The long-run effects of environmental reform in open economies

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Abstract

We compare the short- and long-run effects of environmental reform and harmonization under autarky and free trade. When trade is driven by environmental distortions rather than real relative advantages, harmonization of environmental policies, even if achieved by lowering standards in one country, can improve short-run aggregate welfare. With the possibility of multiple steady states, harmonization can improve long-run welfare, especially when the environment is fragile. Further, long-run considerations favor upward harmonization even when it is equivalent to downward harmonization in the short run. For a country trapped in a low (or bad) steady state, environmental reform may not move it to a high (or good) steady state under autarky. However, under trade, harmonization of policies may enable this country to reach the high steady state. Conversely, reforms that increase the relative differences in distortions may, under trade, cause economies to move to a low steady state.

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1. Introduction

Environmentalists’ distrust of international trade contributed to the failure of the November 1999 WTO meetings in Seattle, the inability of President Clinton to obtain fast-track negotiating authority, and the difficulty of passing NAFTA. Environmentalists fear that competitive pressures, heightened by trade liberalization, create a danger of a “race to the bottom” in environmental standards. They conclude that the international harmonization of policies is important to prevent this race.

Economists recognize that the harmonization of distortions such as tariffs improve welfare under plausible circumstances. However, they tend to oppose pressures for harmonization of environmental policies across nations, arguing that policy differences reflect differences in income, tastes, capital stocks, resource endowments, or a variety of other factors that contribute to inter-industry trade. In this case, harmonization is an attempt to thwart the efficient workings of the market.²

Several recent papers, including Chichilnisky [6,7], Copeland and Taylor [8,9], Brander and Taylor [3,4], and Karp et al. [13], emphasize that differences in environmental regimes (or market failures) can provide an impetus for trade. Property rights may be weaker in some countries, and some countries may have been more successful in dealing with externalities. If this is the correct explanation for different standards, and if these different standards have a significant effect on trade flows, then harmonization may increase welfare.

Econometric studies find little evidence of a relation between aggregate trade flows and differing levels of environmental protection, although Mani and Wheller [17] present evidence that trade may create transitory pollution havens. At a commodity-specific level, it is easier to see how differing levels of market failure can influence trade flows. For example, in response to serious deforestation, China restricted logging in 12 provinces in 1998, and in 18 provinces in the year 2000. This logging ban, together with continued economic growth and a reduction in tariffs, has caused China to become one of the world’s largest importers of logs. Burma, where logging is controlled by warlords, and where the market failure is probably more severe than in pre-reform China, has become a primary source of supply. The environmentally beneficial policy in China could worsen the regional environmental problems by increasing the pressure on Burmese resources [18]. Aggregate trade statistics, of the sort typically used in econometric studies, might not identify this kind of causal relation. However, this is the type of scenario that concerns environmentalists who oppose liberal trade.

Tariffs provide a useful analogy for the effect of harmonization. Welfare is likely to improve whether harmonization is achieved by raising low tariffs or lowering high ones. This equivalence is due to the fact that welfare depends on relative, not on absolute prices [10]. To the extent that trade is driven by relative rather than absolute environmental standards, a similar equivalence is likely to hold. In this case, the environmentalists’ goal of harmonization could

²The arguments for and against harmonization are presented in many articles, including: Bhagwati [1], Bhagwati and Srinivasan [2], Charnovitz [5], Hoel [12], Levinson [16], Klevorick [14], Robertson [19], and Wilson [20]. Krugman [15] summarizes many of these arguments.
be achieved by weakening standards where they are strict (i.e. by “downward” rather than “upward” harmonization). However, absolute environmental standards—unlike prices—have real effects, making it unlikely that upward and downward harmonization are exactly equivalent.

The opposing views regarding harmonization of environmental policies is at least partly explained by contradictory views about the reasons for the policy differences. Do they reflect different levels of distortions, or different tastes and endowments? This paper emphasizes the first explanation, so the model is biased in favor of harmonization.

We study the differing effects of absolute and relative levels of environmental distortions and environmental reform in both the short and long run. We use a dynamic North–South trade model where a renewable environmental stock affects production costs. The change in the stock depends on production decisions, and these decisions depend on the trade regime (free trade or autarky) and on the absolute and relative levels of the environmental distortions. There may be multiple steady states in this model. Under both trade and autarky the steady state may be unique, in which case it may be either low (“bad”) or high (“good”); alternatively, both types of steady states might simultaneously exist. Environmental reform (and the trade regime) may affect the properties of these steady states—including their existence. Under trade, upward and downward harmonization equally improve aggregate welfare in the short run. However, in the long run the two types of harmonization may have very different effects: upward harmonization increases the likelihood that the economies reach the good steady states.

The trade regime influences the effects of environmental reform. In some cases, an autarkic country is trapped by tastes and technology at a low steady state: environmental reform does not enable it to reach a high steady state. However, in the presence of trade, upward or even downward harmonization of policies sometimes enables the country to escape to a high steady state. In other cases, reform against harmonization moves an autarkic economy to a high steady state, but moves a trading economy to a low steady state. Thus, reform can have very different effects under autarky and free trade.

In addition to illustrating these (and other) possibilities, the model identifies the factors that determine the various outcomes. For example, under trade, reform in the least distorted economy (a movement away from harmonization) is likely to be beneficial if the initial difference in environmental distortions in the countries is not great, or if the environmental problem is not severe. The same reform is likely to have perverse effects if the natural rate of growth of the environmental stock is small (i.e. if the environment is “fragile”).

Our focus on the long run is particularly relevant for trade involving renewable resources and stock pollutants. In these cases, trade and welfare in different time periods are connected, requiring a dynamic model. Over time, some countries seem to have been trapped in vicious cycles of low resource stocks and low standard of living, while others enjoy high stocks and high welfare. Our emphasis on multiple steady states helps to explain this phenomenon and to show the role of trade and harmonization in breaking the vicious cycles.

Section 2 explains why multiple steady states arise, and illustrates possible effects of environmental reform in a general setting. Section 3 describes the analytic model and the equilibria under autarky and trade. This section summarizes results derived in Karp, Sacheti and Zhao [13] (hereafter KSZ), which we use in Sections 4 and 5 to analyze the effects of reform in the
short and the long run.\textsuperscript{3} We discuss the generality of our model in Section 6. Section 7 summarizes and concludes.

2. Multiple steady states: a graphical illustration

Our major results hinge on the possibility of multiple steady states in autarky and trade—not on specific assumptions about functions. This section uses a general model to illustrate the differing steady state effects of reform under autarky and trade.

Suppose that the production of final goods requires environmental services ($E$), the supply of which is endogenous. The cost of producing $E$ decreases with the environmental stock $Z$. The $E$-producing industry has a market failure that leads to an inefficiently high exploitation of the environmental stock and an inefficiently high supply of environmental services, for a given stock level. The magnitude of this distortion is measured by $\delta$; a larger value of $\delta$ implies a greater market failure.

The equilibrium supply of $E$ depends on both the market failure and the current stock, $E = E(Z, \delta)$. A larger environmental stock decreases the cost of supplying environmental services but at a decreasing rate, so $E_Z \geq 0$ and $E_{ZZ} \leq 0$.\textsuperscript{4} A larger market failure increases the equilibrium supply of environmental services for a given stock, so $E_{\delta} \geq 0$. Environmental reform means that the distortion is reduced, or $\delta$ is reduced.

In order to obtain a specific functional form for $E(Z, \delta)$ we need to specify the market failure and the nature of the producer’s optimization problem (among many other things). At this stage,

\textsuperscript{3} We use the same analytic model as in KSZ, but that paper focused on a comparison between free trade and autarky. Here we study the effect of environmental reform in general, and on harmonization of environmental policies in particular.

\textsuperscript{4} The concavity of $E(Z)$ is satisfied under a variety of situations. For example, if $p(E)$ is the inverse demand for services and $c(E, Z; \delta)$ is the marginal cost (inclusive of user cost—i.e. the producers’ shadow value of the stock), then the equilibrium level of $E$ is given by $p(E) = c(E, Z; \delta)$. If $c$ is convex in $Z$ and $p$ is not “too convex”, then $E_{ZZ} \leq 0$. 

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\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{Possibilities of steady states: (a) autarky; (b) trade.}
\end{figure}
we do not need that degree of detail. The intuition for our results depends on the assumed properties of the extraction function: \( E_Z \geq 0 \), \( E_{ZZ} \leq 0 \), and \( E_\delta \geq 0 \). Many plausible models give rise to an equilibrium supply function with these properties.

To complete the description of the model we assume that the natural growth rate of the environmental stock (absent extraction) is a strictly concave function \( G(Z) \) that increases for small \( Z \), reaches a maximum, and then decreases to 0 (at the natural carrying capacity). The steady state of the autarkic economy depends on the relation between \( G(Z) \) and \( E(Z, \delta) \). Fig. 1a illustrates three possible configurations. In Case I, there is a unique low steady state and in Case II, there is a unique high steady state. Case III describes a situation where \( G(Z) \) and \( E(Z, \delta) \) intersect at three points, a low steady state, \( Z_1 \), a high steady state, \( Z_h \) and an intermediate (unstable) steady state, \( Z_u \). The economy moves toward either the high or the low steady state, depending on whether the initial level of \( Z \) is above or below \( Z_u \).

Reform corresponds to a decrease in \( \delta \), leading to a downward shift in the curve \( E(Z, \delta) \). A small reform causes small increases in the stable steady states, and thus has only a quantitative effect. A large reform, or a sufficiently large downward shift in \( E(Z, \delta) \), can eliminate the low steady state, leaving the high steady state as the unique long-run equilibrium (to Case II). We consider the changes between the three cases in Fig. 1a as representing the qualitative effects of the reform. Similarly, increasing the distortion can lead to an upward shift in \( E(Z, \delta) \), leaving only a low steady state (to Case I). Since a higher environmental stock reduces extraction costs, reform improves steady-state welfare. It may also lead to static welfare benefits, raising welfare at a given stock level. The market failure means that the environmental sector absorbs more inputs than is socially optimal. Static welfare might increase if these inputs were devoted to other activities.

In a trade equilibrium, two countries, North and South, exchange commodities which use environmental services as inputs. At a point in time, their environmental stocks are \( Z_N \) and \( Z_S \). As was the case under autarky, \( E_i \) (the equilibrium supply of environmental services in country \( i \)) depends on \( Z_i \) via its direct effect on production costs, and on the market failure, \( \delta_i \); \( E_i \) also depends on the price of environmental services, which depends on the aggregate (world) supply of services. Thus with trade, the supply of environmental services \( E_i \) in country \( i \) depends on the environmental stocks and the market failures in both countries. An increase in \( Z_N \), for example, decreases North’s relative costs of producing environmental services. Under plausible circumstances higher \( Z_N \) increases the equilibrium supply of \( E_N \) and decreases the equilibrium supply of \( E_S \).

Fig. 1b illustrates one of many possibilities for the long-run effects of reform. The heavy line labeled \( \dot{Z}_S = 0 \) shows the set of stocks in \( Z_N - Z_S \) space at which South’s stocks are in long-run equilibrium. This curve can be non-monotonic, as shown. For \( Z_N < Z_1 \) (i.e. for low levels of \( Z_N \)), South produces environmental services not only for domestic use, but also for export (embodied in final products). Thus, under trade, a low level of \( Z_N \) implies that the graph of \( E_S \) (as a function of \( Z_S \)) is high. Then Case I in Fig. 1a applies: \( \dot{Z}_S = 0 \) has a unique solution at a low steady state. Increases in \( Z_N \) shift down the graph of \( E_S \) (as a function of \( Z_S \)), thus increasing South’s low steady state. For \( Z_N > Z_2 \), South produces a smaller flow of environmental services. Case II in Fig. 1a applies and there is a unique solution to \( \dot{Z}_S = 0 \), the high steady state. Again, increases in \( Z_N \) raise this steady state. For intermediate levels \( Z_1 < Z_N < Z_2 \), Case III in Fig. 1a applies and there are two stable and one unstable solutions to \( \dot{Z}_S = 0 \). Over this region, an
increase in $Z_N$ increases both of South’s stable steady states and decreases the unstable steady state.

Environmental reform in one country reduces its comparative advantage in the environment-intensive good. Reform thus decreases its extraction level and tends to increase extraction of the other country. For any level of $Z_N$, reform in South increases (both) the stable steady state(s). The mechanism is the same as under autarky. Southern reform therefore shifts the $\dot{Z}_S = 0$ isocline up to the left (the thin solid curve in Fig. 1b). Reform in North shifts production of environmental services to South, lowering South’s stable steady states. Northern reform shifts the $\dot{Z}_S = 0$ isocline down to the right (the dashed curve in Fig. 1b).

The curve labeled $\dot{Z}_N = 0$ shows the pre-reform set of steady states for North. For this example, the heavy solid curves intersect at a single point, so the unique pre-reform steady state under trade has high stocks in both countries. Reform in North shifts the $\dot{Z}_N = 0$ curve to the right, to the dotted curve. (This change is analogous to the change in $\dot{Z}_S = 0$ caused by South’s reform.) The previous paragraph explains why North’s reform shifts down the $\dot{Z}_S = 0$ curve. Fig. 1b illustrates a situation in which North’s reform leads to two stable steady states. If the initial condition is such that the equilibrium moves toward the low steady state, reform leads to lower stocks and lower welfare in the long run.

This example merely shows that, in the presence of two distortions, a reduction in one distortion can lower welfare—an illustration of the theory of the second best. At this level of generality we can only conclude that the effect of reform is ambiguous. In order to understand how the economic and environmental fundamentals are likely to determine the effects of reform, we need a less general model.

3. Special model: the autarkic and trade equilibria

In this section, we define the special model and describe the autarky and trade equilibria. The detailed derivation of these equilibria can be found in KSZ and we provide a sketch in Appendix A. This model leads to a simple formula for the equilibrium extraction function $E(Z)$. We equate $E(Z)$ to the natural growth function of the resource and characterize the steady states under autarky and trade.

3.1. Description of the model

Fig. 2 shows a flow chart of the autarkic economy. The first arrow shows that the stock and flow in the previous period $(Z_{-1}, E_{-1})$ affect the current stock, $Z$. There are two goods: the “subsistence good” $A$, which we choose as the numeraire, and the “composite good” $B$, which has price $p$. These goods are competitively produced using labor $L$ and environmental services $E$ with Leontief technology:

$$A^* = \min\left\{ \frac{E_A}{a_1}, \frac{L_A}{b_1} \right\}, \quad B^* = \min\left\{ \frac{E_B}{a_2}, \frac{L_B}{b_2} \right\}. \quad (1)$$

$B$ is relatively environment-intensive, i.e. $\frac{a_2}{a_1} > \frac{b_2}{b_1}$. 


The representative consumer attempts to consume $A^*$ units of $A$. If her income, $y$, is less than $A^*$, she spends everything on good $A$, receiving utility $y$ (equal to the consumption of $A$). If her income exceeds $A^*$, she buys $A^*$ units of good $A$ and $(y - A^*)/p$ units of $B$, resulting in utility $A^* + (y - A^*)/p$. These preferences provide a simple way to describe a situation where the income elasticity for the subsistence good is very high at low income and is very low at high income. We assume that the representative consumer’s income exceeds $A^*$. Similar preferences have been used by Eswaran and Kotwal [11]. Section 6 discusses the assumptions regarding technology and preferences in greater detail, and explains the effect that they have on our results.

The supply of labor is exogenously fixed at $\bar{L}$. Environmental services, $E$, are “extracted” from the environmental stock $Z$ using $B^*$ units of good $B$ with a decreasing returns to scale technology. Larger stocks decrease the costs of producing $E$.

Imperfect property rights take the following form: There are a fixed number, $n$, of $E$-producers who choose their input level and receive a share of output proportional to their share of total inputs. They ignore the dynamic effects of their extraction activity. The aggregate production function is assumed to be $E = (B^*Z)^{0.5}$. The Nash equilibrium supply function is

$$E = \delta Z p^e / p.$$  \hspace{1cm} (2)

Here $p^e$ is the price of $E$ and the (fixed) parameter $\delta = 1 - 1/(2n)$ is positively related to the magnitude of the environmental distortion (or negatively related to the degree of property rights). If there is open access with no property rights (i.e. $n = \infty$), $\delta = 1$; for perfect static property rights (i.e. $n = 1$), $\delta = 0.5$. Hereafter we assume $\delta > 0.5$. We refer to $\delta Z$ as the apparent stock of this economy. A larger distortion or a larger physical stock both increase the apparent stock and therefore increase extraction.

The assumption that income exceeds $A^*$ implies that the consumption of $A$ is fixed at $A^*$. In this case, the economy’s welfare is measured by the consumption of $B$, which equals the production of
For $d_0 = 5$, the value of marginal product of $B$ used in the production of $E$ is less than the price of $B$. In this case, there is a static efficiency gain from increasing the consumption of $B$ and using less of it to produce environmental services.

To help fix ideas, we can think of good $A$ as food, good $B$ as steel, $Z$ as the stock of water in lakes, and $E$ as the flow of water used in production. Food is a pure consumption good, and its income elasticity falls as income increases. Steel can be consumed (in the form of cars) or used for pipes to transport water from lakes to agricultural and steel production. A low income economy uses steel only for pipes, but a richer economy also consumes cars. Water in lakes is a renewable resource that provides benefits only as a source of a factor of production. (The consumer does not fish or swim.) A larger stock of water means that supplies are closer to production, so less steel is needed to obtain usable water.

The two economies, North and South, are identical except for their values of $d$ and (possibly) their stock levels. We assume that $d_S > d_N$, so the environmental distortion is worse in South. For the trade equilibria we restrict attention to the case where both economies are diversified in production, so that factor prices are equal.

Throughout the paper we use the following:

**Definition 1.** Environmental reform in country $i$ means a reduction in $d_i$. Harmonization of environmental policies means a reduction in $d_S/d_N$. Upward harmonization means a reduction in $d_S/d_N$ caused by a decrease in $d_S$. Downward harmonization means a reduction in $d_S/d_N$ caused by an increase in $d_N$.

Downward harmonization is consistent with a “race to the bottom”, while upward harmonization is consistent with a “race to the top.”

### 3.2. Description of the equilibria

The Leontief technology, fixed labor supply, and utility function imply that labor is fully employed if and only if the endogenous supply of $E$ is sufficiently high, or if and only if $Z \geq Z_c(\delta)$, where $Z_c(\delta)$ is a decreasing function of $\delta$. The level of employment is an incidental feature of this model. However, the two cases, full employment and less than full employment, provide a simple equilibrium extraction function $E(Z)$ and a convenient means of describing the equilibria. When labor is fully employed (i.e. for $Z \geq Z_c(\delta)$), the Leontief technology and fixed labor supply determine the amount of $E$ demanded by the production sectors of $A$ and $B$. Therefore, $E(Z)$ is a constant for $Z \geq Z_c(\delta)$. When labor is partially employed, $E(Z)$ is proportional to $Z$: $E = \delta Z / a_2$. Thus, environmental reform affects the level of extraction only for $Z < Z_c(\delta)$: there, reform reduces $E$ and raises $Z_c$. For $Z > Z_c(\delta)$, reform affects neither the extraction nor the flow of welfare. Fig. 3 graphs the extraction function for two levels of property right: $\delta_1 > \delta_2$.

Now we consider free trade. We assume that both countries are incompletely specialized, so factor prices are equal under trade. Consequently, labor is unemployed (i.e. its price is 0) either in both countries or in neither country. Since the equilibrium depends on stocks in the two regions, the assumption of incomplete specialization restricts our analysis to a certain region of the state space—in particular, a region where the two stocks are not extremely different. This restriction
greatly reduces the number of types of static equilibria that we need to study. Unemployment (in both countries under autarky and trade) corresponds to a region where both stocks are fairly small (but still large enough to support consumption of $A^*$); full employment (in both countries under autarky and trade) corresponds to a region where both stocks are quite large.

When labor is unemployed, there is only one constraining factor of production, $E$, resulting in the standard Ricardian model. In view of the assumption that countries have the same technology, the autarkic and free trade equilibria are identical. In this case, the aggregate supply of $E$ is the same under free trade and autarky. This supply is increasing in both $\delta_S$ and $\delta_N$: reform in either country reduces the supply of $E$ in that country.

If labor is fully employed, the technologies and utility function imply that the total amount of $E$ used in the world production of $A$ and $B$ is again fixed. In addition, aggregate $E$ under trade equals the sum of the autarky full employment levels of $E$. However, the distribution of the aggregate level depends on the apparent stocks, and thus on the property rights. Using Eq. (2) and the fact that factor prices are equal, we have $\frac{E_S}{E_N} = \frac{\delta_S Z_S}{\delta_N Z_N}$. Reform in either country affects extraction levels in both countries, but not aggregate extraction. However, reforms in both countries that leave the relative property rights $\delta_S/\delta_N$ unchanged do not affect extraction in either country.

The countries are the same except for $\delta$ and $Z$, and the supply of $E_i$ is proportional to the apparent stock $\delta_i Z_i$ (see Eq. (2)). The country with higher $\delta Z$ exports $B$. South exports $B$ if and only if $Z_N < \frac{\delta_S}{\delta_N} Z_S$. However, South’s real costs of producing the resource intensive good are lower than North’s if and only if $Z_N < Z_S$. Thus, for $Z_S < Z_N < \frac{\delta_S}{\delta_N} Z_S$, South has an “apparent” but not a “real” comparative advantage in the production of $B$. In this case, world welfare would be raised if North increased, and South decreased, their extraction of $E$ and their production of $B$.

Under autarky, the distortion leads to excessive extraction and a welfare loss (i.e. environmental reform matters in the short run) if and only if the resource stock is low ($Z < Z^*$). This result is an extreme version of the empirical observation that property rights matter most when the resource base is weak. However, whenever trade occurs, property rights matter. In this sense, trade makes market imperfections more important.
4. Short-run effects of reform

We are chiefly interested in the long-run effects of environmental reform. To provide a basis for comparison, we first consider the short-run (static) effects of reform. We noted that under autarky, environmental reform affects the economy only when $Z < Z^c(\delta)$. In this case, reform reduces the supply of $E$ and improves welfare. To see the welfare effects, note that the value of marginal product of $B$ in extracting $E$ is lower than the price of $B$. National income equals rents in the $E$-producing sector, since for $Z < Z^c(\delta)$ the price of labor is zero. A lower value of $\delta$, leading to a lower equilibrium supply of $E$, increases profits in the extraction sector. This increase in national income raises static welfare. Therefore, in autarky, environmental reform has no static welfare effect when the environmental stock is large, but improves welfare when the stock is small.

If stocks in both countries are small (i.e. there is unemployment in both countries), there is no incentive to trade, and reform has the same effects as under autarky. If stocks are large (i.e. labor is fully employed in both countries), upward or downward harmonization (smaller $\delta_S/\delta_N$) improves aggregate welfare equally, while an increase in $\frac{\delta_S}{\delta_N}$ decreases aggregate welfare. For example, Northern reform increases South’s production of $E$ and leaves unchanged the aggregate supply of $E$, $A$ ($= 2A^*$) and $B$. Since $\delta_S > \delta_N$, the marginal value of product of $B$ is lower in South. North’s reform, by increasing the Southern extraction, decreases the amount of $B$ available for consumption (since more is used for the production of the input $E$), and lowers world welfare. Reform in South increases world welfare.

Consider now an individual country’s welfare. A country’s reform reduces the inefficiency in its extraction sector, increases its partner’s inefficiency, and raises the world price of the resource intensive good $B$. Thus, reform in a $B$-exporting country benefits that country and harms its trading partner. If the reforming country is an importer of $B$, its terms of trade deteriorate and the welfare changes in both countries are ambiguous.

Equal-proportionate reform in the two countries which leaves relative distortions unchanged (i.e. reform, without harmonization) does not alter $E_i$ or aggregate welfare. However, this reform reduces world apparent resource stocks and thus raises the price of $B$. Equal-proportionate reform thus benefits the exporter of the environmentally intensive good and harms the importer. Therefore, when evaluating a policy change which leaves $\delta_S/\delta_N$ unaltered, exporters of the resource intensive good ($B$) prefer a “race to the top,” and importers of $B$ prefer a “race to the bottom.”

5. Long-run effects of reform

In the dynamic model, we assume a logistic growth function for $Z$, given by $\dot{Z}_i = \eta Z_i - \gamma Z_i^2 - E_i$, for $i = N, S$. The parameter $\gamma$ captures the congestion effect of the stock; $\gamma > 0$ insures that $Z$ is bounded. The non-congested growth rate of the environment, $\eta$, provides a measure of environmental resilience. When $\eta$ is large, the environmental stock recovers quickly from low levels. We associate a large value of $\eta$ with a resilient environment, and a small value of $\eta$ with a fragile environment. The carrying capacity of the stock is $\frac{\eta}{\gamma}$ and the stock that maximizes the
sustainable yield is \( \frac{\eta}{Z^2} \). The level of \( E_i \) is the amount of extraction (the flow of environmental services) at a point in time.

The dynamic equilibria for autarky and trade are sequences of the static equilibria studied in the last section, corresponding to the evolving stock levels. Fig. 4 shows examples of autarkic (panel a) and trade (panel b) equilibria. These graphs are specializations of those in Fig. 1. We show only examples of the situation where there are multiple equilibria under autarky and free trade. Under both trade regimes, there could be a single equilibrium, which might involve either full employment or unemployment. The magnitude of \( \eta \) relative to critical values, \( \hat{\eta}^a \), \( \eta^*a \) for autarky and \( \hat{\eta} \) and \( \eta^* \) for free trade, determines the nature and the multiplicity of the steady states (Appendix B.1).

We define a low steady state \( Z_l \) as one that is less than \( \frac{\eta}{Z^2} \) (the stock level that maximizes sustainable yield), and the high steady state \( Z_h \) as one above this level; the economy converges to \( Z_\infty \) (a steady state) and the initial condition is \( Z_0 \). An unstable steady state is \( Z_u \). Recall that \( Z_c \) is the critical stock level, above which reform has no short-run effect. Table 1 summarizes the relation under autarky between the value of \( \eta \) and the type of steady state.

We obtain a similar taxonomy under free trade by replacing the critical values \( \hat{\eta}^a \) and \( \eta^*a \) with \( \hat{\eta} \) and \( \eta^* \), and the scalars \( Z_l \) \( Z_h \) and \( Z_u \) with vectors \( Z_l = (Z_{Nl}, Z_{Sl}) \) (a low steady state with unemployment in both countries), \( Z_h = (Z_{Nh}, Z_{Sh}) \) (a high steady state with full employment in

Table 1

<table>
<thead>
<tr>
<th>Value of ( \eta )</th>
<th>Type of steady state</th>
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<tbody>
<tr>
<td>Case (I) ( \eta &lt; \hat{\eta}^a )</td>
<td>Unique low: ( Z_\infty = Z_l &lt; Z_c )</td>
</tr>
<tr>
<td>Case (II) ( \eta &gt; \eta^*a )</td>
<td>Unique high: ( Z_\infty = Z_h &gt; Z_c )</td>
</tr>
<tr>
<td>Case (III) ( \hat{\eta} &lt; \eta &lt; \eta^*a )</td>
<td>Multiple: ( { Z_\infty = Z_h &gt; Z_c \text{ if } Z_0 &gt; Z_u } ) ( Z_\infty = Z_l &lt; Z_c \text{ if } Z_0 &lt; Z_u )</td>
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</table>
both countries), and $Z_u$ (the unstable steady state). In the case of multiple steady states, the description of the basins of attraction to the steady states is only slightly more complicated than in Table 1.

A marginal change in $d_i$ could cause a qualitative change in the steady state, or change the type of steady state the economy approaches, only for “knife-edge” cases where the parameters of the model or the initial value of $Z$ are at critical levels. In general, a qualitative change in the steady state requires a non-marginal change in $d_i$; i.e. a large reform. We are able to use comparative statics to analyze large reforms because the most of the critical values (at which a change in regime occurs) are monotonic in $d$, for $d \in [0.5, 1]$.

5.1. Long-run effects of reform under autarky

We use the following derivatives (calculated in Appendix B.2) to determine the effect of reform in each of the three cases identified in Table 1:

\[(a) \frac{d\eta^{*a}}{d\delta} = 0; \quad (b) \frac{dZ_l}{d\delta} < 0; \quad (c) \frac{d\eta^{*a}}{d\delta} > 0; \quad (d) \frac{dZ_u}{d\delta} = \frac{dZ_h}{d\delta} = 0.\]  

Case I: If $\eta < \eta^{*a}$, environmental reform does not enable the economy to escape from a low steady state (Eq. (3a)). However, reform increases the level of the low steady state (Eq. (3b)) and increases welfare along the trajectory whenever $Z < Z^c$.

Case II: If $\eta > \eta^{*a}$, the economy always reaches the high steady state, and reform has neither a qualitative nor a quantitative effect (Eqs. (3c) and (3d)). Reform’s only effect is that during a period when $Z < Z^c$, welfare is higher, and the environment recovers more rapidly.

Case III: For the intermediate case, $\eta^{*a} < \eta < \eta^{*a}$, the magnitude of the reform is important. If the reform is “moderate”, in the sense that the two stable steady states remain, then the effect of reform depends on the initial condition, $Z_0$. When $Z_0 < Z_u$ (which is independent of $\delta$, by Eq. (3d)), reform has the quantitative effect as described in Case I. When $Z_0 > Z_u$, reform has no effect, as in Case II (with $Z_0 > Z^c$). If the reform is very large, it causes Case II to hold as $\eta^{*a}$ decreases sufficiently (Eq. (3c)). This large reform causes a qualitative change for small initial stocks, since the stock approaches a high rather than a low steady state. For large initial stocks, a large reform has neither a quantitative nor a qualitative effect.

The autarkic economy can trap at the low steady state if the resilience of the resource is very low (Case I), or if the resource is moderately resilient but the initial stock is low (Case III). Environmental reform can help an autarkic economy escape from the trap only in the second situation. We will show that with trade, reform has the potential to help the economy escape from the low steady state even under the first situation.

5.2. Long-run effects of reform under free trade

We determine the long-run effects of reform in the free-trade equilibrium by studying how changes in $\delta_S$ and $\delta_N$ affect the steady states and the critical levels of $\eta$. At the low steady state $Z_l$, the autarkic and trade equilibria are identical; reform in country $i$ has the same effect on the low steady state $Z_l$ under trade and autarky (so Eq. (3b) applies). A reduction in $\delta_i$ has indeterminate
effects on the high steady state $Z_h$. The only possibility we can exclude is that reform in North decreases $Z_{Nh}$ and increases $Z_{Sh}$.

The qualitative effects of environmental reform depend on the change in $\eta$ and $\eta^*$. From Eq. (B.3) in Appendix B.1, the critical value $\eta$ depends on relative distortions, measured by $\delta_S/\delta_N$, but not on absolute distortions:

$$\frac{d\eta}{d(\delta_S/\delta_N)} > 0.$$ (4)

Harmonization of policies, achieved by either an improvement in Southern standards, or a deterioration in Northern standards, reduces $\delta_S/\delta_N$. If $\eta < \eta^*(\delta_S/\delta_N)$ prior to harmonization, the trade equilibrium is Case I (a unique low steady state). Harmonization may reduce the critical $\eta$ by enough that it is less than $\eta$, thereby creating a high steady state (i.e. moving the economy to Case III, with multiple equilibria). If the initial stocks, $Z_0$, are sufficiently large, harmonization causes the economies to move toward the high steady state. In this case, harmonization benefits both North and South in the long run, even if either of them suffers the kinds of short-run welfare losses described in Section 4. Here, harmonization shifts production of the resource-intensive good away from South, possibly altering the nations’ apparent comparative advantage and reversing the direction of trade for a time. The lower level of exploitation enables South’s stocks to recover. In the long run, South exports commodity $B$ and North’s stocks also recover.

Unilateral reform in North, which represents a movement away from harmonization, could cause $\eta$ to exceed $\eta$. Suppose, for example, that pre-reform $\eta > \eta^*$ and $Z_0$ lies above the convergent saddle path through $Z_u$, so that the economy is moving toward $Z_h$. If after Northern reform, $\eta < \eta^*$, the economy approaches the low steady state $Z_l$. In this case, even if North and/or South benefit from Northern reform in the short run, both lose in the long run.

If the economies are initially close to the low steady state, they remain trapped at $Z_l$ even if harmonization causes the regime to change from Case I to Case III. In this situation, harmonization has no qualitative effect. Reform in either country increases its steady state welfare, without altering the other country’s steady state welfare.

For $\eta > \eta^*$ only a high steady state exists. Eq. (B.4) in Appendix B.1 shows that the critical value $\eta^*$ depends on both the relative and absolute values of $\delta_i$. (In contrast, $\eta$ depends only on relative $\delta$ values.) Southern reform, which decreases both the absolute distortion in South and its distortion relative to North, reduces the range of values of $\eta$ at which the low steady states exist; that is (Appendix B.3),

$$d\eta^*/d\delta_S > 0.$$ (5)

Southern reform may cause the regime to change from Case III (multiple steady states) to Case II (only a high steady state). If this occurs, Southern reform causes the economies to move to a high steady state even if they were previously trapped at a low steady state.

Northern reform increases the relative distortions but decreases an absolute distortion. The effect of this reform on $\eta^*$ depends on which of the two influences is stronger. In particular, it

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5 The location of the high steady state $Z_h$ depends only on the relative property rights, while the location of the low steady state $Z_l$ depends on both individual property rights. Thus, $\eta$, which affects the existence of $Z_h$, depends only on the relative property rights, while $\eta^*$, which affects the existence of $Z_l$, depends also on the absolute property right levels.
depends on the initial difference between $\delta_S$ and $\delta_N$ and the severity of the environmental problem. The relative distortion matters more if the initial difference $\delta_S - \delta_N$ is large, and when the environmental problem is “more severe”.

We define the index $g = \gamma a_2 \psi^c$ as a measure of the severity of the environmental problem. This index depends on the physical/biological process, and on the economic variables which describe production and preferences, but not on $\delta_i$. The index is an increasing function of the congestion parameter $\gamma$. Greater congestion tends to make the environmental problem more severe. The parameter $a_2$ is the amount of the environmental factor needed to produce a unit of commodity $B$. An increase in $a_2$ means that the environment becomes more important to production, and low environmental stocks become more damaging. Finally, $\psi^c$, which is a function of all of the economic parameters except $\delta_i$, is the minimum aggregate apparent stock needed for full employment. An increase in $\psi^c$ also means that the environment, and thus environmental problems, are more important.

The effect on $\eta^*$ of $\delta_N$ depends on whether the index $g$ exceeds a critical level, defined as $g^* \equiv \delta^2_N + 2 \delta_N - 1$, and on whether $\delta_S$ exceeds a critical value $\delta^*_S(\delta_N, g)$. This function is increasing in $\delta_N$ and decreasing in $g$, with $\delta_N < \delta^*_S < 1$ (Appendix B.3). We have

$$
\frac{d\eta^*}{d\delta_N} = \begin{cases} 
0 & \text{if } g < g^*, \\
> 0 & \text{if } g > g^* \text{ and } \delta_S < \delta^*_S(\delta_N, g), \\
< 0 & \text{if } g > g^* \text{ and } \delta_S > \delta^*_S(\delta_N, g).
\end{cases}
$$

Eq. (6) states that if the environmental problem is not “severe” ($g < g^*$), then the absolute effect of Northern reform always dominates the relative effect, and Northern reform decreases the critical value $\eta^*$. If, on the other hand, the environmental problem is “severe” ($g > g^*$), then either the absolute or relative effect may dominate. If the difference between the property rights is large ($\delta_S > \delta^*_S$), the relative effect dominates, and Northern reform increases the critical value $\eta^*$. If the difference between the distortions is small ($\delta_S < \delta^*_S$), the absolute effect dominates, and Northern reform decreases the critical value $\eta^*$.

The fact that upward harmonization (through reducing $\delta_S$) certainly decreases $\eta^*$, but downward harmonization (through increasing $\delta_N$) may increase $\eta^*$, argues in favor of upward rather than downward harmonization. This argument is based on the long-run effects of reform.

5.3. The different effects of reform

The dynamic and static effects of reform differ under free trade. If stocks are large enough to support full employment of labor (so that “trade matters”) and if factor prices are equalized, instantaneous aggregate welfare depends only on relative distortions. Harmonization, whether achieved by upgrading Southern standards or degrading Northern standards, has the same instantaneous effect on aggregate welfare. In the long run, however, absolute as well as relative levels of standards are important. Harmonization upwards is more likely than harmonization downwards to increase long-run stocks and welfare.

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\[ \text{If the difference between the distortions is small (} \delta_S < \delta^*_S \text{), the absolute effect dominates, and Northern reform decreases the critical value } \eta^*. \]

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The dynamic effects of reform depend on the trade regime. In a closed economy, reform does not alter the critical value \( \eta^a \), below which only a low steady state exists. If a country under autarky is trapped in a steady state with low environmental stocks, technology and preferences determine its destiny. Environmental reform cannot lead to a qualitative improvement (high stocks). In contrast, if open economies are trapped in a low steady state, harmonization of environmental policies (which reduces \( \eta \)) may enable them to escape to a high steady state.

In a closed economy, reform always reduces the critical value \( \eta^a \), above which only a high steady state exists. Therefore, if both the high and low steady states exist in an autarkic economy, reform might eliminate the low steady state, ensuring that the economy reaches the high steady state. With open economies, reform in the more distorted economy reduces the critical value \( \eta^* \) above which only a high steady state exists. Reform in the less distorted economy, which reduces harmonization, may increase this critical value. Therefore, in open economies, reform against harmonization can either increase or decrease the danger that environmental stocks move to a low steady state.

In addition to illustrating these possibilities, the model shows how the plausibility of a particular outcome is related to the intrinsic growth rate of the environment. If \( \eta \) is small then the environment is “fragile”, in the sense that it regenerates slowly. In this situation, changes in \( \eta \) and \( \eta^a \) are more important than are changes in \( \eta^* \) or \( \eta^{*a} \) (since the actual value of \( \eta \) is more likely to be close to the first pair of critical values). Thus, “fragility” of the environment makes it more likely that harmonization—even if achieved by lower standards in North—improves the environment and welfare in both the short and the long run. If, on the other hand, the environment is “resilient” (\( \eta \) is close to \( \eta^* \) or \( \eta^{*a} \)), unilateral reform in North may lead to long-run improvements, at the cost of short-run welfare losses.

6. Discussion of the model

Since our model is not completely standard, it is worth discussing the plausibility of its assumptions, and the bearing these have on our results. Our major assumptions involve the lack of substitutability in consumption and in production of the final goods A and B. These assumptions are perhaps as realistic as those in other analytic models (e.g. the assumptions of constant expenditure or factor shares). The major advantage of these assumptions is that they lead to a simple representation of an extraction function \( E(Z) \) that is concave: \( E(Z) \) is piecewise linear. We are then able to obtain an analytical characterization of the multiple steady states. The lack of substitutability has nothing to do with the possibility of multiple steady states, as Section 2 shows. Models with smooth production and consumption functions often do not lead to simple and yet concave harvest functions. For example, in Brander and Taylor (1997, 1998), with Cobb–Douglas utility function and linear production functions, the equilibrium extraction function \( E(Z) \) is linear, leading to unique steady states. Zhao [21] uses Cobb–Douglas utility and production functions, and the resulting \( E(Z) \) is concave but too complicated to permit analytic results in a dynamic model.

The lack of substitutability, and the resulting piece-wise linear form of \( E(Z) \), does determine a particular feature of our model: the existence of two distinct types of static equilibria. If environmental stocks are low, the fixed factor (labor) does not constrain production—there is
unemployment. If the environmental stocks are high, the fixed factor is a binding constraint—there is full employment. If a country is not allowed to trade, reform affects the static equilibrium only in the first type of equilibrium (low stocks). With free trade, there is an incentive for trade to take place only in the second type of equilibrium (high stocks). The fact that there is a sharp distinction between the two types of equilibria is incidental. However, the assumptions does lead to a significant and plausible implication that environmentally related market failures—and therefore reform—are especially important when the environmental stock is low.

The lack-of-substitutability assumption helps to distinguish between the short- and the long-run effects of reform. It also emphasizes the role of relative rather than absolute distortions in the trade equilibrium. When trade matters (i.e. when the stocks are high), upward and downward harmonization equally improve the aggregate static welfare, because the lack of substitutability implies that only the relative distortions in the two countries matter. Further, harmonization is more important as the environment becomes more fragile. However, in the long run, upward harmonization dominates, especially when the environmental problem is severe and the property rights are much different in North and South.

The lack-of-substitutability assumption leads to the possibility of unemployment, a feature that is tangential to our model, and that is not needed for the multiplicity of steady states. Unemployment in the low steady state does reduce welfare, because part of the labor force is not used to produce goods. Our analysis requires that we rank welfare in different equilibria, not that we study a decomposition of the welfare change.

Another simplifying assumption of our model is that the producers in the extraction industry are myopic. They do not optimize dynamically; instead, they respond to the existing resource stock and prices to choose the current extraction level. Introducing dynamic behavior is not likely to alter the features of \( E(Z) \) presented in Section 2, such as the monotonicity and concavity of \( E(\cdot) \). However, it would be difficult to obtain analytic results with a model of forward-looking agents. Given that environmental evolution typically takes place on a much longer time scale than human activities (especially human planning), myopic behavior may be a better approximation of reality than forward looking behavior. The assumption does limit the kind of environmental reform we study in this paper: reform is represented by a reduction of the number of extractors, rather than by increasing the farsightedness of the extractors.

7. Conclusions

We studied the differing effects of environmental reform in the short and the long run, under both free trade and autarky. Under autarky there is a single distortion, which causes a real effect only when the stock is low. Environmental reform either increases welfare or has no effect.

Under trade, there are two distortions in the two countries, so both the absolute and relative levels of these distortions may be important. In line with the theory of the second best, decreasing a single distortion does not necessarily improve welfare. In the short run, the race to the bottom and the race to the top increase (or have no effect on) aggregate welfare: only relative distortions matter. In the long run, the absolute levels of distortion are also important. Reform in the less distorted economy ameliorates an absolute distortion but worsens the relative distortion, and has ambiguous welfare effects. The net effect of this reform is more likely to be positive if the initial
gap between the distortions is not large, if the environment is not important, and if the environment is resilient. Thus, long-run considerations tend to favor upward harmonization, relative to downward harmonization.

Under autarky, the environmental distortion has no real effect when stocks are large, but under trade the environmental distortion always has real effects. Consequently, reform always has real effects under trade, but not necessarily under autarky. Trade increases the ability to use environmental reform to improve welfare. However, trade also makes it possible that environmental reform has perverse results.

Appendix A. Model details

The derivation of the autarky and trade equilibria is straightforward and rather standard. We refer readers to Chichilnisky [7] and KSZ for details. There are two instantaneous autarky equilibria depending on whether labor is fully employed. With full employment, the price of $B$ is $b_2 \phi / (\phi \cdot b_2)$ and the amount of environmental extraction is $\phi \cdot b_2$, where $\phi = a_2 L - A^* D > 0$, and $D = a_2 b_1 - a_1 b_2 > 0$. With unemployed labor, the price of $B$ is $\phi / a_1$ and the amount of extraction is $\delta Z / a_2$. Labor is fully employed if and only if $Z > Z^c$, where

$$Z^c = a_2 \phi / b_2 \delta.$$  \hfill (A.1)

Similarly, there are two instantaneous free trade equilibria. When labor is fully employed, the world price of $B$ is $b_2 \psi / (b_2 \psi - 2 \psi D)$, and the amount of resource extracted in country $i$ is $E_i^\psi = 2 \delta_i Z_i \phi / b_2 \psi$, where $\psi = \delta N Z_N + \delta S Z_S$. With unemployment, the world price and extraction are the same as under autarky. Labor is fully employed if and only if $\psi \geq \psi^c$, where

$$\psi^c = 2 a_2 \phi / b_2.$$  \hfill (A.2)

Appendix B. Derivations

B.1. Critical $\eta$ values

The critical values $\eta^a$ and $\eta^{*a}$ are determined by checking the existence (and nonexistence) of solution to $\dot{Z} = 0$ for $E$ functions associated with partial and full labor employment. $\eta^*$ and $\eta^{*a}$ are determined by checking the existence of solution to the simultaneous equations $\dot{Z}_S = 0$ and $\dot{Z}_N = 0$. We refer readers to KSZ for derivation details. The following gives the specific values.

$$\eta^a = 2 \sqrt{\gamma \phi / b_2},$$  \hfill (B.1)

$$\eta^{*a} = \delta / a_2 + \gamma Z^c,$$  \hfill (B.2)
\[
\eta = \frac{2\sqrt{\delta_S^2 + \delta_N^2}}{\delta_S + \delta_N} \sqrt{2\gamma b_2}, \tag{B.3}
\]

\[
\eta^* = \frac{\delta_N^2 + \delta_S^2 + a_2 \gamma \psi^c}{a_2 (\delta_N + \delta_S)}. \tag{B.4}
\]

We can further show that (Appendix I of KSZ)

\[
\eta^{*a} < \frac{2\delta}{a_2}. \tag{B.5}
\]

**B.2. Proof of Eq. (3)**

Eq. (3a) is straightforward from (B.1). Differentiating (B.2) with respect to \(\delta\) and using (B.5), we get (3c). Eqs. (3b) and (3d) are clear from Figs. 3 and 4a.

**B.3. The signs of \(\frac{\partial g}{\partial \delta_N}\) and \(\frac{\partial g}{\partial \delta_S}\)**

From (B.4), \(\frac{\partial g}{\partial \delta_N} = Hf_N(\delta_S, \delta_N)\) and \(\frac{\partial g}{\partial \delta_S} = Hf_S(\delta_S, \delta_N)\), where \(H > 0\) is a constant independent of \(\delta_S\) and \(\delta_N\), \(f_N(\delta_S, \delta_N) = (-\delta_S^2 + 2\delta_N \delta_S + \delta_N^2 - g)\) and \(f_S(\delta_S, \delta_N) = (\delta_S^2 + 2\delta_N \delta_S - \delta_N^2 - g)\), with \(g = \gamma a_2 \psi^c\). \(\delta_S > \delta_N\) implies \(f_N < f_S\), thus \(\frac{\partial g}{\partial \delta_N} < \frac{\partial g}{\partial \delta_S}\).

We first show Eq. (6). \(f_N\) is a quadratic equation in \(\delta_S\), and we are concerned with its sign for the relevant range of \(\delta_S\), \([\delta_N, 1]\). The two roots of \(f_N = 0\) are \(\delta_{S1} = \delta_N - \sqrt{2\delta_N^2 - g}\) and \(\delta_{S2} = \delta_N + \sqrt{2\delta_N^2 - g}\), and \(f_N > 0\) for \(\delta_S \in (\delta_{S1}, \delta_{S2})\). It is straightforward to show that \(\delta_{S1} < \delta_N\) and \(\delta_{S2} > 1\) when \(g \leq g^* \equiv \delta_N^2 + 2\delta_N - 1\), establishing the first part of Eq. (6). This result is shown in Fig. 5a.
For $g > g^*$, $\delta S_2 < 1$. To determine the position of $\delta S_1$, we argue that $f_N$ is positive at $\delta S = \delta N$. To show this, we use (B.5), from which we can show that $g < 2\delta S_1^2$. That is, $f_N(\delta N, \delta S) > 0$. Therefore, $\delta S_1 < \delta N$. This scenario is depicted in Fig. 5b, which also shows that $\delta S_1^*$, described above (6), is defined as the larger root of $f_N$: $\delta S_1^* \equiv \delta S_2$.

Now we show $\frac{d\delta S^*}{d\delta S} > 0$. It is straightforward to show that $f_S$ is increasing in $\delta S$ for $\delta S \in [\delta N, 1)$. From the fact that $f_S > f_N$ and $f_N > 0$ when $\delta S = \delta N$, we know $\frac{d\delta S^*}{d\delta S} > 0$. This result is illustrated in both panels of Fig. 5.

References


