to delay its irreversible investment until a more favorable situation arises (such as when the permit price is higher), or until uncertainties are reduced by more information. As a result, the investment incentive goes down in response to the uncertainties. However, in a general equilibrium, if other firms also face this investment opportunity, our firm may lose its opportunity if it delays the investment for too long and other firms instead invest (since then the permit price will be lower and our firm may never invest). Leahy (1993), Caballero and Pindyck (1996) and Baldursson and Karatzas (1997) showed that this concern does not matter in models of firms making entry and exit decisions facing common demand shocks in a competitive equilibrium. The firms may be ‘myopic’ by pretending that the future price will not be affected by other firms’ future entry/exit, and the resulting entry/exit decisions are in fact optimal. Our model extends the literature by showing that the myopic rule is optimal even under firm specific uncertainties, and for a different decision framework where firms make continuous investment decisions rather than discrete entry and exit decisions.

We then consider the firms’ investment strategies facing an emissions charge/subsidy that is constant overtime. Following Milliman and Prince (1989) and Jung et al. (1996), and without loss of generality, we choose the charge policy to be ‘comparable’ to the permit policy in that they lead to the same abatement levels in the current period. In a deterministic model, as shown by the above authors, the firms’ investment incentive is weaker under TEPs because the equilibrium permit price goes down as the firms invest. With cost uncertainties, we show that firms’ investment incentive decreases in these uncertainties under both policies, but more so under a charge system. Further, with uncertainties, investment incentive under the two policies can diverge even without the general equilibrium consideration since tax is a price tool and permit is a quantity tool (Weitzman, 1974). After controlling for the curvature of the payoff functions (we assume that the random variables enter the abatement cost function linearly), we find that uncertainties again reduce the investment incentive more under emissions charges. Thus TEPs help maintain firms’ investment incentive under uncertainty relative to charges.

The paper is organized as follows. We construct the general equilibrium model of permit trading in Section 2. We solve for the firms’ optimal investment strategies under permits in Section 3, and under an equivalent charge policy in Section 4. We discuss the generality of our model in Section 5, and conclude the paper in Section 6.

2. Model setup: investment under permits

Irreversible investment models under uncertainty can quickly become intractable, even without the added difficulty of handling a rational expectations general equilibrium. We
thus assume special functional forms in order to obtain analytical results. We will discuss the implications of these assumptions in Section 5, arguing that they are not likely to change our major conclusions.

Consider a tradable emissions permit market consisting of \( N \) price taking risk neutral firms with rational expectations about permit prices.\(^3\) We focus on emissions trading and ignore firms’ output decisions.\(^4\) Let the total abatement cost (TAC) of firm \( n \) be

\[
C(a_n, K_n, n, \epsilon_n, \epsilon_0) = \frac{1}{2} c(K_n, n) \epsilon_0 a_n^2 + d(K_n, n) \epsilon_n, \quad n = 1, \ldots, N, \quad (1)
\]

where \( a_n \geq 0 \) is the abatement level, \( K_n \geq 0 \) the stock of abatement capital or technology, \( \epsilon_n \geq 0 \) firm \( n \)'s specific shock, and \( \epsilon_0 \geq 0 \) the industry shock affecting all firms. By allowing TAC to depend on \( n \), we account for the heterogeneity of the firms, a major advantage of tradable permits. \( c(K_n, n) \epsilon_0 \) is the unit marginal abatement cost, and \( d(K_n, n) \epsilon_n \) is the fixed cost of abatement. The industry shock affects both the total and marginal costs of abatement, while the firm specific shock only affects the total cost. As we show later on, not allowing \( \epsilon_n \) to affect the marginal abatement cost enables us to obtain a clean and intuitive solution to the optimization problem. We assume that \( c_{Kn} < 0, c_{KnKn} > 0, d_{Kn} < 0 \) and \( d_{KnKn} > 0 \). Since \( \epsilon_0 \geq 0 \), the cost \( C(\cdot) \) is increasing and convex in the abatement level \( a_n \). Stock \( K_n \) reduces the cost, but at a decreasing rate.

We consider firm decisions in continuous time over \( [0, \infty) \). We assume that firm specific and industry shocks follow independent geometric Brownian motions:

\[
de \epsilon_n = \alpha_n \epsilon_n(t) dt + \sigma_n \epsilon_n(t) dz_n(t), \quad n = 0, 1, \ldots, N, \quad (2)
\]

where \( dz_n(t) \) is the incremental Wiener process, with \( E(dz_n(t)) = 0 \), \( \text{var}(dz_n(t)) = dt \), and \( \text{cov}(dz_n, dz_m) = 0 \) for \( n \neq m \). The term \( \alpha_n \) measures the expected growth rate of the shock \( \epsilon_m \), and \( \sigma_n \) measures its degree of variability. To make the problem interesting, we assume \( \alpha_n < r \), \( n = 0, 1, \ldots, N \). Otherwise, the cost of abatement would increase too quickly to allow any investment.

At time \( t \), firm \( n \) observes \( K_n(t), \epsilon_n(t), \) and \( \epsilon_0(t) \) and thus knows its own TAC function. Based on the TAC functions, or the marginal abatement cost (MAC) functions, firms trade permits until the MACs are equalized across all firms. (We assume that the trading is efficient.) Let \( \bar{\epsilon} \) be the total number of permits distributed by the government, \( \bar{\epsilon}_n \) firm \( n \)'s free permits (i.e., \( \bar{\epsilon} = \sum_n \bar{\epsilon}_n \)), and \( \epsilon^0_n \) its emission without abatement, all constant overtime. Let \( \bar{a} \) be the total industry abatement: \( \bar{a} = \sum_n a_n(t) = \sum_n \epsilon^0_n - \bar{\epsilon}, \forall t \geq 0 \). The equilibrium

\(^3\) Our result does not depend on the assumption of risk neutrality. When the firms are risk averse, we can either use the risk adjusted discount rate or use risk neutral probabilities and the riskless discount rate if there are traded assets that can span the risks.

\(^4\) Firms may be in different industries and produce different kinds of outputs. Requate (1998) studies specifically the relationship between output choice and permit trading decisions.
permit price \( p^* \) and abatement level \( a^*_n \) satisfy the following instantaneous permit market equilibrium condition:

\[
c(K_n, n)a^*_n = c(K_m, m)a^*_m = \frac{p^*}{e_0}, \quad m, n = 1, \ldots, N; \quad \sum_n a^*_n = \bar{a},
\]

(3)

Thus, given \( K = \{K_1, \ldots, K_N\} \), firm specific shocks do not affect the permit price or the abatement levels. When an industry shock occurs, the abatement levels do not change, but the equilibrium permit price adjusts proportionally (so that \( p/e_0 \) remains unchanged). Of course, both shocks affect the firm’s total costs, including abatement and permit costs. As a result, firm \( n \) may find it necessary to invest in \( K_n \) in response to high values of \( e_0 \) or \( e_n \).

For tractability, we assume that capital stocks do not depreciate. The investment cost is linear in the investment level, with the unit cost given by \( \kappa \). Linearity implies that the stock can be non-differentiable (although continuous) in time: if the current stock is too low, firm \( n \) can instantaneously adjust the stock to its desired level. Optimal investment depends on the future permit prices, as well as the firm’s current information about future shocks. In the intertemporal equilibrium, described in Appendix A, firms have rational expectations about the future permit price paths, and consequently decide their investment strategies. These strategies in turn generate the price path the firms have expected.

Directly solving the competitive equilibrium proves to be too hard a problem. Instead, we rely on the equivalence between the competitive equilibrium and a fictitious ‘social planner’s problem’ of maximizing the total firm payoffs subject to the shocks and permit policy (Lucas and Prescott, 1971; Baldursson and Karatzas, 1997). The social planner is fictitious in that it is not maximizing the social welfare, which would include the pollution damage (or even the choice of an appropriate policy). Rather, the social planner is only a convenient way of solving for the competitive equilibrium, and consequently it maximizes the firm payoffs only.

Proposition 1, derived in Appendix A, presents the planner’s optimal decisions.

**Proposition 1.** Given \( K, \epsilon = \{\epsilon_1, \ldots, \epsilon_N\} \), and \( e_0 \), the firms’ optimal stocks \( K' \) satisfy the following complementary slackness conditions:

\[
\begin{align*}
J_{K_n'}(K', \epsilon, e_0) - \kappa \leq & 0, \quad K_n' - K_n \geq 0, \\
(J_{K_n'}(K', \epsilon, e_0) - \kappa)(K_n' - K_n) = & 0, \quad \forall n.
\end{align*}
\]

(4)

The function \( J(\cdot) \), given in Appendix A, describes the expected discounted present value of the maximum payoff of the fictitious planner. Appendix A shows that \( J(\cdot) \) is concave in \( K \). Thus, whenever shocks occur so that \( J_{K_n} > \kappa \), more abatement capital is needed (because its marginal value exceeds its marginal cost \( \kappa \)), and firm \( n \) should instantaneously invest until the new stock \( K_n' \) satisfies \( J_{K_n'}(K', \epsilon, e_0) = \kappa \). If \( J_{K_n} < \kappa \), the stock is too high but irreversibility restricts that the stock will not be reduced. As shocks \( \epsilon \) and \( e_0 \) change \( J_{K_n} \) overtime, \( J_{K_n}(K, \epsilon, e_0) = \kappa \) acts as a barrier to capital adjustment: \( J_{K_n} \) can never exceed \( \kappa \) for a strictly positive length of time. Whenever the shocks raise \( J_{K_n} \) above \( \kappa \), instantaneous investments are undertaken to restore the equality. This type of barrier...
control policy is typical of investment problems with linear investment costs (Dixit and Pindyck, 1994). Since \( e(t) \) and \( \epsilon_0(t) \) are not differentiable, the resulting \( K_n(t) \) is not differentiable whenever firm \( n \) invests.

3. Optimal investment under TEPs

Proposition 1 describes the optimal strategies of the fictitious planner. In this section, we solve for \( J(\cdot) \) and the optimal investment barriers, and translate the planner’s strategies to the firms’ equilibrium strategies.

Intuitively, given \( K \), the fictitious planner would like firm \( n \) to invest when the firm receives strong positive shocks in \( \epsilon_n \) or \( \epsilon_0 \). Appendix B shows that in the socially optimal solution, the investment barriers for firm \( n \) are implicitly given by

\[
h(K, \epsilon_0, \epsilon_n) = \left[\frac{1}{O_0^1} \frac{-\partial L(K, \alpha)}{\partial K_n} \frac{1}{r - \alpha_0}\right] \epsilon_0 + \left[\frac{1}{O_n^1} (-d_{K_n} \frac{1}{r - \alpha_n})\right] \epsilon_n = \kappa, \tag{5}\]

where \( r \) is the discount rate, and \( L(\cdot) \), described in Appendix A, is decreasing and convex in \( K_n \). In (5), \( O_i^1 = \beta_i^1/\beta_i^0 - 1 \) with \( \beta_i^1 > 1 \) being a constant decreasing in \( \sigma_i^2 \), \( i = 0, n \). Thus \( O_i^1 \) increases in \( \sigma_i^2 \). Further, \( O_i^1 = 1 \) if \( \sigma_i^2 = 0 \) and \( \sigma_i \leq 0 \), \( O_i^1 = r/(r - \alpha_i) \) if \( \sigma_i^2 = 0 \) and \( \alpha_i > 0 \), and \( \lim_{\sigma_i \to -\infty} O_i^1 = \infty \), \( i = 0, n \).

Since both \( L(\cdot) \) and \( d(\cdot) \) are convex in \( K_n \), \( h(\cdot) \) is decreasing in \( K_n \). Then (5) describes the following investment rule: If either \( \epsilon_0 \) or \( \epsilon_n \) is high so that \( h(K, \epsilon_0, \epsilon_n) > \kappa \), \( K_n \) should be raised to restore the equality. Higher shocks call for more investment because they raise the marginal value of investment (or the marginal reduction in the total abatement cost). However, if shocks occur so that \( h(\cdot) < \kappa \), the stock \( K_n \) cannot be reduced to raise \( h(\cdot) \) due to irreversibility. Thus, the equality in (5) defines only the barrier, or critical values of \( \epsilon_0 \) and \( \epsilon_n \), given \( K \), so that investment occurs only when \( \epsilon_0 \) and \( \epsilon_n \) are above the critical values.

Appendix B further shows that \( \partial / \partial K_i (-\partial L(\cdot)/\partial K_n) < 0 \), \( i = 1, \ldots, N \). Thus \( h_{K_i} < 0 \), \( h_{K_i \epsilon_0} < 0 \) and \( h_{K_i \epsilon_n} < 0 \), \( i = 1, \ldots, N \). As the industry stock \( K \) gets higher, a positive shock to \( \epsilon_0 \) or \( \epsilon_n \) reduces \( h(\cdot) \) by a smaller amount. In other words, when the industry stock is higher, the social planner would have less incentive to let firm \( n \) invest when positive shocks occur, even if the higher industry stock is due to the increased stock of firms other than \( n \).

Eq. (5) has an intuitive interpretation. If \( O_0^1 = O_n^1 = 1 \), the equation simply says that the marginal cost of investment, \( \kappa \), should equal the marginal benefit, which is the reduction in all future costs of abatement, including \( -\epsilon_0 \partial L/\partial K_n/r - \alpha_0 \) (the reduction in the variable cost) and \( -\epsilon_n d_{K_n}/r - \alpha_n \) (the reduction in the fixed cost). The terms \( O_0^1 > 1 \) and \( O_n^1 > 1 \) measure the option value effects: for firm \( n \) to invest, the needed industry shock is higher by the factor \( O_0^1 \) and the needed firm specific shock is higher by the factor \( O_n^1 \). Since \( O_i^1 \) increase in \( \sigma_i \), \( i = 0, n \), higher uncertainties about industry and firm specific shocks raise the barrier to invest.

Now we move from the social planner’s problem to those of individual firms. The investment barrier for firm \( n \) in (5) still applies in the competitive equilibrium, but it is
expressed as a function of the stocks of all firms. This is natural for a social planner with information about all firms. But an individual firm typically only observes its own stock, its own abatement level (i.e., its trading of the permit), and the market price of permits. The investment barrier in the competitive equilibrium should reflect this information constraint. Based on (5), Appendix B derives the following investment barrier for firm $n$ in the competitive equilibrium:

$$f(K_n, p, \epsilon_n; O_0^1, O_n^1, n) = \frac{1}{O_0^1} \frac{1}{2(r - \alpha_0)} \frac{n_{K_n}^c a_n}{K_n} p + \frac{1}{O_n^1} \frac{1}{r - \alpha_n} \frac{n_{K_n}^d d(K_n, n)}{K_n} \epsilon_n = \kappa,$$

(6)

where $n_{K_n}^c = -c_K K_n / c > 0$ is the elasticity of the variable abatement cost coefficient with respect to firm $n$'s stock, and $n_{K_n}^d = -d_K K_n / d > 0$ is that of the fixed abatement cost. To save notation, we will write $f(\cdot)$ as $f(K_n, p, \epsilon_n)$ when no confusion arises. Given $K_n$, we denote the solution to (6) in terms of $p$ and $\epsilon_n$ as $(p_b^1(K_n), \epsilon_b^1(K_n))$ and call it the investment barrier under permits (Fig. 1). We use the following convention to compare different barriers.

**Definition 1.** (Ordering of investment barriers). For a given level of $K_n$, firm $n$'s investment barrier $(p_b^1, \epsilon_b^1)$ is said to be higher (or lower) than barrier $(p_b^2, \epsilon_b^2)$ if $f(K_n, p_b^1, \epsilon_b^1) >$ (or $<$) $f(K_n, p_b^2, \epsilon_b^2)$.

Since $f_p > 0$, $f_{\epsilon_n} > 0$, barrier $(p_b^1, \epsilon_b^1)$ is higher than $(p_b^2, \epsilon_b^2)$ if $p_b^1 > p_b^2$ and $\epsilon_b^1 > \epsilon_b^2$. Definition 1 goes beyond the ordering of the two corresponding elements of the barriers, and enables a complete ordering of all possible barriers.

![Diagram](Fig. 1. Investment barrier under TEPs.)
Appendix B shows that \( \eta^{e}_{K_n} \alpha_n/K_n \) and \( \eta^{d}_{K_n} d/K_n \) are both decreasing in \( K_n \). Then \( f_{K_n} < 0 \), \( f_{K_n} \alpha_n < 0 \) and \( f_{K_n} \epsilon_n < 0 \). At any moment, the permit price is determined in (3) through efficient permit trading. When a new industry shock occurs, and before the firms invest, permit price \( p \) changes in proportion to the change in \( \epsilon_n \). Eq. (6) thus indicates that whenever industry and/or firm shocks occur so that \( p \) and \( \epsilon_n \) are such that \( f(K_n, p, \epsilon_n) > \kappa \), or if \((p, \epsilon_n)\) is higher than the firm’s investment barrier, instantaneous investment is undertaken by firm \( n \) to restore the equality to (6). No investment occurs when \((p, \epsilon_n)\) is lower than the barrier. It is intuitive that the firm wishes to invest when the industry shock raises \( p \). Investment allows it to abate more and sell more (or buying fewer) permits, thus it is more willing to invest if the permit price is high. Similarly, when \( \epsilon_n \) is high, higher stock \( K_n \) reduces the fixed abatement cost by a bigger amount.

Let \( \hat{K}_n(p, \epsilon_n) \) be the optimal stock level when firm \( n \) invests given \( p \) and \( \epsilon_n \). That is, \( f(\hat{K}_n, p, \epsilon_n) = \kappa \) and \( \hat{K}_n > K_n \), \( \hat{K}_n(p, \epsilon_n) - K_n \) measures the level of firm \( n \)’s investment when \( p \) is fixed (we will show later that \( p \) will change with strictly positive probability). Appendix B shows that

Proposition 2. (i) Under TEPs, firm \( n \)’s investment barrier is increasing in the cost of capital \( \kappa \), the current stock \( K_n \), and the uncertainty levels of both the industry and its own shocks. (ii) For two firms \( n \) and \( m \) with the same initial stocks, shocks and firm specific uncertainties, i.e., \( K_n = K_m, \epsilon_n = \epsilon_m \), and \( \sigma^2_{n} = \sigma^2_{m} \), firm \( n \)’s barrier is higher if in equilibrium \( a_n < a_m, \eta^e_{K_n} < \eta^e_{K_m}, \eta^d_{K_n} < \eta^d_{K_m}, \) or \( d(K_n, n) < d(K_m, m) \). (iii) Suppose \((p, \epsilon_n)\) is such that firm \( n \) invests. Its level of investment \( \hat{K}_n(p, \epsilon_n) - K_n \) is inversely related to its barrier, i.e., its responses to the factors in (i) and (ii) are opposite to the responses of the investment barrier.

As an example, Fig. 1 shows how the barrier shifts or rotates out when \( \kappa \) or \( \sigma^2_\epsilon \) is higher. As the barrier increases, the set of possible values of \( p \) and \( \epsilon_n \), for which firm \( n \) invests is smaller. If the rise in the barrier is caused by factors other than higher uncertainties \( \sigma^2_\epsilon \) or \( \sigma^2_\eta \), the distribution functions of \( p \) and \( \epsilon_n \) are unchanged and the probability that firm \( n \) invests becomes lower. However, higher \( \sigma^2_\epsilon \) or \( \sigma^2_\eta \) may not reduce a firm’s probability of investing even though they increase its investment barrier (through raising the option value coefficients \( O^0_\epsilon \) and \( O^1_\epsilon \)). Sarkar (2000), and Dixit and Pindyck (1994) showed that as the uncertainties in \( \epsilon_0 \) or \( \epsilon_n \) rise, \( \epsilon_0 \) or \( \epsilon_n \) becomes more random and the investment barrier may be ‘hit’ more frequently even though the barrier itself is higher, raising the probability of investment. That is, \( \Pr\{f(K_n, p, \epsilon_n) > f(K_n, p^\perp(K_n), \epsilon^\perp_n(K_n))\} \) may actually rise as \( \sigma^2_\epsilon \) or \( \sigma^2_\eta \) increases. Whether or not this scenario arises depends on the functional forms and parameter values of our model. Since our paper is motivated by the concern that uncertainty reduces investment, we restrict ourselves to cases where this probability is decreasing in \( \sigma^2_{i}, i = 0, n \). That is, we only consider scenarios where as the shocks become more volatile, the probability that firm \( n \) will invest decreases. For these scenarios, the investment barrier is always monotonically related to the probability of investment. That is, (i) and (ii) in Proposition 2 translate directly into how the firm’s probability of investing responds to the exogenous factors.

Proposition 2(iii) shows how the firm’s investment responds to the exogenous factors given the permit price. However, the price, and thus firm \( n \)’s actual level of investment,
depend on other firms’ investment. If many firms invest, the industry marginal abatement cost decreases, lowering \( p \), thereby reducing firm \( n \)’s investment level. Let \( \mathcal{I} \subset \{1, \ldots, N\} \) denote the set of firms which invest and \( K \) be the stock levels before any firm invests. The new equilibrium is reached when \( f(\hat{\mathcal{I}}, p^*, \epsilon_j) = \kappa, i \in \mathcal{I}, \) and \( f(K_j, p^*, \epsilon_j) \leq \kappa, j \notin \mathcal{I}, \) where \( p^* \) is the new equilibrium permit price. Firm \( n \)’s investment is then \( \hat{K}_n(p^*, \epsilon_n) - K_n \).

At this point, we formally define the concept of investment incentive used in this paper. Given \( K_n \) and the shocks \( \epsilon_0 \) and \( \epsilon_n \) firm \( n \)’s equilibrium level of investment depends on the investment of other firms, or other firms’ shocks \( \epsilon_{-n} \) and stocks \( K_{-n} \). Its expected investment level is given by \( EI_n(K_n, \epsilon_0, \epsilon_n) = E_{\epsilon_{-n}}(\hat{K}_n(p^*, \epsilon_n) - K_n)1(\epsilon \in \mathcal{I}), \) where 1(·) is the indicator function, taking the value of one if its argument is true and zero otherwise. Then

**Definition 2.** (Investment incentive)). Given \( K_n, \epsilon_0 \) and \( \epsilon_n \), firm \( n \)’s investment incentive is represented by \( EI_n(K_n, \epsilon_0, \epsilon_n) \). The incentive is said to be higher as \( EI_n(K_n, \epsilon_0, \epsilon_n) \) increases.

If an exogenous factor only affects firm \( n \)’s investment, price \( p \) will not change and Proposition 2 applies. For example, if \( \sigma_n^2 \) rises, firm \( n \)’s investment barrier rises, or its probability of investment decreases, and the level of investment also goes down when it does invest. Thus, its investment incentive decreases in \( \sigma_n^2 \). However, if the factor affects all firms, such as an increase in \( \kappa \) or \( \sigma_0^2 \), the equilibrium price \( p^* \) will change and results in Proposition 2 will not apply. For example, since firms are heterogeneous, it is conceivable that in equilibrium, responding to the shocks, certain firms may actually invest more if \( \sigma_0^2 \) is higher, if other firms invest much less due to the higher \( \sigma_0^2 \).

Readers familiar with the real options literature will notice that the option value coefficients \( O_0 \) and \( O_n \) in (5)–(6) are similar to those in an individual firm’s optimal investment rules facing exogenously given random price processes. In fact, as we will show in the next section, the barrier in (6) is the same as the one in (9) where the firm faces the price process \( p(t) = C(K_n, n)\alpha_n \epsilon_0(t) = C(K_m, m)\alpha_m \epsilon_0(t) \) with \( K_n \) and \( K_m \) fixed through time (cf. (3)). That is, in determining its investment strategy, the firm can ignore the effects of other firms’ investment on the permit price in the future, and act as if the price is determined entirely by the random variable \( \epsilon_0(t) \), starting at the current price level. Of course, as other firms invest, the current permit price changes. Then the firm adjusts its ‘perceived’ price process so that it starts at the new current level, with the future price still affected only by \( \epsilon_0(t) \). In the terminology of Leahy (1993), Caballero and Pindyck (1996), and Baldursson and Karatzas (1997), the firm can be myopic in that it ignores the role of other firms in the future. The literature found that this kind of myopic behavior is optimal when the firms in a general equilibrium model face common uncertainties. Our results indicate that the optimality also carries over to independent idiosyncratic uncertainties (i.e., \( \epsilon_n \)’s).

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5 Leahy (1993) observed that ‘it is an open question as to whether a version of the myopic result still holds’ with idiosyncratic shocks (p. 1125). He further commented that much depends on how the firm specific shocks enter the model.
Intuitively, the possibility of other firms investing reduces a firm’s option value of being able to delay the investment and gathering more information about the shocks: Other firms may invest and grab the investment opportunity if this firm waits for too long. However, this possibility also reduces the firm’s payoff if it actually invests. The option value relative to the value of the investment remains unchanged, even though both go down in absolute terms. As a result, the investment barrier, which depends on the comparative values of the option and the investment, is unaffected by the existence of other firms. In fact, the myopic behavior is parallel to firms’ optimal decisions in static general equilibrium models. In a static world, an individual firm can ignore other firms and respond only to the current market price. Of course, others affect this firm, but only indirectly through influencing the market price. Similarly, in our model, a firm can also ignore others and respond only to a price process. The current period actions of other firms affect this firm only through changing the ‘starting point’ of the price process.

4. Optimal investment under emission charges

We now turn to the policy of an abatement subsidy (or a tax) that remains constant over time, and compare firms’ investment barrier and investment incentive under the tax and permit policies. Under the tax/subsidy system, each firm’s abatement and investment decisions are independent of those of other firms, since the payoff from abatement (through increased subsidy) is determined by the fixed subsidy rate. The model is simpler without the general equilibrium requirement: we only need to study how a representative firm responds to the shocks $e_0$ and $e_n$.

Let $\tilde{\alpha}_n$ be the rate of abatement subsidy. In each period, given $K_n$, firm $n$’s decision on abatement level is given by

$$\max_{\alpha_n} \frac{1}{2} c(K_n, n)\alpha_n^2 - d(K_n, n)\alpha_n + \tau\alpha_n,$$

which implies that $\tilde{\alpha}_n = \tau/c(K_n, n)e_0$. To make the tax and permit policies comparable, we set $\tau = p^e(K(0), \epsilon_0(0), \tilde{\alpha})$, where $p^e(\cdot)$, described in (A.4) in Appendix A, is the equilibrium permit price given the initial stocks $K(0)$. From (3), we know $\tilde{\alpha}_n = a^*_n$: given the current stocks and shocks, the two policies lead to the same abatement level for all firms. We argue later that this particular way of normalization does not affect our major results.

Since $\tau$ is fixed, a shock in $\epsilon_0$ would change the abatement level $\tilde{\alpha}_n$ even without affecting the stock $K_n$. However, we noted in (3) that under the permit policy, an industry shock leads to a proportional permit price change and does not affect the abatement level $a^*_n$ prior to the firm $n$’s investment. The difference arises because tax is a price tool and permit is a quantity tool (Weitzman, 1974). Under different tools, the firm reacts differently to shocks in its marginal abatement cost. Consequently, the firm would have different incentive to invest. To compare the investment incentives, we construct a fictitious subsidy policy where the subsidy rate would fluctuate directly with the industry shock: $s(t) = b\epsilon_0(t)$ with $b = \tau/\epsilon_0(0)$ being fixed. Whenever $\epsilon_0(t)$ changes, the fictitious subsidy rate $s$ changes proportionally, similar to the permit price in (3). However, since $b$ is
fixed, firms’ investments will not affect the level of \( s \). Thus, the only difference between policy \( s \) and the permit policy lies in the fact that under permits, firms’ investment and abatement determine the permit price \( p^* \) in a general equilibrium. We thus say that the difference in firms’ investment incentives under \( \tilde{\alpha} \) and \( s \) is due to the general equilibrium effect. On the other hand, \( s \) and \( \tau \) are similar in that their levels are independent of firms’ abatement and investment. Their difference lies in that the constant \( b \) fixes the ‘real’ marginal cost, and thus the abatement level, of each firm, regardless of the industry shock. That is, the fictitious policy \( s \) is in fact a quantity tool. We thus say that the difference in investment incentives under \( s \) and \( s \) is due to the price vs. quantity effect. Notice that \( s \) is entirely fictitious: it does not reflect any real world policies, and is constructed only to help distinguish between the two effects.

4.1. The general equilibrium effect

Substituting \( \tilde{\alpha}_n = b/c(K_n, n) \) into (7), we obtain the firm’s per period payoff under subsidy \( s = b\epsilon_0 \) as

\[
S_n(K_n, \epsilon_n, \epsilon_0, b) = \frac{1}{2} \frac{b^2}{c(K_n, n)} \epsilon_0 - d(K_n, n) \epsilon_n.
\]  

The payoff increases in \( \epsilon_0 \). \( s = b\epsilon_0 \) means that a higher industry shock raises the subsidy rate \( s \) the firms receive. Adopting the same approach as the social planner’s problem in the last section, we get the firm’s investment barrier

\[
f(K_n, s, \epsilon_n; O_0^1, O_n^1, n) = \kappa,
\]  

where \( f(\cdot) \) is given in (6). Given \( K_n \), the solution to (9) represents the investment barrier under the fictitious policy, denoted as \( (s^*(K_n), \epsilon_n^*(K_n)) \). When \( (s, \epsilon_n) \) is higher than the barrier, i.e., if \( s = b\epsilon_0 \) and \( \epsilon_n \) are such that \( f(K_n, b\epsilon_0, \epsilon_n) > \kappa \), firm \( n \) will invest to restore the equality. Let \( \tilde{K}_n(s, \epsilon_n) \) be the optimal level of capital stock when firm \( n \) invests, i.e., \( f(\tilde{K}_n, b\epsilon_0, \epsilon_n) = \kappa \), and let \( \tilde{K}_n(s, \epsilon_n) - K_n \) be the investment incentive under the fictitious subsidy. Since the level of \( s \) is fixed regardless of firms’ investment, Proposition 2 can be directly adopted for policy \( s \), where the level of investment is also the investment incentive. That is,

**Proposition 3.** Firm \( n \)’s investment barrier is the same under permits and the fictitious tax: given \( K_n \), \( p^b = s^b \) and \( \epsilon_n^b = \epsilon_n^f \), \( n = 1, \ldots, N \). Further, the barrier \( (s^b, \epsilon_n^b) \) and the investment incentive \( \tilde{K}_n(s, \epsilon_n) - K_n \) have the characteristics specified in Proposition 2.

The proposition confirms our earlier discussion that under permits, the firm can ‘pretend’ that the permit price is exogenously set at a level proportional to \( \epsilon_0 \) and ignore the effects of the firms’ investment on the permit price. However, identical investment barriers do not lead to the same investment levels under the two policies. Under TEPs, when some or all firms invest, the current period permit price \( p \) decreases to the new level \( p^* < p \), reducing \( \tilde{K}_n \), while \( s \) remain unchanged. In fact, if abatement costs are constant over time (i.e., no uncertainty), the coefficients \( O_i^1 = 1 \) and \( \alpha_i = 0 \), for \( i = 0, n \). Then
\( \hat{K}_n(s, \varepsilon_n) - \hat{K}_n(p^*, \varepsilon_n) \) corresponds precisely to the deterministic analysis in Milliman and Prince (1989) and Jung et al. (1996). That is, the difference in investment levels under \( s \) and \( \bar{a} \) is due entirely to the fact that \( p^* \) is determined endogenously in a general equilibrium. Our purpose is to investigate how this difference depends on the uncertainty levels of \( \varepsilon_0 \) and \( \varepsilon_n \).

At time \( t \), suppose shocks occur and \( f(K_n, p, \varepsilon_n) = f(K_n, s, \varepsilon_n) > \kappa \) so that firm \( n \) decides to invest. Since the firm specific shocks are independent, at this instant there is a strictly positive probability that some other firms will invest (as long as \( K < \infty \)). Strictly speaking, the probability of investment by other firms is

\[
\Pr \{ f(K_i, p, \varepsilon_i) = f(K_i, s, \varepsilon_i) > \kappa, \text{ for some } i \neq n, i = 1, \ldots, N \} = 1 - \prod_{i \neq n} \Pr \{ f(K_i, p, \varepsilon_i) = f(K_i, s, \varepsilon_i) \leq \kappa \} > 0. \tag{10}
\]

That is, whenever firm \( n \) needs to invest, there is a strictly positive probability that other firms also invest, reducing the permit price \( p \). As \( p \) goes down to the new equilibrium price \( p^*, f(K_n, p^*, \varepsilon_n) < f(K_n, s, \varepsilon_n) \), and the investment level of firm \( n \) will be smaller under permits than under policy \( s \) with strictly positive probability. Thus,

**Proposition 4.** Given \( K_n, \varepsilon_0 \) and \( \varepsilon_n \), firm \( n \)'s investment incentive under permits, \( EI_n(K_n, \varepsilon_0, \varepsilon_n) \), is smaller than that under the fictitious policy, \( \hat{K}_n(b\varepsilon_0, \varepsilon_n) - K_n \).

Suppose the uncertainty level \( \sigma_i^2 \) increases for all \( i = 0, 1, \ldots, N \). Proposition 2 and subsequent discussion imply that the investment barriers of all firms rise and the probability in (10) decreases. Then firm \( n \)'s (expected) investment level under permits will be closer to that under fluctuating subsidy.\(^6\) In the extreme, if \( \sigma_i^2 \to \infty, i = 0, 1, \ldots, N \), no firm will invest and the investment paths are identical under the two policies. Uncertainty thus reduces the magnitude of \( (\hat{K}_n(b\varepsilon_0, \varepsilon_n) - K_n) - EI_n(K_n, \varepsilon_0, \varepsilon_n) \).

**Proposition 5.** The higher investment incentive under the fictitious subsidy over the permits is reduced, but not eliminated, by the industry and/or firm-specific cost uncertainties.

### 4.2. The price-vs.-quantity effect

Substituting \( \hat{a}_n = \tau/c(K_n, n)\varepsilon_0 \) into (7), we know firm \( n \)'s instantaneous payoff rate under subsidy \( \tau \) is

\[
T_n(K_n, \varepsilon_n, \varepsilon_0, \tau) = \frac{\tau^2}{2 (c(K_n, n)\varepsilon_0)} - d(K_n, n)\varepsilon_n. \tag{11}
\]

\(^6\) In fact, as discussed earlier, \( \hat{K}_n(p^*, \varepsilon_n) \) may in fact increase when \( \sigma_0^2 \) is higher because \( p^* \) is higher, even though \( \hat{K}_n(s, \varepsilon_n) \) is always lower as \( \sigma_0^2 \) rises.
In contrast to \( s_n(\cdot) \) under policy \( s \) (Eq. (8)), here under \( s \) the payoff is decreasing and convex in the industry shock. Further, higher industry shock \( \epsilon_0 \) reduces the effectiveness of investment in raising the payoff. The difference in how \( \epsilon_0 \) affects the costs in (8) and (11) is due to our assumption that \( \epsilon_0 \) enters the abatement cost linearly (Eq. (1)). Adopting the same approach as in the last section, we find firm \( n \)'s investment barrier as

\[
g(K_n, \epsilon_0, \epsilon_n; O_0^2, O_n^1, \sigma_0^2, n) = \frac{O_0^2}{2(r - \sigma_0^2 + \alpha_0)} \frac{\tau^2}{c(K_n, n)} \frac{\eta_{K_n}^c}{K_n} \frac{1}{\epsilon_0} + \frac{1}{O_n^2} \frac{1}{r - \alpha_n} \frac{\eta_{K_n}^d}{K_n} \frac{d(K_n, n)}{\epsilon_n} = \kappa \quad (12)
\]

where \( O_0^2 = \beta_0^2 + 1/\beta_0^2 > 0 \) is the option value coefficient, and \( g_K < 0, g_{\epsilon_0} < 0 \) and \( g_{\epsilon_n} > 0 \). Let \((\epsilon_0^*(K_n), \epsilon_n^*(K_n))\) denote the solution to (12) given \( K_n \), or the investment barrier (Fig. 2). When \((\epsilon_0, \epsilon_n)\) exceed the barrier or when negative industry and/or positive firm shocks occur so that \( g(K_n, \epsilon_0, \epsilon_n) > \kappa \), instantaneous investment is undertaken to restore the equality. Let \( \tilde{K}_n(\epsilon_0, \epsilon_n) \) be the optimal level of capital stock when firm \( n \) invests, and let \( \tilde{K}_n - K_n \) be the corresponding investment incentive. Again, since the subsidy rate \( \tau \) is fixed, there is a negative relationship between the investment barrier and the incentive, as in the case of policy \( s \).

For the investment to be finite, we impose the condition that \( r > \sigma_0^2 - \alpha_0 \) (Appendix C). Then we can show that \( \beta_0^2 < -1 \) and is increasing in \( \sigma_0^2 \). That is, \( 0 < O_0^2 < 1 \) and is decreasing in \( \sigma_0^2 \). From (12), we know

![Fig. 2. Investment barrier under tax \( \tau \).](image-url)
Proposition 6. Under policy \( \tau \), firm \( n \)'s investment incentive is decreasing in the cost of capital \( \kappa \), the current stock \( K_n \), and the uncertainties in both the industry and firm shocks. The incentive is increasing in the subsidy level \( \tau \), and the effectiveness of investment in reducing the costs.

To compare how investment incentives \( \Delta K_n = K_n - \tilde{K}_n \) and \( \Delta \tilde{K}_n \) respond to uncertainties, it suffices to compare how \( \Delta K_n(\epsilon_n, \epsilon_n) \) and \( \Delta \tilde{K}_n(\epsilon_n, \epsilon_n) \) change as \( \sigma_n^2 \) and/or \( \sigma^2 \) increase. In particular, we compare two elasticity measures \( \eta_{\sigma^2}^{K_n} = (d\Delta K_n/d\sigma^2_n) / (\Delta K_n / \sigma^2_n) \) and \( \eta_{\sigma^2}^{\tilde{K}_n} = (d\Delta \tilde{K}_n/d\sigma^2_n) / (\Delta \tilde{K}_n / \sigma^2_n) \), \( i = 0, \ldots, n \).\(^7\) The magnitude of the elasticities depend on \( K_n, \epsilon_0 \) and \( \epsilon_n \). Similar to Weitzman (1974), we conduct a local rather than a global comparison. In particular, we compare at the point where \( \Delta K_n = \Delta \tilde{K}_n \), i.e., when the shocks are such that firm \( n \) invests the same amount under the policies:

\[
f(\Delta K_n, b\epsilon_0, \epsilon_n; O_0^1(\sigma_0^2), O_n^1(\sigma_n^2), n) = g(\Delta \tilde{K}_n, \epsilon_0, \epsilon_n; O_0^2(\sigma_0^2), O_n^2(\sigma_n^2), \sigma_n^2, n) = \kappa
\]

Appendix D shows that

\[
\eta_{\sigma^2}^{\Delta K_n} = \eta_{\sigma^2}^{\Delta \tilde{K}_n}, \quad \eta_{\sigma_n^2}^{\Delta K_n} = \eta_{\sigma_n^2}^{\Delta \tilde{K}_n} \propto \eta_{0}^\tau - \eta_0^\tau
\]

where \( \eta_0^\tau = \eta_0^\tau - \sigma_0^2 / (r - \sigma_0^2 + \chi_0) \), with \( \eta_0^\tau = (\partial O_0^1 / \partial \sigma_0^2) / (O_0^1 / \sigma_0^2) \) and \( \eta_0^\tau = -(\partial O_0^2 / \partial \sigma_0^2) / (O_0^2 / \sigma_0^2) \) being the elasticities of the two option value coefficients \( O_0^1 \) and \( O_0^2 \). Since firm shock \( \epsilon_n \) does not affect the abatement level, the impacts of \( \sigma_n^2 \) on the two investment incentives are the same. Since \( \sigma_n^2 \) affects function \( f(\cdot) \) through \( O_0^1 \) and \( g(\cdot) \) through \( O_0^2 / (r - \sigma_0^2 + \chi_0) \), at \( \tilde{K}_n = \tilde{K}_n \) the elasticities of the two terms with respect to \( \sigma_n^2 \) fully capture the responses of the investment incentives to the industry uncertainty. Intuitively, under \( s \), the only reason that a risk neutral firm (whose payoff is linear in \( \epsilon_0 \)) cares about the cost uncertainty is the existence of the option value of delaying the investment. For policy \( \tau \), there is an added effect due to the ‘curvature’ of the payoff function: it is convex in \( \epsilon_0 \). Thus higher uncertainty raises a firm’s investment payoff through this curvature effect, offsetting (partially) the option value effect.

It is difficult to compare \( \eta_0^1 \) and \( \eta_0^2 \) analytically, even though we know their functional forms. Numerical examples indicate that \( \eta_0^1 < \eta_0^2 \), especially when uncertainty level is high.\(^8\) Fig. 3 shows the three elasticity measures for the case of \( r = 0.085 \) and \( \chi_0 = 0.02 \).

\(^7\) In comparing the incentives under permits and policy \( s \), we can show that Proposition 5 directly extends to comparison of elasticities.

\(^8\) The numerical result is robust to different parameter values, as long as \( \epsilon_0(t) \) follows a geometric Brownian motion. It may change if other stochastic processes are assumed. Intuitively, \( O_0^1 \) measures the option value effect of an irreversible investment to obtain a project of uncertain value, and \( O_0^2 \) measures that of irreversibly abandoning this project. Thus, \( O_0^1 \) depends on the ‘downside risk’ that the project’s value will be too low, and \( O_0^2 \) hinges on the ‘upside risk’ that the value is high. Since the randomness described by the Wiener process \( dz(t) \) is symmetric, the absolute responses of \( O_0^1 \) and \( O_0^2 \) to uncertainty \( \sigma^2 \) can be similar. However, since \( O_0^2 < O_0^1 \) (they start at the same level when \( \sigma^2 = 0 \) and \( O_0^1 \) increases while \( O_0^2 \) decreases as \( \sigma^2 \) rises), the elasticity of \( O_0^2 \)’s response tends to be higher than that of \( O_0^1 \). Thus, the result that \( \eta_0^1 < \eta_0^2 \) may also carry to other stochastic processes of \( \epsilon_0(t) \).
Panel (a) shows the comparison of $\eta_0^1$ and $\eta_0^2$. Thus, based solely on option values, uncertainty reduces the investment incentive proportionally more under fixed tax \( \tau \) than under fluctuating tax \( s \).

Under \( \tau \), the curvature factor encourages investment, and mitigates the effects of uncertainty in retarding investment. This factor is decreasing in \( r \) and \( a_0 \) and increasing in \( r^2 \). Since we imposed a limit on the uncertainty level (i.e., \( r > \sigma_0^2 - a_0 \)), the curvature factor cannot fully offset the option value effect. But as \( r \) and \( a_0 \) decrease and uncertainty increases, the curvature factor becomes more important (Fig. 3(b)). In summary, we know

**Proposition 7.** The price-vs.-quantity effect exists only for the industry shock which affects the marginal abatement costs. Based on the option value effect, increased uncertainty reduces the investment incentive proportionally more under \( \tau \) than under \( s \). However, under \( \tau \), higher uncertainty may raise the incentive due to the curvature effect, and this effect becomes more significant as the uncertainty level increases and \( r \) or \( a_0 \) decreases.

Combining Propositions 5 and 7, we know that except for the curvature effect of \( \epsilon_0 \) affecting the abatement cost linearly, firms’ investment incentive decreases more under policy \( \tau \) than under permits.

5. Discussion and generality of the model

There are a number of assumptions that helped us obtain the analytical results but also made our model somewhat special. In this section, we show that these assumptions do not change our major conclusions. One special feature of our model concerns how the shocks affect the variable and fixed parts of the abatement cost, shown in (1). In general, industry and firm shocks should affect both the fixed and variable abatement costs. We can easily extend the model to let the industry shock affect the fixed cost as well. We will obtain a similar investment barrier to (6). In fact, if there is perfect correlation among \( \epsilon_0 \) and \( \epsilon_n \), \( n = 1, \ldots, N \), (6) describes the barrier for firm \( n \) facing the industry shock alone that
affects both its variable and fixed cost. We assumed away the fixed cost effect of the industry shock mainly to reduce clutter.

The model becomes much more complicated if we allow the firm shock to affect the variable and marginal abatement costs. The social planner's problem becomes impossible to solve. We can apply Leahy (1993) and Caballero and Pindyck (1996) and solve the myopic firm's investment strategy. Then we obtain an investment barrier similar to (6), except that now the uncertainty’s effect on the investment level becomes ambiguous. In addition to the option value effect captured by the option value coefficient, there is also the price-vs.-quantity effect because each firm takes the permit price as a constant independent of the firm specific shock. If the option value effect dominates the price-vs.-quantity effect, our major results still hold. By assuming away the firm shocks from the variable cost, we are able to eliminate the price-vs.-quantity effect, and highlight the interaction of the option value and the general equilibrium effects.

The variable and marginal abatement costs are assumed to be linear in the industry uncertainty. This assumption influences the price-vs.-quantity effect in comparing the fixed and fluctuating tax policies, since an important part of the effect is driven by the ‘curvature’ of the payoff function. For example, if the payoff function under policy $s$ is convex in $\epsilon_0$, investment will decrease less as uncertainty rises. Therefore, the curvature factor in the price-vs.-quantity effect is not a general result, even though the option value factor can be extended to other functional forms.

We assumed linear investment cost and no capital or technological depreciation. Introducing depreciation complicates the derivation, since even with independent shocks, the optimal investment strategy will be characterized by a partial differential equation with free boundaries, which is notoriously difficult to solve analytically. It will not change our major results, since depreciation will not remove the existence of option values (Abel and Eberly, 1997). Linear investment cost is responsible for the barrier control strategy, and the investment path would be differentiable in time if a convex investment cost function is assumed. Our chief result, however, is not the barrier control strategy itself. Our interest is in the impacts of uncertainty on investment incentive under different policies. These results are not likely to change even if we assume more general cost functions. For example, Abel and Eberly (1994) showed in a partial equilibrium model with a general adjustment cost function that uncertainty reduces investment.

We compared tax and permit policies that are fixed overtime, even as firms invest and the abatement costs change. Such comparison in part reflects the reality that these policies are only infrequently changed in response to the new abatement costs. If the government does adjust the policies whenever firms invest, the firms will anticipate future policy changes and will respond accordingly. The resulting dynamic game is difficult to analyze. Biglaiser et al. (1995) found that first best policies will typically not be dynamically consistent. An interesting future research topic is to compare the two policies in a game setting.

As long as the tax and permit policies are fixed overtime, the specific levels of the tax rate and permit quantity do not matter in comparing the effects of cost uncertainties on investment incentive under the two policies. In (6), (9) and (12), we can change $\tilde{a}$ (or $p$), $b$ (or $s$) and $\tau$, and the directions of the effects of uncertainties remain unchanged. Further, the general equilibrium effect still goes down as the uncertainties increase, as the expected
investment of other firms still decreases, leading to a smaller reduction in the permit price. In our model, we normalized the two policies so that they lead to the same abatement level in the current period. We could normalize them at other quantities, without altering our results. Possible candidates include setting \( \bar{a} \) and \( \tau \) so that the two policies lead to the same expected total emission overtime, or achieve the same expected social benefit, given a pollution damage function.

Following the neoclassical theory of induced innovation (Newell et al., 1999) and the literature on abatement investment, such as Magat (1978), Milliman and Prince (1989), Jung et al. (1996), Farzin et al., 1998) and Farzin and Kort (2000), we only addressed the positive question of firms’ investment incentive under TEPs and taxes, without attempting to answer the normative question of which policy is more efficient. Nevertheless, there seems to be a long-standing consensus among (at least) environmental economists that an efficient environmental policy should encourage firms, in the long run, to invest in abatement capital or technology (see, for example, Kneese and Schultze, 1975; Orr, 1976; Kemp and Soete, 1990). From a purely theoretical standpoint, investment decisions and policy efficiency do not have to be related. After all, it is the environmental externality that the policy is trying to correct. If the policy successfully does so and if there is no distortion in other sectors of the economy, investment decisions should be left to the firms themselves and should be determined by market forces. That is, environmental policy should not even attempt to influence firms’ investment incentive.

To the best of our knowledge, there does not exist a formal investigation into why environmental policies should encourage such investment. There is, however, some evidence that indicates possible explanations. If traditionally environmental externalities have been ‘under-regulated’ in the sense that the policies have corrected only part of the externalities, more investment helps reduce the ‘inefficiency’ of these policies by ameliorating the environmental problem and the need for strict regulation. That is, in the long run, (lax) environmental regulation that encourages more investment should be more efficient. Another possibility is that regulators may be subject to ‘hold-up’ by firms who anticipate less strict regulation if they do not invest and thus keep their abatement expensive (Gersbach and Glazer, 1999). In this case, policies that encourage investment help reduce this hold-up problem, and tend to be more efficient. Further, there are typically market failures in the technology adoption and diffusion process, and policies that encourage adoption help overcome these failures (Jaffe et al., forthcoming). For example, when information spillover exists from adopters of new technologies to potential adopters, there is less than socially optimal adoption. Empirically, firms have been perceived not to be willing to invest up to the socially optimal level, leading in part to the introduction of ‘technology-forcing’ regulation in certain cases (such as mobile source air pollution). The relevance of our paper for policy analysis should be viewed in this broad context of

\[\text{Kneese and Schultze (1975) argued that in the long run, ‘perhaps the most important single criterion on which to judge environmental policies is the extent to which they spur new technologies toward the efficient conservation of environmental quality’ (p. 82).}\]

\[\text{While people may disagree about whether we have too much or too little regulation, the fact that many environmental problems are getting worse over time and new regulations are constantly being introduced does point to the possibility of insufficient regulation.}\]
regulation that targets the environmental externality itself and (indirectly) the long-run investment incentive.

6. Conclusion

Abatement cost uncertainties reduce firms’ investment incentive under both TEPs and emission charges, but more so under the charges, if the investment is irreversible and increased investment barrier reduces the investment probability, and after we control for the assumption that the uncertainties enter the cost function linearly. Since with cost certainty, the investment incentive is higher under charges, we conclude that the advantage of charges over TEPs in encouraging more investment decreases in the uncertainties. In other words, tradable permits in fact help maintain firms’ investment incentive under uncertainty.

If the permit trading itself is imperfect and is subject to significant random shocks, investment incentive will be adversely affected under tradable permits. This effect is over and above that of abatement cost uncertainty that we have identified in this paper. It is an interesting and important empirical question to determine, for particular emissions and permit markets, the relative magnitude of the various sources of shocks.

We have ignored the normative issue of optimal policy design, taking the (most likely inefficient) fixed permits or fixed charge policies as given. Therefore, a policy that encourages investment incentive is not necessarily the more efficient policy. Of course, if there is no distortion in the capital and R&D sectors, the permit policy is efficient if the damage function of the emissions increases from sufficiently low levels to sufficiently high levels at the permit amount $\bar{e}$. The charge policy is efficient if the marginal damage is constant at the charge level $\tau$. An interesting extension of our model is to investigate the optimal policies when the damage function is of a more general form.

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Appendix A

Permit market and investment equilibria

Competitive equilibria

Given any permit price $p$, firm $n$’s total cost (including TAC and permit cost) is given by

$$D(p, K_n, n, \epsilon_n, \epsilon_0) = C(a_n(p, K_n, n, \epsilon_0), K_n, n, \epsilon_n, \epsilon_0)$$

$$+ p(\epsilon_n^0 - a_n(p, K_n, n, \epsilon_0) - \bar{\epsilon}_n).$$  \hspace{1cm} (A.1)
The firm can invest to increase its stock $K_n$ and thus to reduce the total cost $D(\cdot)$. To specify the inter-temporal competitive equilibrium of the investment, suppose there is a permit price process $\{p(t), t \geq 0\}$. Firm $n$’s optimal investment decision is given by

\[
V(p(t), K_n(t), n, e_n(t), e_0(t)) = \max_{K_n} - E \int D(p(\tau), K_n(\tau), n, e_n(\tau), e_0(\tau)) e^{-r(\tau-t)} d\tau - \sum_w \kappa(K_n(w^+) - K_n(w^-)) e^{-r(w-t)}, \tag{A.2}
\]

subject to (2), the price process $p(t'), t' \geq t$, and $K_n(w^+) > K_n(w^-)$. The discount rate is $r$, and $w$’s are the instants when investment occurs, with $w^-$ and $w^+$ representing the instants just before and after the investment.

Given $K_n(t)$, (A.2) generates the optimal investment strategies

\[
K_n^*(t) = K_n^*(p(t), K_n(t), e_n(t), e_0(t)), \quad n = 1, \ldots, N. \tag{A.3}
\]

From (3), the rational expectations competitive equilibrium price is given by

\[
p^*(t) = p^*(\{K_n^*(t)\}_{n=1}^N, e_0, \bar{a}). \tag{A.4}
\]

Eqs. (A.3) and (A.4) completely characterize the competitive equilibrium. Note that since $e_n(t)$ and $e_0(t)$ are Markovian, the resulting $\{p^*(t), K_n^*(t)\}$ are also Markovian.

The fictitious social planner’s problem

The fictitious planner maximizes the aggregate firm payoffs. From (A.1) we know

\[
\sum_n D(p, K_n, n, e_n, e_0) = \sum_n C(a_n, K_n, n, e_n, e_0).
\]

That is, when all permits \( \bar{e} \) are freely distributed by the government, the social planner can simply minimize the total expected abatement cost:\footnote{If some of the permits are auctioned at the market price, the equivalent fictitious planner’s objective function must include the cost of purchasing these permits. The analysis becomes more complicated because the marginal abatement cost (representing the permit price) enters the objective function directly.}

\[
\max_{a, \bar{a}} -E \int_0^\infty \sum_n C(a_n(t), K_n(t), n, e_n(t), e_0(t)) e^{-rt} dt
- \sum_w \sum_n \kappa(K_n(w^+) - K_n(w^-)) e^{-rw}
\]
\[
\text{subject to } \sum_n a_n(t) = \bar{a}, \quad \text{Eq.(2)}, \quad K(w^+) > K(w^-),
\tag{A.5}
\]

where $a = \{a_1, \ldots, a_N\}$ represents the firms’ abatement levels. Time indices $w$’s are the instants at which at least one firm invests.
The social planner’s optimization involves two steps. First, at each moment \( t \), given \( K \), \( \varepsilon = \{ \varepsilon_1, \ldots, \varepsilon_N \} \), and \( \varepsilon_0 \), the planner allocates the \( \bar{a} \) permits among the \( N \) firms:

\[
S(K, \varepsilon, \varepsilon_0, \bar{a}) = \min \left\{ \sum_n C(a_n, K_n, n, \varepsilon_n, \varepsilon_0) \mid \sum_n a_n = \bar{a} \right\}
\]

\[
= L(K, \bar{a})\varepsilon_0 + \sum_{n=1}^N d(K_n, n)\varepsilon_n,
\]

where \( L(K, \bar{a}) = \sum_n \frac{1}{2} c(K_n, n)a_n(K, \bar{a})^2 \). The optimal allocation \( a_n(K, \bar{a}) \) satisfies (3), where \( p \) should now be interpreted as as the shadow value of \( \bar{a} \). In the second equality in (A.6), we have substituted in \( a_n(K, \bar{a}) \).

To guarantee that the subsequent dynamic optimization problem is well behaved, we impose the condition that \( L(\cdot) \) is convex in \( K \); note that \( C(\cdot) \) is convex in \( K_n \). Since \( d(\cdot) \) is convex in \( K_n \), the assumption guarantees that \( S(\cdot) \) is convex in \( K \). Further, applying the envelope theorem to the minimization problem in (A.6), we know \( \partial S/\partial K_n = C_{K_n} = \frac{1}{2} c_{K_n}a_n^2+ d_{K_n}\varepsilon_n \). But from the second equality in (A.6), \( \partial S/\partial K_n = L_{K_n}\varepsilon_0+d_{K_n}\varepsilon_n \). Thus

\[
\frac{\partial L(K, \bar{a})}{\partial K_n} = \frac{1}{2} c_{K_n}(K_n, n)a_n(K, \bar{a})^2.
\]

In the second step, we rewrite the problem in (A.5) by substituting in the optimal permit allocation:

\[
J(K(t), \varepsilon(t), \varepsilon_0(t)) = \max K - E \int_{t}^{\infty} S(K(\tau), \varepsilon(\tau), \varepsilon_0(\tau), \bar{a})e^{-r(\tau-t)}d\tau
\]

\[
- \sum_{w} \sum_{n} \kappa(K_n(w^+) - K_n(w^-))e^{-r(w-t)}
\]

subject to Eq. (2) and \( K(w^+) > K(w^-) \).

We solve (A.8) following Dixit and Pindyck (1994). Suppose at state \( \{ K, \varepsilon, \varepsilon_0 \} \), at least one firm needs to increase its stock. Applying Bellman’s Principle of Optimality to (A.8), we get

\[
J(K, \varepsilon, \varepsilon_0) = \max_{K'} S(K, \varepsilon, \varepsilon_0, \bar{a})dt - \kappa \sum_n (K_n - K_n')
\]

\[
+ e^{-r dt} E[J(K', \varepsilon + d\varepsilon, \varepsilon_0 + d\varepsilon_0)],
\]

where the expectation \( E \) is conditional on \( \varepsilon \) and \( \varepsilon_0 \). Since \( S(\cdot) \) is convex in \( K \), or \( -S(\cdot) \) is concave in \( K \), we can show that \( J(\cdot) \) is concave in \( K \).\(^{12}\) Thus the necessary and sufficient condition for the maximization problem on the right hand side is given by the following Kuhn–Tucker conditions:

\[
1 - r dt E[J_{K_n}(K', \varepsilon + d\varepsilon, \varepsilon_0 + d\varepsilon_0)] - \kappa \leq 0, K_n' - K_n \geq 0,
\]

\[
((1 - r dt) E[J_{K_n}(K', \varepsilon + d\varepsilon, \varepsilon_0 + d\varepsilon_0)] - \kappa)(K_n' - K_n) = 0,
\]

\( n = 1, \ldots, N \)

\(^{12}\) Chapter 11 of Dixit and Pindyck (1994) showed this point for the case of \( N = 1 \). Their approach can be directly generalized to \( N > 1 \). Theorem 9.8 of Stokey and Lucas (1989) strictly proved a case of \( N = 1 \) for discrete time optimization. Again, their proof can be generalized to \( N > 1 \) and continuous time.
As \( dt \to 0 \), \( d\epsilon \to 0 \) and \( d\epsilon_0 \to 0 \) with probability one. Thus we can remove the expectation operation and obtain (4) in Proposition 1.

The remaining task is to determine the function \( J(\cdot) \). Suppose the state \((K, \epsilon, \epsilon_0)\) is such that no investment is needed for any firm (the continuation region). The Bellman equation is

\[
J(K, \epsilon, \epsilon_0) = -S(K, \epsilon, \epsilon_0, \tilde{a})dt + e^{-r dt} \{ E[J(K, \epsilon + d\epsilon, \epsilon_0 + d\epsilon_0)] \}.
\]

Applying Ito’s lemma and using the fact that the shocks are independent, we know \( J(\cdot) \) satisfies

\[
\sum_{n=0}^{N} \left\{ \frac{1}{2} \sigma_n^2 \epsilon_n^2 J_{\epsilon \epsilon_n}(K, \epsilon, \epsilon_0) + \gamma_n \epsilon_n J_{\epsilon}(K, \epsilon, \epsilon_0) - rJ(K, \epsilon, \epsilon_0) \right\} - S(K, \epsilon, \epsilon_0, \tilde{a}) = 0,
\]

(A.9)

The optimality conditions in (4) imply the following boundary conditions

\[ (\text{Value} - \text{matching}) \quad J_{K_n}(K, \epsilon, \epsilon_0) = \kappa, \quad n = 1, \ldots, N, \quad (A.10) \]

\[ (\text{Smooth} - \text{pasting}) \quad J_{K, \epsilon_n}(K, \epsilon, \epsilon_0) = 0, \quad n = 1, \ldots, N, \quad m = 0, 1, \ldots, N. \quad (A.11) \]

Eqs. (A.9)–(A.11) will enable us to solve for \( J(\cdot) \).

Appendix B

Investment barrier under TEPs

From (A.6), the general solution to the differential equation (A.9) is

\[
J(K, \epsilon_0, \epsilon) = \sum_{n=0}^{N} \left[ B_n^1(K) \epsilon_n^p + B_n^2(K) \epsilon_n^p \right] - \frac{L(K, \tilde{a}) \epsilon_0}{r - \alpha_0} - \sum_{n=1}^{N} \frac{d(K_n, n) \epsilon_n}{r - \alpha_n}, \quad (B.1)
\]

where \( B_n^i(K), i = 1, 2, n = 0, \ldots, N \), are constants of integration to be determined by the boundary conditions in (A.10) and (A.11), and \( \beta_n^1 > 1 \) and \( \beta_n^2 < 0 \) are roots of the fundamental quadratic

\[
\frac{1}{2} \sigma_n^2 \beta(\beta - 1) + \gamma_n \beta - r = 0. \quad (B.2)
\]

We can show that \( \partial \beta_n^1 / \partial \sigma_n < 0 \).

If \( \epsilon_n = 0, n = 0, 1, \ldots, N \), the total abatement cost is zero (cf. (1) and (A.6)). No investment is needed and we are in the continuation region, and (B.1) applies. Further, the total abatement cost is \( J(K, 0, 0) = 0 \). Since \( \beta_n^2 < 0 \), \( \lim_{\epsilon_n \to 0} \epsilon_n^p = \infty \). Thus to have \( J(K, 0, 0) = 0 \), we must have \( B_n^2(K) = 0, n = 0, \ldots, N \). To determine \( B_n^1(K), n = 0, \ldots, N \),
as well as the investment barrier, we use the two barrier equations (A.10) and (A.11). Substituting (B.1) (with shocks only affect the fixed abatement costs. (If the parameter B enters firm m’s variable cost part, the function L(·) would depend on e, and B1j(·) would be a function of K, rather than Kn only.) Thus we can replace B1j(K) by B1j(Kn) in (B.3)–(B.5). Solving for ∂B0j(K)/∂Kn and ∂B1j(K)/∂Kn in (B.4) and (B.5), and substituting into (B.3), we find that \( J_{Kn} = h(\cdot) \), and thus obtain the optimal investment rule in (5).

From (A.7), we know ∂/∂Kn (−∂L/∂Kn) = −cKn aKn/∂(Kn) Kn. Efficient permit trading means that ∂aKn/∂Kn < 0, since as Kn increases, firm m’s marginal abatement coefficient \( c(Kn, m) \) decreases. Firm m will then abate more, and consequently firm Kn will abate less. Thus ∂/∂Kn (−∂L/∂Kn) < 0. Since L(·) is convex in K, we know ∂/∂Kn (−∂L/∂Kn) < 0.

Next we derive (6). From (3), we know \( aKn(K, \tilde{\alpha}) = p(K, e0, \tilde{\alpha})/c(Kn, n)e0 \). Substituting aKn to (A.7), we get

\[
\frac{\partial L(K, \tilde{\alpha})}{\partial Kn} = \frac{1}{2} \frac{cKn(Kn, n) p(K, e0, \tilde{\alpha})^2}{cKn(Kn, n)^2 e0^2}.
\]

Substituting this expression into (5) and using the two elasticity definitions, we get (6).

Now we show both \( \etaKn aKn/Kn \) and \( \etaKn d/Kn \) are decreasing in Kn. Firm m’s optimal abatement decision is \( aKn = p/c e0 \). Thus \( \etaKn aKn/Kn = −aKn cKn/c = −(p/e0) (cKn/c) = −(\partial L(K, \tilde{\alpha})/\partial Kn) (2e0/p(K, e0, \tilde{\alpha})) \), where the last equality follows from (B.7). Since L(·) is convex in Kn and p(·) is decreasing in Kn, we know \( \etaKn aKn/Kn \) decreases in Kn. Since \( \etaKn d/Kn = −dKn \) and d(·) is convex in Kn, we know \( \etaKn d/Kn \) decreases in Kn.

**Proof of Proposition 6.** Step 1: Let \( \kappa^2 > \kappa^1 \), and \( (p^{\mu i}, \epsilon_0^{\mu i}) \) be the barrier when \( \kappa = \kappa^i, i = 1, 2 \), for the same level of Kn. Further, suppose the current price p and firm shock \( \epsilon_0 \) be higher than both barriers, and let \( \tilde{K}^{\mu i} \) be the optimal stock level given \( \kappa^i, i = 1, 2 \). Since \( f(Kn, p^{\mu i}, \epsilon_0^{\mu i}) = \kappa^2 > \kappa^1 = f(Kn, p^{\mu i}, \epsilon_0^{\mu i}) \), from Definition 1, we know \( (p^{\mu 2}, \epsilon_0^{\mu 2}) \) is
higher than \((p^{b1}, e^{b1})\), or the barrier is increasing in \(\kappa\). Further, since \(f(\tilde{K}^2_n, p, \epsilon_n) = \kappa^2 > \kappa^1 = f(K_n, p, \epsilon_n)\) and \(f_K < 0\), we know \(\tilde{K}^2_n < \tilde{K}^1_n\), or \(\tilde{K}^2_n - K_n < \tilde{K}^1_n - K_n\).

**Step 2:** Let \((p^{bi}, e^{hi})\) and \(\tilde{K}^i_n\) be the barrier and optimal stock level associated with \(K^i_n\), \(i = 1, 2\), with \(K^2_n > K^1_n\). Thus \(f(K^2_n, p^{h2}, e^{h2}) = f(K^1_n, p^{b1}, e^{b1}) = \kappa\). Since \(f_K < 0\), we know \(f(K^1_n, p^{b1}, e^{b1}) > f(K^2_n, p^{h2}, e^{h2}) > f(K^2_n, p^{b1}, e^{b1})\). Thus the barrier is increasing in \(K_n\). Further, \(f(\tilde{K}^2_n, p, \epsilon_n) = f(\tilde{K}^1_n, p, \epsilon_n) = \kappa\). Thus \(\tilde{K}^2_n = \tilde{K}^1_n\), or \(\tilde{K}^2_n - K_n < \tilde{K}^1_n - K_n\).

**Step 3:** Let \((p^{bi}, e^{hi})\) and \(\tilde{K}^i_n\) be the barrier and optimal stock level associated with \(\sigma^0(i)\), \(i = 1, 2\), with \(\sigma^2(2) > \sigma^2(1)\), or \(O^0(2) > O^0(1)\). Since \(f(K_n, p^{h2}, e^{h2}; O^0_0(2), O^1_n, n) = \kappa = f(K_n, p^{b1}, e^{b1}, O^1_0(1), O^0_n, n)\), and \(f(\cdot)\) is decreasing in \(O^0_n\), we know \(f(K_n, p^{h2}, e^{h2}; O^0_0(i), O^0_n, n) > f(K_n, p^{h1}, e^{h1}, O^1_0(i), O^0_n, n), i = 1, 2\). That is, the barrier is increasing in \(\sigma^0(i)\).

Further, suppose the firm invests under both uncertainty levels given \(p\) and \(\epsilon_n\) then we know \(f(\tilde{K}^2_n, p, \epsilon_n; O^0_0(2), O^1_n, n) = f(\tilde{K}^1_n, p, \epsilon_n; O^0_0(1), O^1_n, n)\). Since \(f(\cdot)\) is decreasing in both \(K_n\) and \(O^0_n\), we know \(\tilde{K}^2_n < \tilde{K}^1_n\), or \(\tilde{K}^2_n - K_n < \tilde{K}^1_n - K_n\).

Similarly, we can show that the barrier (the level of investment) is decreasing (increasing) in \(a_n\), \(d\), and the two elasticities, and increasing (decreasing) in \(\sigma^2_n\). ☐

**Appendix C**

**Reason for condition \(r > \sigma^2_0 - \alpha_0\)**

Let \(y = 1/\epsilon_0\). Applying Ito’s Lemma, we know the stochastic process for \(y\) is
\[
dy = (\sigma^2_0 - \alpha_0)y \, dt - \sigma_0 \, dy \, dz.
\]
If \(r \leq \sigma^2_0 - \alpha_0\), the expected payoff to the firm (cf. (11)) would be infinite since part of the objective function is increasing at a faster rate than the discount rate. Firms would have incentive to invest without bounds.

**Appendix D**

**Derivation of (14)**

Applying the Implicit Function Theorem to (13), we know \(\eta_{\sigma_i}^{K_n} = -(f_{O^i} / f_{K_n})(dO^i / d\sigma^i)\)
\[
(\sigma^2_0 / \tilde{K}_n), i = 0, n; \eta_{\tilde{K}_n}^{\tilde{K}_n} = -((g_{O^i} \cdot (dO^2_0 / d\sigma^2_0) + g_{O^i} / g_{K_n})(\sigma^2_0 / \tilde{K}_n), and \eta_{\tilde{K}_n}^{\tilde{K}_n} = -(g_{O^i} / g_{K_n})(dO^i / d\sigma^2_0)(\sigma^2_0 / \tilde{K}_n).\]

Let \(A_n(K_n) = (1/c(K_n, n))(\eta_{K_n}^{K_n} / K_n).\) From the functional forms of \(f(\cdot)\) and \(g(\cdot)\) in (6) and (12), we know the second terms of \(f(\cdot)\) and \(g(\cdot)\) are equal. This in turn implies that \(f_{O^i} = g_{O^i}\). Further, (13) implies that the first terms of \(f(\cdot)\) and \(g(\cdot)\) are also equal:
\[
(1/O^0_0)(1/(2(r - \alpha_0)))b^2e_0A_n(\tilde{K}_n) = (O^0_0/2(r - \sigma^2_0 + \alpha_0))/\epsilon_0A_n(\tilde{K}_n),
\]
\[
\frac{1}{O^0_0} \frac{1}{2(r - \alpha_0)}b^2 \epsilon_0 = \frac{O^2_0}{2(r - \sigma^2_0 + \alpha_0)} \epsilon_0 = B.
\]

(D.1)

Therefore, at \(\tilde{K}_n = \tilde{K}_n, f_{K_n} = g_{K_n}.\) Thus \(\eta_{\tilde{K}_n}^{\tilde{K}_n} = \eta_{\tilde{K}_n}^{\tilde{K}_n}.\)
From (6) and (12), we can show that $\eta^\text{K}_n^{\tilde{K}_n} = (1/f_{\tilde{K}_n}K_n)BA_n(K_n)\eta^1_0$, and $\eta^{\tilde{K}_n}_n = (1/g_{\tilde{K}_n}K_n)BA_n(K_n)\eta^1_0$. $\tilde{K}_n = \tilde{K}_n$ and $f_{\tilde{K}_n} = g_{\tilde{K}_n} < 0$ then imply (14).

References

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