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What would it take to Change an Inference?:

Using Rubin’s Causal Model to Interpret the Robustness of Causal Inferences

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Abstract

We contribute to debate about causal inferences in educational research in two ways. First, we quantify how much bias there must be in an estimate to invalidate an inference. Second, we utilize Rubin’s causal model (RCM) to interpret the bias necessary to invalidate an inference in terms of sample replacement. We apply our analysis to an inference of a positive effect of Open Court Curriculum on reading achievement from a randomized experiment, and an inference of a negative effect of kindergarten retention on reading achievement from an observational study. We consider details of our framework, and then discuss how our approach informs judgment of inference relative to study design. We conclude with implications for scientific discourse.

Keywords: causal inference; Rubin’s causal model; sensitivity analysis; observational studies

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**Introduction**

Education is fundamentally a pragmatic enterprise (e.g., National Research Council, 2002; Raudenbush, 2005), with the ultimate goal of educational research to inform choices about curricula, pedagogy, practices, or school organization (e.g., Bulterman-Bos, 2008; Cook, 2002).

To achieve that goal, educational researchers must pay careful attention to the basis for making causal inferences (e.g., Schneider et al., 2007). In Holland’s (1986) language if educational researchers do not infer the correct causes of effects then policy manipulations based on their research will not produce the intended results.

But study results can be ambiguous. As a result, debate about the general bases for causal inferences in the social sciences dates back to the 1900’s (e.g., Becker, 1967; Rubin, 1974; Thorndike & Woodworth, 1901; see Abbott, 1998 or Oakley, 1998 for reviews), with some heated as in the Cronbach vs Campbell exchanges of the 1980’s (e.g., Cook & Campbell, 1979; Cronbach, 1982). Debates have also emerged about specific causal inferences. For example, analyzing data from the federal longitudinal database High School and Beyond, Coleman, Hoffer and Kilgore (1982) estimated that students attending Catholic schools had higher achievement than similar students attending public schools leading to an inference that Catholic schools educate students better than public schools (Chubb and Moe 1990; Coleman, Hoffer and Kilgore 1982). Controversy ensued over the internal validity of the results: Despite controlling for background characteristics, can we ever be sure that the Catholic and public students being compared were really similar? Indeed in a critique of the Coleman findings, Alexander and Pallas (1983) noted that “… the single greatest burden of school effects research is to distinguish convincingly between outcome differences that reflect simply differences in the kinds of students who attend various schools from differences that are attributable to something about the schools themselves” (p. 170).

Given concerns about inferences from observational studies, several institutions, such as the What Works Clearinghouse (Eisenhardt & Towne, 2008) and the US Department of Education’s National Center for Education Research (NCER), have drawn on the medical model to call for a sound, scientifically rigorous basis for making causal inferences in educational research. In particular these institutions have emphasized the importance of random assignment to treatment conditions for making causal inferences; if subjects are randomly assigned to treatments then any preexisting differences between treatment groups will be eliminated in the long run (Fisher, 1970[1930]). Prominent examples of randomized experiments in educational research include evaluations of Sesame Street (Bogatz & Ball, 1972); the Perry Preschool Project (Schweinhart, Barnes, & Weikart, 1993); small classes (Finn & Achilles, 1991); and Comer’s School Development Program (see Cook, 2003, p. 123, for a review).

Despite their many virtues, even perfectly executed randomized experiments do not preempt debate about causal inferences. This is because it is rare when an educational researcher can randomly sample subjects from the desired target population and then also randomly assign subjects to meaningful treatment conditions (Cook, 2003). For example, imagine a researcher randomly sampling students and then telling some they had been randomly assigned a treatment, such as to attend a Catholic school. Consequently, randomized experiments are open to the critique that their external validity is limited by the representativeness of the sample on which the experiment was conducted. As a result most if not all educational research leaves the door open to debate because of a non-random sample and/or non-random assignment to treatments.

In this article, we put forth a framework that informs debate about causal inferences in educational research. As a foundation, we draw on Rubin’s causal model (RCM) (Rubin, 1974) to express concerns about bias in terms of the characteristics of unobserved data. In particular, we use RCM to characterize how one could invalidate inferences by replacing observed cases with unobserved cases in which there was no treatment effect. The underlying intuition is straight-forward: How much would a study sample have to change in order to change the inference? We answer this question using a framework that quantifies sources of bias rooted in either restricted sampling or non-random treatment assignment.

Equally importantly, our framework enables researchers to identify a “switch point” (Behn and Vaupel, 1982) where the bias is large enough to undo one’s belief about an effect (e.g., from inferring an effect to inferring no effect). Using the switching point, we transform external validity concerns such as: “I don’t believe the study applies to my population of interest” to questions such as “How much bias must there have been in the sampling process to make the inference invalid for a population that includes my population of interest?” Similarly with respect to internal validity, we transform statements such as “But the inference of a treatment effect might not be valid because of pre-existing differences between the treatment groups” to questions such as “How much bias must there have been due to uncontrolled pre-existing differences to make the inference invalid?”

Importantly, our analysis contributes to a process and discourse of inference for particular studies. Quantifying a switch point and interpreting in terms of sources of bias is a crucial step. Considered together with the capacity of the study design to reduce or eliminate bias, our framework can help researchers better evaluate whether bias is large enough to invalidate the inference of a study.

In the next Section we elaborate on the idea of a “switch point” for an inference, and provide a more formal definition of the robustness of an inference. In Section 3, using Rubin’s causal model (Rubin, 1974), we develop our framework in terms of missing data for interpreting the bias necessary to invalidate an inference. In Section 4 we apply the framework to Borman, Dowling and Schneck’s (2008) inference of a positive effect of the Open Court Curriculum from a randomized experiment on a volunteer population, and to Hong and Raudenbush’s (2005) inference of a negative effect of kindergarten retention from a random sample in an observational study. We consider choices for thresholds and then discuss how our approach informs judgment of inference relative to study design, compare with other approaches to quantifying discourse about inferences, and characterize other sources of bias. We conclude with implications for scientific discourse.

**The Robustness of an Inference: Comparing Evidence against a Threshold**

The starting point for our analysis is when one makes an inference about the effect of a policy because empirical evidence exceeds a given threshold. The threshold defines the point at which evidence from a study would make one indifferent to the policy choices. If the evidence were more in favor of the policy one would choose the policy, if the evidence were less one would not choose the policy. Given the pragmatic emphasis of educational research, the threshold could be the effect size where the benefits of a policy intervention outweigh its costs for either an individual or community. For example, a policy-maker might have a specific threshold at which the evidence is strong enough in favor of a curriculum to outweigh the costs of introducing that curriculum into her school. Or as is commonly the case in academic research, the threshold can be defined by statistical significance – the threshold is an estimate just large enough to be interpreted as unlikely to occur by chance alone (for a given a null hypothesis).

Regardless of the specific threshold, one can compare an estimate with a threshold to represent how much bias there must be switch the inference. The more the estimate exceeds the threshold, the more robust the inference with respect to that threshold. Therefore, we refer to the evaluation of the estimate against the threshold as the “robustness” of the inference.

Consider Figure 1, in which the treatment effects from hypothetical studies A (estimated effect of six) and B (estimated effect of eight) each exceed the threshold of four. If the threshold of four represents an effect large enough to infer that the benefits of a study outweigh its costs, then in both cases one would draw the inference that the effect of the treatment was strong enough to implement. But the estimated effect from study B exceeds the threshold by more than does the estimate from A. Assuming that the estimates were obtained with similar levels of control for selection bias in the design of the study and similar levels of precision, the inference from study B is more robust than that from A because a greater proportion of the estimate from B must be due to bias to invalidate the inference.

Insert Figure 1 here

The relative robustness of an inference can be explicitly quantified in terms of the difference between an estimate and a threshold, expressed relative to the size of the estimate:

(*estimate−threshold*)/*estimate* = 1-*threshold*/*estimate*. (1)

Equation (1) simply implies that the robustness of an inference is a function of the percentage of the estimate that exceeds the threshold. For study A, (*estimate−threshold*)/*estimate*=(6−4)/6=1/3, or 33%. Thus 33% of the estimate from A would have to be due to bias to invalidate the inference. In contrast, 50% of the estimate for study B would have to be due to bias to invalidate the inference (8-4)/8=50%.

More formally, define a population effect as δ, the estimated effect as, and the threshold for making an inference as δ#. For example, to account for sampling error δ # might be the threshold for statistical significance (δ# is associated with a p value of exactly .05). An inference about a positive effect is invalid if:

 > δ # > δ. (2)

That is, an inference is invalid if the estimate is greater than the threshold while the population value is less than the threshold (a symmetric argument applies for negative effects). For example, the inference from hypothetical study A is invalid if 6 > 4 >δ.

The expression in (2) can be used to quantify how much bias there must be in an estimate to invalidate an inference. Subtracting  from each side in (2) and multiplying by -1 yields:

−δ> − δ # > 0.

Defining bias as β= −δ, (2) implies an estimate is invalid if and only if:

β >− δ #. (3)

An inference is invalid if bias accounts for more than the difference between the estimate and the threshold.

To express (3) as a proportion of the original estimate, divide the right hand side by:

(−δ#)/  =1 −δ#/ . (4)

This is equivalent to (1); the proportion of bias necessary to invalidate the inference is equivalent to the graphical comparison of an estimate to a threshold for inference. If an unbiased test statistic is used and assuming no random sampling error, (3) and (4) express how much bias due to the design components there must be to invalidate an inference based on. The challenge then is to interpret the expressions in (3) and (4) in a framework that can be applied to observational studies or randomized experiments. For this we turn to Rubin’s causal model in the next Section.

**Rubin’s Causal Model (RCM) and Sources of Bias**

*Potential Outcomes*

RCM is best understood through the counterfactual sequence: I had a headache; I took an aspirin; the headache went away. Is it because I took the aspirin? One will never know because we do not know what I would have experienced if I had not taken the aspirin. One of the potential outcomes I could have experienced by either taking or not taking an aspirin will be counter to fact, termed the counterfactual within RCM (for a history and review of RCM see Holland, 1986; or Morgan & Winship, 2007, chapter 2). In one of the examples in this study, it is impossible to observe a single student who is simultaneously retained in kindergarten and promoted into the first grade.

Formally expressing the counterfactual in terms of potential outcomes shows how RCM) n bed as the er to fact. erpreted using ated effect.lid ofsive leavn Instead, Frank'absorb impact, where absorption=tm can be applied to represent bias from non-random assignment to treatments or non-random sampling. Define the potential outcome  as the value on the dependent variable (e.g., reading achievement) that would be observed if unit *i* were exposed to the treatment (e.g., being retained in kindergarten); and as the value on the dependent variable that would be observed if unit *i* were in the control condition and therefore not exposed to the treatment (e.g., being promoted to the first grade). i  If SUTVA (Rubin, 1986, 1990) holds – that there are no spillover effects of treatments from one unit to another – then the causal mechanisms are independent across units, and the effect of the treatment on a single unit can be defined as

δi= −. (5)

The problems of bias due to non-random assignment to treatment are addressed in RCM by defining causality for a single unit– the unit assigned to the treatment is identical to the unit assigned to the control. Similarly, there is no concern about sampling bias because the model refers only to the single unit *i.*

Of course, RCM does not eliminate the problems of bias due to non-random assignment to treatments or non-random sampling. Instead, it recasts these sources of bias in terms of missing data (Holland, 1986), because for each unit, one potential outcome is missing. We use this feature to describe characteristics of missing data necessary to invalidate an inference.

*Application to Non-Random Assignment to Treatment*

Consider a study in which the units were randomly sampled but were not randomly assigned to treatments (e.g., an observational study of the effects of kindergarten retention on achievement). In this case we would focus on interpreting the bias necessary to invalidate an inference due to non-random assignment to treatment, a component of internal validity (Cook & Campbell, 1979). Using notation similar to that of Morgan and Winship (2007), let X*=t* if a unit received the treatment and X*=c* if a unit received the control. Y*t*| X*=t* is then the value of the outcome Y for a unit exposed to the treatment, and Y*c*| X*=t* is the counterfactual value of Y under the control condition for a unit that was exposed to the treatment. For example Yretained| X=retained is the observed level of achievement for a student who was retained in kindergarten, while Ypromoted| X=retained is the unobserved level of achievement for the same student if he had been promoted.

Using this notation, and defining bias as β= E[]−E[], in technical appendix A we show the bias due to nonrandom assignment to treatments, βa, is:

βa=π{ E[Y*c*|X*=t*]−E[Y*c*|X*=c*]}+(1−π){E[Y*t*|X*=t*]−E[Y*t*|X*=c*]} . (6)

In words, the term E[Y*c*| X*=t*]−E[Y*c*| X*=c*] represents bias introduced by comparing members of the treatment group with members of the observed control (Y*c*| X*=c*) instead of their counterfactual: members of the treatment group if they had received the control (Y*c*|X*=t*). Similarly, E[Y*t* |X*=t*] − E[Y*t* | X*=c*] represents bias introduced by comparing members of the control with members of the observed treatment (Y*t*| X*=t*) instead of their counterfactual: members of the control if they had received the treatment (Y*t*| X*=c*). The bias attributed to the incorrect comparison for the treatment group is weighted by the proportion in the treatment group, π; and the bias attributed to the incorrect comparison for the control group is weighted by 1−π.

*Application to a Non-Random Sample*

Now consider a study in which the units were randomly assigned to treatments but were *not* randomly sampled from the population to which one would like to generalize. In this case the target population consists of both those directly represented by the sample as well as those not directly represented by the sample – one might be concerned with statements about general causes across populations, known as external validity (Cook & Campbell, 1979, page 39). As an example in this paper, one might seek to make an inference about the effect of the Open Court curriculum beyond the population of schools that volunteered for a study of Open Court (e.g., Borman et al., 2008).

To quantify robustness with respect to external validity we adapt RCM to focus on bias due to non-random sampling. Instead of the unobserved data defined by the counterfactual, consider a target population as comprised of two groups, one that has the potential to be observed in a sample, *p*, and one does not have the potential to be sampled but is of interest, *p´*. For example, consider population *p* to consist of schools that volunteered for a study of the Open Court curriculum, and population *p´* to consist of schools that did not volunteer for the study. Although the study sample can only come from those schools that volunteered for the study, one might seek to generalize to the broader population of schools including *p´* as well as *p.*

To formally adapt RCM to external validity, decompose the combined population treatment effect,, into two components: *p*, the treatment effect for the population potentially sampled, and *p´*, the treatment effect for the population not potentially sampled (Cronbach, 1982; Fisher, 1970[1930]; Frank & Min, 2007). Assuming the proportion of units receiving the treatment is the same in *p* and *p´*, an expression for an unbiased treatment estimate across both populations is

= π*p**p*+(1− πp) *p´*, (7)

where π*p* represents the proportion from *p* in the sample representative of *p* and *p´*.

We can use (7) to develop expressions for bias in estimated treatment effects due to a non-random sample. Let Z=*p* if a unit is from *p*, and Z=*p´* if a unit is from *p´*. Combining this notation with our earlier notation (e.g., Y*t*| Z=*p* represents the treatment outcome for a school in the population of schools that volunteered for the study), and defining bias due to non-random sampling as βs, in technical appendix A we show:

βs =(1−π p){( E[Y*t*|Z=*p*]−E[Y*c*|Z=*p*])− (E[Y*t*|Z=*p´*]−E[Y*c*|Z=*p´*])} . (8)

In words, the bias is due to an estimate based on 1−π p units of *p* witheffect E[Y*t*|Z=*p*]−E[Y*c*|Z=*p*] instead of units from *p´* with effect E[Y*t*|Z=*p´*]−E[Y*c*|Z=*p´*]. Equations (7) and (8) show how RCM can be used to generate expressions for bias due to non-random sampling as well as non-random assignment to treatments. ii

*Limiting Condition: No Treatment Effect*

Expressions (6) and (8) can be used to quantify the robustness of an inference by considering replacing observed data with hypothetical data under the limiting condition of no treatment effect in the hypothetical data. Interpreting bias in terms of replacement data expresses validity in terms of the exchangeability of observed and unobserved cases. External validity concerns about bias due to a non-random sample can be cast in terms of the proportion of cases in the observable population *p*  that are unexchangeable with the unobservable population *p´* in the sense that one must replace the proportion in *p* to make a causal inference that applies to *p* and *p´*. Similarly, internal validity concerns about bias due to non-random treatment assignment can be cast in terms of the proportion of the observed cases that are unexchangeable with the optimal counterfactual cases such that one must replace the proportion of observed cases to make a causal inference in the sample.

*Non-random sampling.* Bias due to non-random sampling in (8) can be expressed by assuming the null hypothesis of zero effect holds in the replacement units as might be claimed by a skeptic of the inference (Frank & Min, 2007). In this case, E[Y*t*|Z=*p´*] = E[Y*c*|Z=*p´*] and substituting into (8) yields:

βs =(1−π*p*){ E[Y*t*|Z=*p*] − E[Y*c*|Z=*p*]} = (1−π*p*) . (9)

Setting βs > −δ# and solving (9) for (1−π*p*), the inference is invalid if:

(1−π*p*) >1− δ# /. (10)

Equation (10) is particular example of the bias necessary to invalidate an inference shown in (4), in this case in terms of bias due to non-random sampling associated with π*p* (this replicates the result in Frank & Min, 2007).

*Non-random assignment to treatments.* We can use equation (6) to isolate the conditions that could invalidate an inference due to bias from non-random assignment to treatment conditions. Following Morgan and Winship (2007, page 46), (6) can be rewritten as:

βa= {E[Y*c*|X*=t*]−E[Y*c*|X*=c*]}+

(1−π){(E[Y*t*|X*=t*]−E[Y*c*|X*=t*]) – (E[Y*t*|X*=c*]−E[Y*c*|X*=c*])} . (11)

Thus bias is a function of expected differences at baseline, or in the absence of a treatment E[Y*c*|X*=t*]−E[Y*c*|X*=c*]), and differential treatment effects for the treated and control (E[Y*t*|X*=t*]−E[Y*c*|X*=t*])− (E[Y*t*|X*=c*]−E[Y*c*|X*=c*])=δ*t*− δ*c*.

Assuming δ*t*− δ*c*=0 as in the limiting case when there is no treatment effect (δ*t*=δ*c*=0) and setting βa >−δ# implies an inference is invalid if:

βa =(E[Y*c*|X*=t*]−E[Y*c*|X*=c*]) > −δ#. (12)

Equation (12) indicates the bias due to differences in the absence of treatment necessary to invalidate the inference.

To interpret (12) in terms of sample replacement, note that bias due to the difference in expected values in the absence of treatment can be re-expressed in terms of the expected value of the differences in the absence of treatment:

(E[Y*c*|X*=t*]−E[Y*c*|X*=c*]) > −δ#

 E[(Y*c*|X*=t*)−(Y*c*|X*=c*)] > −δ# . (13)

Now separate the sample into the proportion in which there is no bias (α), and the proportion in which there is bias (1−α):

(α)(E[(Y*c*|X*=t*)−(Y*c*|X*=c*)] + (1-α)E[(Y*c*|X*=t*)−(Y*c*|X*=c*)] > −δ# . (14)

Setting E[(Y*c*|X*=t*)−(Y*c*|X*=c*)=0 for those cases in which there is no bias because the expected value of the counterfactuals equals the expected value of the observed controls, and E[(Y*c*|X*=t*)−(Y*c*|X*=c*)= − δ =  for those cases in which the estimated effect is entirely due to bias (because δ =0) yields:

1-α > 1−δ# /. (15)

From (15), an inference would be invalid if (1−δ# /) or more of the cases in which there was bias were replaced with counterfactual data for which there was no treatment effect. Thus equation (15) is particular example of the bias necessary to invalidate an inference shown in (4), in this case in terms of bias due to non-random assignment to treatments, associated with 1-α.

In the next Section we use our framework to quantify the robustness of inferences from empirical studies. For each study we quantify the extent of bias necessary to invalidate the inference and interpret using our framework based on RCM. We then compare with one prominent and one recent study of similar design. As a set, the studies address issues of assignment of students to schools, school policy and structure, curriculum, and teacher knowledge and practices. In technical appendix B we then apply our framework to all of the studies available on-line (as of Dec 19, 2012) for upcoming publication in this journal.

**Examples**

*The Effect of Open Court Reading on Reading Achievement*

Borman et al. (2008) motivated their randomized experiment of the Open Court Reading (OCR) curriculum on reading achievement by noting “The Open Court Reading (OCR) program, published by SRA/McGraw-Hill, has been widely used since the 1960s and offers a phonics-based K-6 curriculum that is grounded in the research-based practices cited in the National Reading Panel report (National Reading Panel, 2000)” (page 390). Furthermore, the program is quite popular: “…to date, a total of 1,847 districts and over 6,000 schools have adopted the OCR program across the United States” (page 390). And yet from the perspective of internal validity, Borman et al. stated “despite its widespread dissemination, though, OCR has never been evaluated rigorously through a randomized trial” (page 390).

Based on an analysis of 49 classrooms randomly assigned to OCR versus business as usual, Borman et al. (2008) found OCR increased students’ composite reading score across all grades by 7.95 points (in Table 4 of Borman et al). This effect was about 1/7 of the standard deviation on the achievement test, equivalent to 1/8 of a year’s growth and was statistically significant (p <.001, t-ratio of 4.34). Borman et al. concluded that OCR affects reading outcomes: “The outcomes from these analyses not only provide evidence of the promising one-year effects of OCR on students’ reading outcomes, but they also suggest that these effects may be replicated across varying contexts with rather consistent and positive results” (page 405).

In making their inference, Borman et al. (2008) were explicitly concerned with how well their sample represented broad populations of classrooms. Thus they randomly sampled from schools that expressed interest to the Open Court developer SRA/McGraw-Hill. Partly as a result of the random sample, schools in Borman et al.’s sample were located across the country (Florida, Georgia, Indiana, Idaho, North Carolina, and Texas) and varied in socioeconomic status. Furthermore, Borman et al. carefully attended to sample attrition in their analyses.

Even given Borman et al. (2008)’s attention to their sample, there may be important concerns about the external validity of Borman et al.’s inference. In particular, schools that had approached SRA/McGraw-Hill prior to Borman et al.’s study may have had substantive reasons for believing OCR would work particularly well for them (e.g., Heckman, 2005). At the very least, if Borman et al.’s study had any effect on adoption of OCR, then one could not be certain that the post-study population (*p´*)was represented by the pre-study population (*p*), because the pre-study population did not have access to Borman et al.’s results. This is a fundamental limitation of external validity; a researcher cannot simultaneously change the behavior in a population and claim that the pre-study population fully represents the post-study population. Given the limitation for a sample in a randomized experiment to represent non-volunteer or post-study populations, debate about a general effect of OCR is inevitable.

To inform debate about the general effect of OCR, we quantify the proportion of Borman et al.’s estimate that must be due to sampling bias to invalidate their inference. We begin by choosing statistical significance as a threshold for inference because it reflects sampling error (although we will comment at the end of this Section on how other thresholds can be used). Given Borman et al.’s sample size of 49 (and 3 parameters estimated) and standard error of 1.83, the threshold for statistical significance is δ# = se x tcritical, df=46 =1.83 x 2.013=3.68. Given the estimated effect () was 7.95, to invalidate the inference bias must be greater than 7.95−3.68=4.27, which is 54% of the estimate.

Drawing on the general features of our framework, to invalidate Borman et al.’s inference of an effect of OCR on reading achievement one would have to replace 54% of the cases in study, and assume the limiting condition of zero effect of OCR in the replacement cases. Applying (10), the replacement cases would come from the non-volunteer population (*p´*). That is, 54% of the observed volunteer classrooms in Borman et al.’s study must be unexchangeable with the unobserved non-volunteer classrooms in the sense that it is necessary to replace those 54% of cases with unobserved cases for the inference to be valid a population that includes non-volunteer cases.

To gain intuition about sample replacement, examine Figure 2 which shows distributions for business as usual and Open Court before and after sample replacement. The dashed line represents the observed data based on parameter estimates from Borman et al.’s multilevel model, including pre-test as a covariate (data were simulated with mean for business as usual = 607; mean for Open Court =615 –see Borman et al., 400-402). The replacement data were constructed to preserve the original mean of roughly 611 and standard deviation (assumed to be 6.67 based on Borman et al.’s results and assuming equal variance within groups). iii

Insert Figure 2 here

The black bars in Figure 2 represent the 46% (n=19) of classrooms that were not replaced, and the gray bars represent the 54% (n=30) replacement classrooms randomly selected from a hypothetical population of classrooms that did not volunteer for the study (*p´*). iv For business as usual, the mean for the replacement data was about 4 points greater than the observed data. For OCR the mean for the replacement data was about 3 points less than the observed data, narrowing the difference between the curricula by about 7 points (the data with the replacement values were associated with t-ratio of 1.86 with a p-value of .068). The graphic convergence of the distributions represents what it means to replace 54% of the data with data for which there is no effect.

We now compare the robustness of Borman et al.’s inference with the robustness of inferences from two other randomized experiments: Finn and Achilles’ (1990) prominent study of the effects of small classes on achievement in Tennessee, and Clements and Sarama’s (2008) relatively recent study of the effects of the Building Blocks curriculum on preschoolers’ mathematical achievement. The latter offers an important contrast to the scripted OCR curriculum because it is molded through continuous interaction between developers and teachers who implement the curriculum.

For comparison of robustness across studies we conduct our analysis using the standardized metric of a correlation. v As shown in Table 1, 47% of Borman et al.’s estimated correlation between OCR and achievement must be due to bias to invalidate their inference. By comparison, 64% of Finn and Achilles’ (1990) estimate must be due to bias to invalidate their inference, and Clements and Sarama’s (2008) inference would be invalid even if only 31% of their estimate were due to sampling bias. Note that these are merely statements about the relative robustness of the causal inferences. In order to inform policy related to curricula or small classes, administrators and policy makers should take into account the characteristics of the study designs (e.g., what Shadish, Cook and Campbell, 2002, refer to as surface similarity), as well as the costs of implementing a particular policy in their contexts.

Insert Table 1 here

*The Effect of Kindergarten Retention on Reading Achievement*

We now quantify the robustness of an inferred negative effect of kindergarten retention on achievement from an observational study. Similar to the Open Court Curriculum, kindergarten retention is a large scale phenomenon, with the US Department of Health and Human Services (2000) estimating that 8% of second graders (more than five hundred thousand) were a year behind their expected grade level as a result of not being promoted, known as retention, in kindergarten or first grade (see also Alexander, Entwisle & Dauber, 2003, p. 1). Furthermore, a disproportionate percentage of those retained are from low socioeconomic backgrounds and/or are racial minorities (Alexander et al., chap. 5). As Alexander et al. wrote: “next to dropout, failing a grade is probably the most ubiquitous and vexing issue facing school people today” (p. 2).

Given the prevalence and importance of retention, there have been considerable studies and syntheses of retention effects (e.g., Alexander et al., 2003; Holmes, 1989; Holmes & Matthews, 1984; Jimerson, 2001; Karweit, 1992; Reynolds, 1992; Roderick et al., 1999; Shepard & Smith, 1989). Yet none of these studies has been conclusive, as there has been extensive debate regarding the effects of retention, especially regarding which covariates must be conditioned on (e.g., Alexander, 1998; Alexander et al., 2003; Shepard, Smith & Marion, 1998).

Because of the ambiguity of results, a study of the effects of kindergarten retention that employed random assignment to conditions at any level of analysis would be welcome. But as Alexander et al. (2003, p. 31) wrote, “random assignment, though, is not a viable strategy [for studying retention] because parents or schools would not be willing to have a child pass or fail a grade at the toss of a coin, even for purposes of a scientific experiment (see Harvard Education Letter, 1986:3, on the impracticality of this approach). Also, human subjects review boards and most investigators would demur for ethical reasons.” This is a specific example of Rubin’s (1974) concerns about implementing randomized experiments as well as Cronbach’s (1982) skepticism about the general feasibility of random assignment to treatments.

In the absence of random assignment, we turn to studies that attempted to approximate the conditions of random assignment using statistical techniques. Of the recent studies of retention effects (Burkam et al., 2007; Jimerson, 2001; Lorence et al., 2002), we focus on Hong and Raudenbush’s (2005) analysis of nationally representative data in the Early Childhood Longitudinal Study (ECLS) which included extensive measures of student background, emotional disposition, motivation, and pretests.

Hong and Raudenbush (2005) used the measures described above in a propensity score model to define a “retained counterfactual” group representing what would have happened to the students who were retained if they had been promoted (e.g., Rubin 1974; Holland 1986). As represented in Figure 3, Hong and Raudenbush estimated that the “retained observed” group scored nine points lower on reading achievement than the “retained counterfactual” group at the end of first grade. vi The estimated effect was about two thirds of a standard deviation on the test, almost half a year’s expected growth (Hong & Raudenbush, p. 220), and was statistically significant (p < .001, with standard error of.68, and t-ratio of −13.67). vii Ultimately, Hong and Raudenbush concluded that retention reduces achievement: “children who were retained would have learned more had they been promoted” (page 200).

Insert Figure 3 here

Hong and Raudenbush (2005) did not employ the “Gold Standard” of random assignment to treatment conditions (e.g., US Department of Education, 2002; Eisenhardt & Towne, 2008). Instead they relied on statistical covariates to approximate equivalence between the retained and promoted groups. But they may not have conditioned for some factor, such as an aspect of a child’s cognitive ability, emotional disposition, or motivation, which was confounded with retention. For example, if children with high motivation were less likely to be retained and also tended to have higher achievement, then part or all of Hong and Raudenbush’s observed relationship between retention and achievement might have been due to differences in motivation. In this sense, there may have been bias in the estimated effect of retention due to differences in motivation prior to, or in the absence of, being promoted or retained.

Our question then is not whether Hong and Raudenbush’s estimated effect of retention was biased because of variables omitted from their analysis. It almost certainly was. Our question instead is “How much bias must there have been to invalidate Hong and Raudenbush’s inference?” Using statistical significance as a threshold for Hong and Raudenbush’s sample of 7639 (471 retained students and 7168 promoted students, page 215 of Hong & Raudenbush), and standard error of .68, δ# = se x tcritical, df=7600=.68 x (−1.96)= −1.33. Given the estimated effect of −9, to invalidate the inference bias must have accounted for −9− −1.33= −7.67 points on the reading achievement measure, or about 85% of the estimated effect (−7.67/−9=.85).

Drawing on the general features of our framework, to invalidate Hong and Raudenbush’s inference of a negative effect of kindergarten retention on achievement one would have to replace 85% of the cases in their study, and assume the limiting condition of zero effect of retention in the replacement cases. Applying (15), the replacement cases would come from the counterfactual condition for the observed outcomes. That is, 85% of the observed potential outcomes must be unexchangeable with the unobserved counterfactual potential outcomes such that it is necessary to replace those 85% with the counterfactual potential outcomes to make an inference in this sample. Note that this replacement must occur even after observed cases have been conditioned on background characteristics, school membership, and pretests used to define comparable groups.

Figure 4 shows the replacement distributions using a procedure similar to that used to generate Figure 2, although the gray bars in Figure 4 represent *counterfactual* data necessary to replace 85% of the cases to invalidate the inference (the difference between the retained and promoted groups after replacement is -1.25, p =.064). The left side of Figure 2 shows the 7.2 point advantage the counterfactual replacement cases would have over the students who were actually retained *retained*|x*=promoted*−*retained*|x*=retained*= 52.2−45.0=7.2). This shift of 7.2 points works against the inference by shifting the retained distribution to the right, towards the promoted students (the promoted students were shifted less than the retained students in order to preserve the overall mean). viii

Insert Figure 4 here

Our analysis appeals to the intuition of those who consider what would have happened to the promoted children if they had been retained, as these are exactly the RCM potential outcomes on which our analysis is based. Consider test scores of a set of children who were retained that are considerably lower (9 points) than others who were candidates for retention but who were in fact promoted. No doubt some of the difference is due to advantages the comparable others had before being promoted. But now to believe that retention did not have an effect one must believe that 85% of those comparable others would have enjoyed most (7.2) of their advantages whether or not they had been retained. This is a difference of more than a 1/3 of a year’s growth. ix Although interpretations will vary, our framework allows us to interpret Hong and Raudenbush’s inference in terms of the ensemble of factors that might differentiate retained students from comparable promoted students. In this sense we quantify the robustness of the inference in terms of the experiences of promoted and retained students and as might be observed by educators in their daily practice.

We now compare the robustness of Hong and Raudenbush’s (2005) inference with the robustness of inferences from two other observational studies: Morgan’s (2001) inference of a Catholic school effect on achievement (building on Coleman et al., 1982), and Hill, Rowan and Ball’s (2005) inference of the effects of a teacher’s content knowledge on student math achievement. Hill et al.’s focus on teacher knowledge offers an important complement to attention to school or district level policies such as retention because differences among teachers are important predictors of achievement (Nye, Konstantopoulos & Hedges, 2004).

As shown in Table 2, Morgan’s (2001) inference and Hill et al.’s inference would not be valid if slightly more than a third of their estimates were due to bias. By our measure, Hong and Raudenbush’s inference is more robust than that of Morgan or Hill et al. Again, this is not a final proclamation regarding policy. In choosing appropriate action, policymakers would have to consider the relative return on investments of policies related to retention, incentives for students to attend Catholic schools, and teachers’ acquisition of knowledge (e.g., through professional development). Furthermore, the return on investment is not the only contingency, as decision-makers should consider the elements of the study designs already used to reduce bias. For example, we call attention to whether the observational studies controlled for pretests (as did Hong and Raudenbush as well as Morgan, 2001) which have recently been found to be critical in reducing bias in educational studies (e.g., Shadish, Clark & Steiner, 2008; Steiner et al., 2010, 2011).

Insert Table 2 here

*Expanded the Details of our Framework*

*Choosing a threshold relative to transaction costs.* The general framework we have proposed can be implemented with any threshold. But given that educational research should be pragmatic, the threshold might depend on the size of the investment needed to manipulate policy or practice. Individuals or families might be comfortable with a lower threshold than policy-makers considering diverting large resources to change the experiences of many people. Therefore, the following is a guide for increasing thresholds based on the transaction costs of program change.

1. Changing beliefs, without a corresponding change in action.
2. Changing action for an individual (or family)
3. Increasing investments in an existing program.
4. Initial investment in a pilot program where none exists.
5. Dismantling an existing program and replacing it with a new program.

Note that the first level does not even constitute a change in action. In this sense it is below a pragmatic threshold for action. The values of the thresholds needed to invalidate an inference increase from levels 2 through 5 as the actions require greater resources. An inference should be more robust to convince a policy maker to initiate a whole scale change in policy than to convince a family to choose a particular treatment.

*Non-zero null hypotheses.* For Hong and Raudenbush’s inference of a negative effect of retention on achievement, consider a null hypothesis adequate for increasing investment in an existing program. For example, define the threshold by δ> −6, where 6 units represents about ¼ of a year of growth, slightly less than half a standard deviation on Hong and Ruadenbush’s outcome. For a null hypothesis defined by δ> −6, the threshold for statistical significance is (se x tcritical, df=7639)=.68 x (−1.645)= −1.12 (using a one tailed test). Therefore δ# = −6−1.12=−7.12, and 1−δ #/=1− (−7.12/−9)=.21. The result is that 21% of the estimated kindergarten retention effect would have to be due to differences between the students before being retained or promoted to invalidate the inference that retention has an effect using a threshold of −6. Thus quantifying the robustness of an inference for non-zero hypotheses can represent uncertainty about qualitative policy decisions based on fixed thresholds.

*Failure to reject the null hypothesis when in fact the null is false*. We have focused on the extent of bias necessary to create a type I error (rejecting the null hypothesis when in fact the null hypothesis is true). It is important note, however, that bias could also hide a substantively important negative effect. This is an example of type II error, failure to reject the null when in fact the null hypothesis is false. Critically, from an ethical or policy perspective type II errors may require different thresholds than type I errors. For example, in medical trials the threshold for discontinuing a trial due to potential harm is not as conservative as criteria used to infer a positive treatment effect (Federal Register, 1998, page 49592).

When there is concern that bias may have hidden a negative effect, one could define δ# as the threshold for inferring a negative effect of a treatment and then quantify the bias necessary to have created a false inference that there is no negative effect. For example, if a value of δ# of −4 would be strong enough to infer a negative effect and the estimate were −3, then the bias necessary to invalidate the inference would be 1−δ# /=1−(−4)/−3)=−1/3. If the original estimate is biased by 1/3 then the inference of no negative effect is invalid. Alternatively, one could report the coefficient by which the estimate would have to be multiplied to exceed the threshold for inference. This is simply the ratio of the observed estimate to its threshold. For example, one would have to multiply an observed effect of −3 by 1.33 to make it exceed the threshold of −4.

*Non-zero effect in the replacement (non-volunteer) population.* In our examples so far, we have assumed that replacement cases have a zero treatment effect. But our general framework also applies to the conditions necessary to invalidate an inference if one assumes a non-zero treatment effect in the replacement population. This can be illustrated in equation (7). Setting the combined population treatment effect to be less than the threshold for inference ( < δ#) and solving for the proportion of the original sample to be replaced (1-π*p*), yields that the inference is invalid if 1-π*p*<(*p*− δ#)/(*p*−*p´*).

In Borman et al.’s example, assume the effect of OCR in the non-volunteer population,, is −2, and that δ#=3.68 and *p*=7.95 (both as in the initial example). Under these conditions, the inference is invalid if 1-π*p*<(7.95– 3.68)/(7.95− −2) =.43; the inference would be invalid if more than 43% of the sample were replaced with cases for which the effect of OCR was −2 *.* Intuitively, to invalidate the inference one would have to replace a smaller percentage (43% versus 54%) if there is a negative versus zero effect in the replacement cases.

**Discussion**

Judgment is required to interpret any causal inference in educational policy research. For instance, Borman et al.’s inference that the effects of OCR “may be replicated across varying contexts with rather consistent and positive results,” (page 405) may not apply to schools that did not volunteer for participation in OCR. Hong and Raudenbush’s (2005) inference that “children who were retained would have learned more had they been promoted” (page 200) may be invalid if some other factor affected the likelihood of retention and achievement.

To inform interpretation of inferences, we have quantified how much bias must be to invalidate an inference. How much of Borman et al.’s (2008) estimate must be due to unusual effectiveness in their sample to invalidate their inference? The answer is 54%. Interpreting in terms of our framework, to infer that OCR does not have a general effect pertaining to a population that includes volunteer and non-volunteer schools, one would have replace 54% of the volunteer classrooms in Borman et al.’s study with non-volunteer classrooms in which there was no effect of OCR. This adds precision to Borman et al.’s language of “may be replicated” and “rather consistent.”

How much of Hong and Raudenbush’s estimate must be due to bias to invalidate their inference? The answer is 85%. Interpreting in our framework in terms of counterfactual data, to believe that retention did not have an effect, one would have to believe that 85% or more of promoted students (who were comparable to retained students in terms of background, emotional disposition, school membership, and pretests) would have held most of their advantage whether or not they repeated a year of kindergarten.

The bias necessary to invalidate an inference should be evaluated relative to the study design. The sampling bias necessary to invalidate Borman et al.’s inference from a randomized experiment on a volunteer sample should be evaluated relative to the characteristics of the sample and desired population, as well as the sampling mechanism. They did include a range of socioeconomic status and region which should make their data representative of broader populations, but all of the schools in their study volunteered for the study, potentially differentiating them from non-volunteer schools. Quantifying this concern, 54% of the volunteer schools would have to be unrepresentative of the volunteer schools to invalidate the inference.

Similarly, the extent of selection bias necessary to invalidate Hong and Raudenbush’s inference should be evaluated relative to the mechanism of treatment assignment, as well as the statistical controls employed to reduce selection bias. Hong and Raudenbush did control for background, emotional disposition and pretests, ruling out many of the most ready explanations for differences between the retained and promoted students. But questions may persist about remaining, uncontrolled differences. Quantifying this concern, 85% of the observed students would have to be replaced with unbiased counterfactual cases (for which there was no treatment effect) to invalidate the inference.

Evaluating the robustness of an inference relative to study design has two important implications. First, there is limited value in directly comparing the bias necessary to invalidate an inference between studies of different designs (e.g., randomized experiments and observational studies). Second, the more bias favoring the treatment the study has accounted for the less robust the inference will appear to be, because adjusting for bias moves the estimate closer to the threshold for inference. In this context, larger explained variance in the outcome (R2) is one indicator of a good model for an observational study because it suggests that many of the most important explanations of the outcome have been accounted for.

*Relation to Other Approaches*

*Bounding an effect.* Altonji et al. (2005, 2010) bounded estimated effects by drawing on information associated with observed confounds (cf. Manski, 1990, who bounds based on the maximum value of the outcome). Consistent with this, Hong and Raudenbush (page 220), reported how their estimate would have changed if it was reduced by an omitted variable comparable to their most important covariate. In our own analysis (reported in endnote *ix*), the estimated retention effect was reduced by .074 when we added emotional disposition to a model already including pretests, background characteristics, and fixed effects for schools. If unobserved confounds accounted for as much reduction, the lower bound of the kindergarten retention effect would be −9.320+.074=−9.246.

Of course the preceding analysis is only as good as the assumption that bias associated with unobserved confounders is no greater than bias associated with observed confounders. Such would not be the case if there were some unique factor associated with kindergarten retention that was not captured in the general controls included in ECLS-K. Or if there were an omitted factor whose effect on being retained was not substantially absorbed by the pretests in ECLS-K. x This forces consumers of research to carefully consider how potential confounders available in a data set were selected from the set of all possible confounders (Altonji et al., 2010; see also Steiner et al.’s 2010 attention to factors affecting choice of treatments).

Generally, while we recognize the great value of bounding an effect, bounding an effect supports different understanding than quantifying the robustness of an inference. The lower bound of −9.246 provides information about a worst (or best) case scenario whereas we incorporate different effects as thresholds in our framework, and then quantify the bias necessary to reduce an estimate below the threshold. Ultimately, we believe both can be useful to researchers and policy-makers.

*Other sensitivity analyses.* Typical sensitivity analyses are expressed in terms of the properties of the omitted variables that could alter an inference either because they affected selection into the sample or assignment to treatments or both (e.g., Copas & Li,1997; Holland, 1989; Lin, Psaty & Kronmal, 1998; Robins, Rotnisky & Scharfstein, 2000; Rosenbaum, 1986, 2002; Rosenbaum & Rubin, 1983; Scharfstein, 2002). For example, Rosenbaum, (1986, page 221) wrote: “The inclusion of an omitted variable U, which is as predictive as the most predictive covariate in the short list of covariates excluding the pretest, would have a relatively modest impact on the estimated effect [of dropout on cognitive test scores] unless that variable had a substantially larger dropout-versus-stayer difference than any covariate under study.”

Rosenbaum’s statement is complex partly because one must consider two relationships associated with an omitted variable: the relationship with the predictor of interest (e.g., drop-out), and the relationship with the outcome (e.g., cognitive test scores). Technically, this can be dealt with by conducting dual sensitivity (Gastwirth, Krieger & Rosenbaum, 1998) or by characterizing sensitivity analysis in terms of the product of the two relationships (Frank, 2000; Hirano & Imbens, 2001; Iami, Keel and Yamamoto, 2010). But the critical limitation of most sensitivity analyses is that they are cast in terms of properties of the variables (e.g., correlations associated with the variables), appealing to those who think in terms of relationships among factors.

Our framework appeals to an alternative intuition than most sensitivity analyses: we express sensitivity in terms of properties of the units of observation (e.g., people or classrooms) instead of variables, and we interpret in terms of motives, experiences, and outcomes of the people responsible for action (Abbott 1998). This may especially appeal to those who engage schools and students in their daily practice. For example, a principal may naturally consider how her school compares with a neighboring school in deciding whether to adopt one of its policies. Just so, she may be able to consider how well the schools which volunteered for a study represent her own, and may draw on our framework to quantify her consideration.

*External validity based on propensity to be in a study*. Hedges and O’Muircheartaigh (2011) used the estimates from a particular propensity stratum to generalize to a corresponding unstudied population. This is a clever approach to exploring the external validity of an inference based on the propensity for being in a study (see also Pearl & Bareinboim, 2011; Stuart et al., 2011). For example, one might use estimated effects of small classes in Tennessee (Finn & Achilles, 1990) for certain strata based on background characteristics to project a score for California.

Our approach differs from that of Hedges and O’Muircheartaigh (2011) in two ways. First, Hedges and O’Muircheartaigh project an estimate to a population outside the study, whereas we consider what would happen if the outside population were brought into the study. In this sense Hedges and O’Muircheartaigh appeal to differential treatment effects (e.g., Cronbach, 1982) and we seek to identify general effects across populations (e.g., Cook & Campbell, 1979).

Second, Hedges and O’Muircheartaigh assume one has all of the information about the unstudied target population that is both relevant for participation in the study and affects the treatment estimate. This is similar to the strong assumption of ignorability (Rosenbaum & Rubin, 1983) – that other factors can be ignored conditioning on the observed factors. Thus the question can still arise as to how much unobserved factors could have affected participation in the study as well as the treatment effect. For example, there may be subtle unobserved factors that affected whether schools volunteered for Borman et al.’s study. And sensitivity to these factors can be assessed using our approach.

*Other Sources of Bias*

We attended to two fundamental sources of bias in using RCM to interpret the bias necessary to invalidate an inference -- restricted samples and non-random assignment to treatment. But bias can come from alternative sources. We briefly discuss three of those sources below: violations of SUTVA, measurement error, and differential treatment effects.

*Violations of SUTVA.* Our analysis concerns how estimates would change if observations were replaced with hypothetical or counterfactual cases. But estimates could also change if observed outcomes changed when others’ assignments were altered. For example, the achievement of the students who were actually retained could change if some promoted students were retained. This could occur if one child’s retention reduced the stigma of another, or competed with the resources, such as the attention of the teacher (Shepard & Smith, 1989). Changes in observed outcomes as a result of others’ assignments to treatment conditions constitute a violation of SUTVA, an assumption made for most inferences. As such, there are some (recent) techniques to inform the implications of violations to SUTVA. For example, one can use agent based models to explore the dynamic implications of changes in treatment assignments (e.g., Maroulis et al., 2010).

*Measurement Error.* Error in measurement, especially of the outcomes, could bias estimates. Generally, measurement error will lead to conservative inferences because it reduces precision of estimates. This might be preferred if one wanted to be cautious in implementing new programs. But measurement error could hide a substantively important negative effect (an example of a type II error, failure to reject the null when in fact the null hypothesis is false).

Measurement error is not a large concern for the two focal examples in the article in which the outcomes were measured with high reliability. The reliability of the Terra Nova comprehensive reading test used by Borman et al was very high (.84 to .93 across grades 3-6 --SRA/McGraw-Hill, 2001), and would likely be higher at the classroom level given fairly large sample sizes per classroom (see Brennan, 1995, for a discussion). Similarly, the test used by Hong and Raudenbush had a reliability of .95 (based on the variance of repeated estimates of overall ability --see ECLS-K user guide, section 3.1.6). Furthermore, both Borman et al., and Hong and Raudenbush increased precision by controlling for a pre-test (which also could reduce the potential for non-normality – see Borman et al., page 400).

*Differential Treatment Effects*. To isolate the bias due to baseline differences, we assumed that the treatment effect for the treated equaled the treatment effect for the control. This would be violated if people chose treatments that are likely to be good for them for idiosyncratic or difficult to observe reasons (Heckman, 2005; Heckman , Urzua & Vytlacil, 2006). For example, Hong and Raudenbush’s estimated effect of kindergarten retention could be upwardly biased if those who were retained might have benefitted more (or suffered less) from retention than those who were promoted for subtle idiosyncratic reasons. In response, one could use propensity score techniques to separately estimate treatment effects for the treated and for the control (e.g. Morgan, 2001). After doing so, one could apply our framework to either estimate (e.g. Frank, Sykes et al., 2008).

**Conclusion**

Causal inference in policy analysis does not depend on a single researcher or study. For education research to inform policy it should emerge through debate among a community of scholars about the relative merits of different studies (e.g., Greco, 2009; Habermas, 1987; Kuhn, 1962; Kvanig, 2003; Sosa, 2007). Campbell’s law (1976) states: “The more any quantitative social indicator is used for social decision-making, the more subject it will be to corruption pressures and the more apt it will be to distort and corrupt the social processes it is intended to monitor” (page 49). Our corollary is “The more any quantitative social indicator is used for social decision-making, the greater will be the intensity of debate about the inference made from the indicator.”

Therefore, we informed debate about causal inferences by quantifying the discourse about the robustness of the inference, and we provided a framework to interpret the robustness. When discussing concerns about an observational study researchers can speak in terms of the proportion of the cases that would have to be replaced with counterfactual data to invalidate the inference; and when discussing concerns about the generality of effects from a randomized experiment researchers can speak in terms of the proportion of the sample that would have to be replaced to represent a population not sampled. Over time, it is our hope that the repeated characterization of robustness in such terms can contribute to a general language for debating inferences from educational research.

**Technical Appendix A (can be made available only on-line)**

**Deriving Expressions for Bias Using RCM**

*Bias due to Non-Random Assignment to Treatments*

To focus on the bias due to non-random assignment to treatments, we use RCM to decompose the population treatment effect, , into two components (e.g., Heckman, 2005; Winship & Morgan, 1999): *t*, the treatment effect for those in the treatment group, and *c*, the treatment effect for those in the control group. Therefore the average treatment effect,  , is

 = π *t* *t* +(1−π *t*)*c*, (A1)

where π*t* represents the proportion in the population who received the treatment. Thus  is a simple weighted average of the two treatment effects *t* and *c*. Taking expectations of (A1) yields:

E[]=E[π *t* +(1−π) *c*]=

π (E[Y*t*|X*=t*]−E[Y*c*|X*=t*])+(1− π)(E[Y*t*|X*=c*]−E[Y*c*|X*=c*]), (A2)

where the notation is as in the main text: X*=t* if a unit received the treatment and X*=c* if a unit received the control. Y*t*| X*=t* is then the value of the outcome Y for a unit exposed to the treatment, and Y*c*| X*=t* is the counterfactual value of Y under the control condition for a unit receiving the treatment.

Using the same notation, the expectation of the standard estimator,  (e.g., the difference in observed means between the treatment and control group) is

E[]=E[Y*t*|X*=t*]−E[Y*c*|X*=c*] . (A3)

and then defining bias, β =E[]−E[], the bias due to non-random assignment of units to treatments, βa, can be defined using the expectations in (A2) and (A3):

βa=E[]−E[]=

E[Y*t*|X*=t*]−E[Y*c*|X*=c*]−{π (E[Y*t*|X*=t*]−E[Y*c*|X*=t*])+(1−π)(E[Y*t*|X*=c*]−E[Y*c*|X*=c*])}. (A4)

Combining terms associated with π and (1−π) yields:

βa=π{ E[Y*c*|X*=t*]−E[Y*c*|X*=c*]}+(1−π){E[Y*t*|X*=t*]−E[Y*t*|X*=c*]} , (A5)

as in the main text.

*Bias due to Non-Random Sampling*

Equation (7) in the main text is:

= π*p**p*+(1− π*p*) *p´*. (7)

As in the main text, let Z=*p* if a unit is in *p*, and Z=*p´* if a unit is in *p´*. Combining this the notation of RCM, Y*t*| Z=*p* represents the treatment outcome for a unit in the population from which the cases were sampled (*p*), and Y*c*| Z=*p´* represents the control outcome for a unit in the population from which cases were not sampled (*p´*). Using this notation, taking expectations of (7) yields:

E[]==

E[π*p* *p* +(1−π*p*) *pʹ*]= π*p* (E[Y*t*|Z=*p*]−E[Y*c*|Z=*p*])+(1−π*p*)(E[Y*t*|Z=*p´*]−E[Y*c*|Z=*p´*]) (A6)

In the same notation the expectation for the standard estimator,  (e.g., the difference in observed means between the treatment and control group) is

E[]=*p* =E[*t*|Z=*p*] –E[*c*|Z=*p*]. (A7)

Again defining bias, β =E[]−E[], the bias due to non-random sampling, βs, can be defined by using the expectations in (A6) and (A7):

βs=E[*t*|Z=*p*] –E[*c*|Z=p]−

{π p (E[Y*t*|Z=*p*]−E[Y*c*|Z=*p*])+(1−π*p*)(E[Y*t*|Z=*p´*]−E[Y*c*|Z=*p´*])}. (A8)

Combining terms yields:

βs =(1−π p){( E[Y*t*|Z=*p*]−E[Y*c*|Z=*p*])− (E[Y*t*|Z=*p´*]−E[Y*c*|Z=*p´*])} , (A9)

as in the main text.

**Technical Appendix B (can be made available only on-line):**

**Application to Studies Available On-line for Upcoming Publication in this Journal**

To gain more of a sense of how to interpret robustness in different contexts, we quantified how much bias would be necessary to invalidate inferences in all of the articles available in on-line preprint in *Education, Evaluation and Policy Analysis* as of Dec 19, 2012 (dates for the articles range from July 24 2012 to Nov 15 2012). As such the studies are of current interest to this readership. All studies were included in which an inference was made by comparing an estimate to a threshold (this eliminated one study by Saunders and Marcelletti that was explicitly “descriptive”). These studies included applications of randomized experiments (e.g., Grigg et al), regression discontinuity (e.g., Mariano & Martorell), meta analysis (Shager et al.,), logistic regression (Stephan & Rosenbaum), fixed effects (Bozick & Dalton), propensity score models (e.g., An), as well as the general linear model (e.g., Engel, Claessens, & Finch).

We applied our analysis to focal inferences from each study, as indicated by a reference in the abstract of the study. In each case, we followed the same procedures as in generating Tables 1 and 2. We first calculated a correlation between the predictor of interest and outcome by converting a reported t-ratio (recognizing that the standard errors form only an approximate basis for certain types of models, such as logistic regression). We then calculated the threshold for statistical significance based on the reported degrees of freedom (each of the studies reported in Tables 3 and 4 either explicitly or implicitly used statistical significance as part of the threshold for inference, for example by only interpreting the policy relevance of those estimates that were statistically significant – see Wainer & Robinson, 2003), and then calculated the % bias necessary to reduce the estimate below the threshold.

We report the results for randomized experiments in Table B1. The % bias necessary to invalidate the inferences ranges from 7% to 33%. While these inferences may appear moderately robust, much of the bias due to internal validity may have been removed by randomly assigning units to treatments. We do note, however, that the robustness is partly a function of sample size (when statistical significance is used as a basis for inference), and the sample sizes are relatively small. This is often the case in randomized experiments which are expensive especially for units of analysis that aggregate individuals (e.g., classrooms in Grigg et al., -- see Slavin, 2008).

Insert Table B1here

We report the results for observational studies in Table B2. The bias necessary to invalidate the inference ranges from 2% to 60%. As in Table B1, the larger percentages are associated with the larger sample sizes, in this case from analysis of federal or city data bases. xi Nonetheless, the implications of an inference for policy purposes should be based on effect sizes (e.g., Wilkinson et al., 1999) as well as the robustness of the inference. For example, the correlation between summer instruction and language arts achievement is only .029 in Mariano’s study, although 60% of the estimate would have to be due to bias to invalidate the inference. The point goes both ways: studies with the largest effect sizes may not be most robust to concerns about sources of bias. For example, the correlation between elements of study design and reported effect size in Shager is .524, but the inference of a relationship would be invalid if only 16% of the estimated effect were due to bias.

Insert Table B2 here

Finally, two of the inferences reported in Tables B1 and B2 were for the lack of an effect because the estimates fell below a threshold (Yuan et al.; Bozick & Dalton). For these two inferences, as in the main text we report the coefficient by which the estimate would have to be multiplied to exceed the threshold for inference. It is simply the ratio of the observed estimate to its threshold, although it can also be considered as 1−% bias necessary to invalidate the inference: (1−1−δ# /= δ# /). For the example of Yuan’s randomized experiment of incentive pay programs, the correlation between incentive pay and extra hours worked of .077 would have to be multiplied by a factor of 2.53 to be positive and statistically significant. Applying similar calculations to Bozick and Dalton’s inference that occupational courses taken do not affect math achievement, the correlation between occupational courses taken in high school and math achievement would have to be multiplied by a factor of 7.18 to become statistically significant.

The analyses reported in Tables B1 and B2 move toward a reference distribution of the robustness of inferences reported in this journal. To fully generate such a distribution researchers will have to analyze inferences from hundreds of studies and gain a sense of how the robustness of inferences varies with the attributes of a study. This is beyond the scope of our study, and ideally should be generated by a research community that reflects diverse interpretations of our framework. Therefore, we hope others will follow the procedures outlined here to generate such distributions against which future researchers characterize may the robustness of new inferences.

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Endnotes

i Rubin (2004) would differentiate possible outcomes using Y(1) for the treatment and Y(0) for the control, but the parenthetical expressions become awkward when part of a larger function. Therefore, we designate treatment and control with a superscript. The use of superscripts is as in Winship and Morgan (1999).

ii Heckman (1979, page 155) also established a relationship between bias due to non-random assignment to treatment conditions and bias due to sample selection. In our terms, Heckman defines Z as depending on attributes of units. In turn, if the attributes that affect the probability of being sampled are not accounted for in estimating a treatment effect, they are omitted variables. In contrast, we characterize bias in terms of the percentage of a sample to be replaced and the expected outcomes of the replacement observations. This allows us to evaluate bias due to non-random sampling (as in 8) or bias due to non-random assignment to treatments (as in 6) using a single framework based on RCM.

iii pooled standard deviation =.

We then corrected for the 1 degree of freedom for the pretest to generate a value of 6.24: 6.24=6.35(48/49).

iv Given our formulation, (E[Y*t*|Z=*p*] − E[Y*c*|Z=*p´*]=0), the replacement classrooms were assigned to have mean value 610.6 *within* each curriculum because 610.6 was the average achievement of the classrooms that were removed (the standard deviations of replacement classrooms within each curriculum were also set equal to 6.67 to reproduce the first two moments of distribution of the replaced classrooms).

v We re-express (4) in terms of 1− r#/r where r is the correlation between treatment and outcome, and r# is defined as the threshold for a statistically significant correlation coefficient. Correlations adjust for differences in scales, and r# depends only on the degrees of freedom (sample size and parameters estimated) and alpha level, not the standard error of an estimate. The correlation was obtained from the ratio of the estimate to its standard error: , and the threshold was obtained using the critical value of the t distribution:  (Frank & Min, 2007). For large samples (e.g., greater than 1,000) r#/r will be equivalent to δ#/ to the second decimal in most circumstances, but in small samples r#/r and δ#/ will not be equivalent because δ is not a scalar multiple of r even though statistical inferences based on  and r are identical. One could also adjust the estimated correlation for estimation bias in small samples (e.g., Olkin & Pratt, 1958).

vi Hong and Raudenbush first estimated the propensity for a student to be retained using a logistic regression of retention on pretreatment personal, classroom, and school characteristics. The predicted values from this regression then became the estimated propensity scores. Hong and Raudenbush then divided their sample into fifteen strata by propensity, and then controlled for the stratum, schools as well as the individual logit of propensity in using a two level model to estimate the average effect of retention on achievement (see pages 214-218). Hong and Raudenbush (2005, Table 4) established common support in terms of balance on propensity scores.

vii We separately estimated the effect of retention is about 2.2 units weaker for an increase of one standard deviation on the pretest, an interaction effect that is statistically significant but not strong enough to overwhelm the large negative effect of retention across the sample so we report only the main effect. Also, at the school level, Hong and Raudenbush conclude that “the average effect of adopting a retention policy is null or very small,” (p. 214).

viii Figure 4 can also be interpreted in terms of a weighting of the data according to the frequency of occurrence in the replaced cases (shaded area) versus the original distribution (dashed line). For example, a retained student with a test score of 60 would receive a weight of about 2 because there are twice as many cases in the shaded bar than in the dashed line at 60. In general, the inference would be invalid if students who were retained and had high test scores received more weight, and students who were promoted and had low test scores received more weight. Such weights would pull the two groups closer together. Intuitively, the inference of a negative effect of retention on achievement would be invalid if the students who were retained and received high test scores counted more, and if the students who were promoted but received low test scores counted more.

ix It can be also valuable to assess the bias necessary to invalidate an inference against the bias reduced by controlling for observed and well recognized covariates (Altonji et al., 2010; Frank, 2000; Rosenbaum, 1986). For example, we estimated the change in effect of kindergarten retention on achievement when including measures of a child’s emotional state and motivation after controlling for schools as fixed effects, pretests and what Altonji et al. (2010) refer to as “essential” covariates: mother’s education, two parent home, poverty level, gender, and eight racial categories (Alexander et al., 2003; Holmes, 1989; Jimerson, 2001; Shepard & Smith, 1989). The estimated retention effect dropped from of −9.394 (n=9298 using the weight C24CW0, standard error .448, R2 of .75) to −9.320 (a drop of .074) when we included measures of the child’s approaches to learning (t1learn), teacher’s and parent’s perceptions of the child’s self control (t1contro, p1contro), and the tendency to externalize problems (t2extern).

x We also follow Altonji et al (2010) in assuming that the unobserved confounds are independent of the observed confounds. Any dependence would reduce the impacts of observed as well as unobserved variables on treatment estimates if all were included in a single model.

xi An (forthcoming) conducts an interesting analysis of the robustness of the results to unmeasured confounders (see Ichino, 2008; as well as Harding’s 2003 adaptation of Frank, 2000).

Table 1

Quantifying the Robustness of Inferences from Randomized Experiments

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Study  (author, year) | Treatment vs Control | Blocking | Population | Outcome | Estimated effect, standard error, source | Effect size  (correlation) | % bias to make the inference invalid |
| A multi-site cluster randomized field trial of Open Court Reading  (Borman et al., 2008) | Open Court curriculum versus business as usual | Within grade and school | 917 students in 49 classrooms | Terra Nova comprehensive reading score | 7.95  (1.83)  Table 4, results for reading composite score | .16  (.54) | 47% |
| Answers and questions about class size: A statewide experiment  (Finn & Achilles, 1990) | Small classes versus all others | By school | 6500 students in 328 classrooms | Stanford Achievement Test, reading | 13.14  (2.34)  Table 5  mean for other classes is based on the regular and aide classes combined proportional to their sample sizes. | .23  (.30) | 64% |
| Experimental Evaluation of the Effects of a Research-based Preschool Mathematics Curriculum  (Clements & Sarama, 2008) | Building Blocks research to practice curriculum versus alternate math intensive curriculum | By program type | 276 children within 35 classrooms randomly sampled from volunteers within income strata | Change in Early Mathematics Childhood Assessment IRT scale score (mean=50, std=10) | 3.55  (1.16)  Building Blocks vs comparison group Table 6 (df of 19 used based on footnote b) | .5  (.60) | 31% |

Table 2

Quantifying the Robustness of Inferences from Observational Studies

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Study  (author, year) | Predictor of interest | Condition on pretest | Population | Outcome | Estimated effect, standard error, source | Effect size  (correlation) | % bias to make inference invalid |
| Effects of kindergarten retention policy on children’s cognitive growth in reading and mathematics  (Hong & Raudenbush, 2005) | Kindergarten retention versus promotion | Multiple | 7639 kindergarteners in 1080 retention schools in ECLS-K | ECLS-K Reading IRT scale score | 9  (.68)  Table 11, model based estimate | .67  (.14) | 85% |
| Counterfactuals, causal effect heterogeneity, and the Catholic school effect on learning,  (Morgan, 2001) | Catholic versus public school | Single | 10835 high school students nested within 973 schools in NELS | NELS Math IRT scale score | .99  (.33)  Table 1, (model with pretest + family background) | .23  (.10) | 34% |
| Effects of teachers’ mathematical knowledge for teaching on student achievement  (Hill et al., 2005) | Content knowledge for teacher mathematics | Gain score | 1773 third graders nested within 365 teachers | Terra Nova math scale score | 2.28  (.75)  Table 7, model 1, (third graders) | NA  (.16) | 36% |

Table B1

Quantifying the Robustness of Inferences from Randomized Experiments in EEPA posted on-line July 24-Nov 15 2012

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Study  (1st author) | Predictor of interest | Population  (df) | Outcome | source | r | % bias to make inference invalid | Multiplier to make estimate significant |
| Incentive Pay Programs Do Not Affect Teacher Motivation or Reported Practices  (Yuan) | Pay-for performance | 5th-8th grade teachers in Nashville  (143) | Test Preparation | Table 3, POINT program | .250 | 33% |  |
| Effects of Two Scientific Inquiry Professional Development Interventions on Teaching Practice  (Grigg) | Professional development for inquiry in science | 4th-5th grade  Classrooms in Los Angeles  (70 schools) | Inquiry used | Table 5 Model 1 | .277 | 15% |  |
| A Randomized Controlled Trial Evaluation of Time to Read, a Volunteer Tutoring Program for 8- to 9-Year-Olds  (Miller) | Time to Read (tutoring) | 8-9 year old children in Northern Ireland  (734) | Future aspiration | Table 5, column 1 | .078 | 7% |  |
| Incentive Pay Programs Do Not Affect Teacher Motivation or Reported Practices  (Yuan) | Pay-for performance | 5th-8th grade teachers in Nashville  (143) | Extra hours worked/ week | Table 3, POINT program | .077 | -154% | 2.53 |

a Degrees of freedom for calculating threshold value of r are defined at the level of the predictor of interest, subtracting the number of parameters estimated at that level. Calculations can be conducted from spreadsheet at <https://www.msu.edu/~kenfrank/research.htm#causal>. Studies listed in order of % robustness.

Table B2

Quantifying the Robustness of Inferences from Observational Studies in EEPA posted on-line July 24-Nov 15 2012

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Study  (1st author) | Random  sample | Predictor of interest | Condition on pretest | Population  (df)a | Outcome | source | r | % bias to make inference invalid | Multiplier to make estimate significant |
| Academic Effects of Summer Instruction and Retention in New York City  (Mariano) |  | Summer instruction and retention | RD | 5th graders in New York  (29,987) | English & Language Arts achievement 6th grade | Table 2, Panel 1, column 1 | .029 | 60% |  |
| Unintended Consequences of an Algebra-for-All Policy on High-Skill Students  (Nomi) |  | schools that increased algebra  enrollment post-policy for low-ability students  by more than 5%) | Interrupted Time Series | 9th graders in Chicago  (17,987) | Peer ability levels for high skill students  (1999 difference for affected schools) | Table 2, column 1 | .030 | 50% |  |
| Impact of Dual Enrollment on College Degree Attainment  (An) | yes | Dual enrollment |  | NELS  (8,754) | Any degree obtained | Table 1, Panel A, column 1 | .043 | 50% |  |
| Life After Vouchers  (Carlson) |  | Student transferred to public school | No | 3rd-10th grade in Milwaukee public schools (1,120) | Math | Table 5, all students | .092 | 36% |  |
| Teaching Students What They Already Know?  (Engel) | yes | Teaching practices | yes | Kindergarten teachers in ECLS-K  (2,174) | Math  (spring of kindergarten) | Table 6, column 1 | .059 | 28% |  |
| Unintended Consequences of an Algebra-for-All Policy on High-Skill Students  (Nomi) |  | schools that increased algebra  enrollment post-policy for low-ability students  by more than 5%) | Interrupted Time Series | 9th graders in Chicago  (17,987) | 9th grade math scores  (1999 difference for affected schools) | Table 3, column 1 | .020 | 26% |  |
| Can High Schools Reduce College Enrollment Gaps With a New Counseling Model?  (Stephan) |  | Coach in school for college going |  | 9th-12th grade schools in Chicago  (54) | Applied to 3+ colleges | Table 4, column 1 | .35 | 23% |  |
| Can Research Design Explain Variation in Head Start Research Results?  (Shager) |  | Elements of study design |  | Studies of Head Start on Cognition  (20) | Reported effect size | Table 4, Model 1 | .524 | 16% |  |
| Can High Schools Reduce College Enrollment Gaps With a New Counseling Model?  (Stephan) |  | Coach in school for college going |  | 9th-12th grade schools in Chicago  (54) | Enrollment in less selective 4-yr college vs 2-yr | Table 3, column 3 | .27 | 2% |  |
| Balancing Career and Technical Education With Academic Coursework  (Bozick) |  | Occupational courses taken in high school | Student fixed effects | ELS: 2002  (7,156) | Math achievement | Table 3, column 1 | .003 | -617% | 7.18 |

a Degrees of freedom for calculating threshold value of r are defined at the level of the predictor of interest, subtracting the number of parameters estimated at that level (including parameters estimated in a propensity model).

Calculations can be conducted from spreadsheet at <https://www.msu.edu/~kenfrank/research.htm#causal>.

Studies listed in order of % robustness.

Figure Captions

*Figure 1*. Estimated Treatment Effects in Hypothetical Studies A and B Relative to a Threshold for Inference

*Figure 2*. Example Replacement of Cases from Non-Volunteer Schools to Invalidate Inference of an Effect of the Open Court Curriculum

*Figure 3*. Effect of Retention on Reading Achievement in Retention Schools (from Hong and Raudenbush, 2005, Figure 2, p. 218).

*Figure 4*. Example Replacement of Cases with Counterfactual Data to Invalidate Inference of an Effect of Kindergarten Retention



*Figure 2*. Example Replacement of Cases from Non-Volunteer Schools to Invalidate Inference of an Effect of the Open Court Curriculum

*Figure 3*. Effect of Retention on Reading Achievement in Retention Schools (from Hong and Raudenbush, 2005, Figure 2, p. 218).



*Figure 4*. Example Replacement of Cases with Counterfactual Data to Invalidate Inference of an Effect of Kindergarten Retention