Irrational Exuberance or Value Creation:
Feedback Effect of Stock Currency on Fundamental Values

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Abstract

In a departure from existing literature, we argue there is a reverse causality between the level of a firm’s stock price and its fundamental value. This is particularly true for capital constrained firms. Higher stock prices relax a firm’s budget constraint by enabling it to sell new shares at higher prices, raise debt at more attractive rates and acquire larger targets through stock-swaps. Higher prices may also signal improved prospects to managers encouraging them to alter their investment decisions. Ascribing a feedback role to stock prices can help explain “excess volatility” and also why trading appears to induce volatility. It also implies large traders need to condition on not only how their trades change prices, but also on how these price changes induce further price changes through their impact on firm’s fundamentals. In this setup, “bubble-like” price movements can be value creating. (JEL G10)

A body of recent research has documented that, contrary to Modigliani-Miller’s irrelevance principle, financial slack appears to impact firm investment decisions. This result has been established in surprisingly varied settings using different experiments and tests. Though not all issues related to endogeneity problems are resolved, the preponderance of evidence speaks clearly to financial slack being important.¹

If financial slack is indeed important, any event, shock or strategy that impacts financial slack will also impact firm value through changes in its investments. This aspect has been studied extensively but in a different context of how financial con-
straints can amplify business cycles. Starting with the assumption that budget constraints are binding, exogenous (usually macroeconomic) shocks which impact these constraints affect firm investment decisions which in turn amplify these shocks further. Bernanke and Gertler (1989), for instance, show that an initial positive shock to the economy improves firms’ profits and retained earnings allowing firms to invest more, further increasing profits and retained earnings and amplifying the upturn.

In this paper we look at a different set up. Instead of assuming exogenous shocks changing budget constraints, we study the impact of trading-induced stock price changes on the budget constraint. It is well documented that firms are able to raise more external funds when stock prices are high. They also use the higher value of their stock currency to make more or bigger acquisitions. This phenomenon was particularly noticeable in the recent dot-com boom. Firms whose stock prices experienced large increases used their stock as currency to make multiple acquisitions during the period of high stock prices. The acquisition activity dried up when stock prices experienced a steep drop.

However, if stock price movements alter a firm’s budget constraint and potentially its investments, decisions to make large trades become more complex. Large traders need to condition on not only how their trade changes price, but also on how these price changes induce further price changes through their impact on firm’s fundamentals. While the dynamics of this process are similar to those in the “accelerator-type” literature, there is a non-trivial difference. Most of that literature studies the amplification of observable macro shocks. In this instance though, a large trade (or a sequence of large trades) possibly conditioned on private information can start the process. This suggests that the inference problem faced by the market maker is more complex than usually modeled. Not only does he need to infer the trader’s private information from the trade, but must also infer the manager’s action in response to the changed budget constraint and its impact on the stock price.
The implication for existing noise trading models (DeLong, Shleifer, Summers and Waldmann (1990)) or behavioral models (Daniel, Hirshleifer and Subrahmanyam (1998), Hong and Stein (1999) and Barberis, Shleifer and Vishny (1998), among others) is that an additional source of uncertainty probably needs to be priced: the price movement away from fundamentals results in changes in real size and underlying riskiness of the firm’s assets in conceivably non-diversifiable ways. The same is true for traders following ‘herd-like’ strategies. With a feedback effect, there are concerns about the destabilizing effect on financial markets of such behavior, (which Keynes referred to as ‘animal spirits’) which is more pronounced and permanent.

To model this effect, we assume the basic market microstructure of Kyle (1985) and Glosten and Milgrom (1985). A rational market maker offers net order flow based price functionals to make zero expected profits per period. Informed traders trade sequentially and prices trend up if they all buy. This relaxes the firm’s budget constraint and also signals to the manager the traders’ good signals. The manager uses the higher valued stock currency to acquire previously unaffordable assets. Traders and the market maker anticipate the manager’s response and condition on it.

If certain patterns of price changes affect firm investment decisions and, thus, firm value, traders may attempt to generate such patterns through trades not justified by their information. For instance, if prices remain high for the duration of time required to raise new funds, or the time required to negotiate an acquisition, the firm is likely to get a better deal. Thus, higher prices during this period could contribute to increased firm value. In this situation, if the first few informed traders get good information about a firm’s prospects and increase the price by buying shares, it may make sense for later informed traders to choose to continue buying (herding) and increase prices even when they have bad (private) signals. This holds as long as, in aggregate, the signals suggest a higher probability of a good outcome.

However, in a market with a rational market maker, the feedback effect alone is not
enough for investors to continue buying to keep prices high. The reason is the market maker is aware that a particular pattern of trades will generate a value-changing feedback effect. He is also aware that the feedback effect generates incentives to herd. Consequently, when setting prices, he conditions on both the feedback effect and the potential for herding. Thus if, by following a number of buy trades with a buy, the later trader attempts to keep the price high in the face of a bad signal, she can do so only by buying at a price which is too high given her own signal. Thus, she makes negative expected profits on her trade, and in order for her to undertake such a trade she must make profits elsewhere.

With a feedback effect, there is a natural assumption which makes this possible. We need only assume that the informed traders trade a portion of their holding in the stock, i.e. they keep a minimum positive inventory in the stock at all times. Since the feedback effect changes the fundamental value of the firm, with a positive inventory an informed trader is prepared to lose on her trade in order to increase the value of her inventory. The larger her inventory and/or the larger the feedback effect, the more likely she is to herd and manipulate prices. It is worth noting, however, that because of a positive inventory, an informed trader will herd to induce the firm manager to increase investment only if it is expected (ex-ante) to add value to the firm.

Ascribing a reverse causality from prices to investments can provide a possible explanation for some perplexing phenomena. For instance, our theory provides an alternate explanation for ‘excessive’ volatility documented by Shiller (1981) and LeRoy and Porter (1981). If prices are used by markets, among other things, to affect changes in real firm investment, there may be little or no relationship between prices and dividends. Stock volatility will in part be a function of how frequently markets try to impact a firm’s investment decisions. It explains why trading may generate volatility. If price movements facilitate changes in firm fundamentals, one would expect higher price volatility when markets are open and trades are affecting prices. Also, it helps
explain why firms appear to be “obsessively” concerned about their stock price, and can justify stock price charting as a seemingly relevant profession.

A number of new results emerge. Herding is more likely when the quality of the informed traders’ information is higher. For higher quality information, the later informed are more likely to trust the information of those who have already traded and are more comfortable disregarding their own signal. For the same reason, the critical level of inventory needed to support herding is inversely related to information quality. With higher information quality, the information loss from herding is less. Thus prices remain close to expected prices. Since trading losses from herding are less, a smaller inventory is needed to recover the losses. Not surprisingly, herding is more likely when the feedback effect is larger. Also the critical level of inventory needed is negatively related to the feedback effect.

It turns out that there is a region in our parameter space where we have multiple equilibria, i.e. both herding and non-herding equilibria can be supported. Though we cannot tell which will occur, we are afforded the opportunity of deriving some welfare implications. It turns out that the time $t=0$ price is higher for the herding equilibrium than for the no-herding equilibrium. The reason is that herding gets the new project accepted with a higher probability, thus increasing expected fundamental value. However, since some information is lost in herding, the probability that the outcome will be good given a pattern of trades is reduced, which decreases fundamental value. In this region, though, the first effect appears to dominate. Interestingly, however, the unconditional expected volatility is also higher under the herding equilibrium. This makes welfare implications ambiguous.

There are also some results about conditional volatility of price paths. On the extreme price-paths, the ones with the highest volatility, herding serves to dampen volatility, thus making price movements less extreme. Since these extreme paths have ‘bubble-like’ characteristics, herding can be argued to be moderating the harm done
Finally, we separate the notions of herding and momentum strategies, that have been interchangeably used in the literature, primarily because of the presumption that herding causes prices to move predictably causing serial correlation in returns. However, in our model the market maker ensures that his price incorporates all the information he has at that point in time. Thus prices are martingales, and any period’s price is an unbiased expectation of all possible future prices. Thus rational herding does not translate into the presence of momentum based trading profits. However, because of the feedback effect, herding can be correlated to future asset prices through increase in inventory value. Thus, while our results are inconsistent with momentum in returns, they are consistent with findings in papers like Wermers (1999) that funds that display herd behavior outperform those that do not. It is also consistent with Falkenstein (1996), that mutual funds herd in stocks with specific characteristics, if some of them proxy for the likelihood of the feedback effect.

The remainder of this paper is organized as follows. Section I outlines the basic structure of the model. Section II identifies parameter spaces and conditions under which herding behavior might be supported in equilibrium. Section III considers another plausible equilibrium. In Section IV, we compare the two equilibria, and discuss some implications. Section V concludes the paper.

I The Model

A Trading Set-up

In the model we try to capture the feedback effect as parsimoniously as possible.

We assume that a firm has existing assets with state and trade dependent per share terminal payoff, v. For simplicity, we assume two states \( \theta = H \) and \( \theta = L \) (for ‘High’ and ‘Low’ respectively). Prior beliefs regarding the state are \( P(\theta = H) = P(\theta = L) = 1/2 \). There are three risk-neutral informed traders: IT 1, 2 and 3, each of whom can...
trade $x_i$ from $X \equiv \{-1,+1\}$. Following Bikhchandani, Hirshleifer and Welch (1992), each informed trader $i$ is endowed with a signal $s_i$, which is an imperfect proxy for the state according to:

\[
\begin{array}{ccc}
\text{P}(s_i = H| \theta) & \text{P}(s_i = L| \theta) \\
\theta = \text{H} & q & 1-q \\
\theta = \text{L} & 1-q & q
\end{array}
\]

We need $q > 1/2$, so that the signal is informative. Thus the probability of getting a H signal if the state is H, $\text{P}(s_i = \text{H} | \theta = \text{H}) = q$, is higher than getting a L signal. Markets open for trading three times and, for tractability, only one informed trader trades each round and each informed can trade only once. The order in which they trade is randomly determined. At each round of trading, each informed trader is accompanied by one noise trader who randomly chooses between two equally likely trades $u_i \in U \equiv \{-1,+1\}$. There is a risk-neutral market maker who observes aggregate demand $y_i = x_i + u_i$ in each round of trading, and sets the price functional such that his per-trade expected profits are zero, i.e. he sets a price equal to the conditional expectation of the value of the traded security, given the current history of information at that time. This is a standard assumption in several models of market-making including Kyle (1985) and Glosten and Milgrom (1985).

**B Security Value and Information Structure**

To capture the feedback effect simply, the state-trade-dependent terminal value of the security $v$, is modeled as follows. We assume that the only trading pattern that results in the firm changing its expected investment strategy is when the informed unambiguously buy one share in each round of trading. This will occur when the trading pattern over three rounds is $\{+2, +2, +2\}$. For all other patterns of trade the firm’s expected investments remain the same.$^{10}$ With unchanged expected investments, $v = 1$ in the H state and $v = 0$ in the L state. However, with the favorable
trading pattern, the firm’s expected investment set increases and the terminal payoffs are \(v = 1+d\) if state is H and \(v = -d\) if the state is L. Given uninformative signals (i.e. with \(q = 1/2\)), the expected value of the firm remains at 0.5, with or without the new investment, but the variance of payoffs is higher with the new investment. However, when signals are informative (i.e. with \(q > 1/2\)), and they indicate a higher probability of the High state, both expected value and payoff variance are higher. The set of final payoffs and beliefs is summarized as follows:

a. \(v = 1+d\), if \(\theta = H\) and the trading pattern is \(\{+2,+2,+2\}\) over the three rounds. This occurs when each informed trader and noise trader buy one share each in all three rounds of trading.

\(v = -d\), if \(\theta = L\) and the trading pattern is \(\{+2,+2,+2\}\) over the three rounds.

\(v = 1\) if \(\theta = H\), and \(v = 0\), if \(\theta = L\), with any other combination of trades over the three rounds.

b. All players know the information regarding the terminal price of the security, and the precise value of \(d\) (>0).

The following timeline summarizes the sequence of actions that take place in our model.

<table>
<thead>
<tr>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>end</th>
</tr>
</thead>
<tbody>
<tr>
<td>All know the values of q and d and the distribution of terminal payoff v across states and paths</td>
<td>\textit{Round 1} of trading</td>
<td>\textit{Round 2} of trading</td>
<td>\textit{Round 3} of trading</td>
<td>State (\theta) and terminal value v of security realized</td>
</tr>
<tr>
<td>Market maker observes (y_1 = x_1 + u_1) and sets (p^1)</td>
<td>Market maker observes (y_2 = x_2 + u_2) and sets (p^2)</td>
<td>Market maker observes (y_3 = x_3 + u_3) and sets (p^3)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
C  Equilibrium

We now search for a herding equilibrium under the current set-up. This will serve two purposes. First, it allows us to illustrate the guts of our model in detail. Second, it also allows us to answer what we believe is a key question: Does the feedback effect result in herd behavior by rational investors? We are especially concerned here with the temptation that might occur to the informed traders to follow one another in hopes of exploiting a future feedback effect from higher prices to fundamentals.

We use the concept of symmetric Nash equilibrium. A sequence of trading strategies and a belief system constitute an equilibrium, if given the belief system and the strategies of other players, no player has an incentive to deviate from her prescribed strategy. For tractability, we confine ourselves to the following definition of a herding equilibrium.\textsuperscript{12}

\textbf{Definition 1} A herding equilibrium is defined as one where every trader follows her own signal, unless she reduces the probability of her being wrong by going against her own signal.

With this definition of herding, it is now easy to prove the following proposition.

\textbf{Proposition 1} It does not benefit the first two informed traders to ever herd while it does benefit the third informed trader to herd when she can conclusively observe that the first two traders have invested \( x_i = +1 \).

Proof: The intuition underlying the proof follows from the fact that herding improves the probability that a trader is right regarding the terminal state, regardless of her own signal, when both previous signals are identical. Hence, with three players, only the third player will ever herd. For a formal proof, see Appendix A.

Proposition 1 suggests a candidate equilibrium that gives rise to herding as we have defined it. Our definition of herding is consistent with naive price-momentum or
trend-chasing strategies as documented in recent literature. However, the suggested candidate equilibrium needs to survive price-setting by a rational market maker. Thus, we first derive prices that would be established under a belief system of herding on part of the market maker as well as all informed traders. Next, we verify whether the informed traders find it profitable to follow conjectured equilibrium strategies under the prices established in the first step of analysis.

Table 1 presents the prices established under the beliefs of herding. See Table 2 for all combinations of signals and trades by the informed traders in a herding equilibrium. Given the derived prices, it remains to check whether the informed traders find it profitable to adopt the conjectured equilibrium strategies rather than deviate. The following result can then be established.

**Proposition 2** *For the above model a herding equilibrium does not exist.*

Proof: See Appendix A.

Proposition 2 is similar to the result established by Avery and Zemsky (1998). In the presence of a rational market maker who sets prices on the basis of net order flow and the belief system, uncertainty regarding the state is not enough to induce herding behavior. In our model, as in theirs, the market maker, anticipating herding behavior on part of the third informed trader, sets prices at a level at which it is unprofitable (in expectation) for the latter to go against an L signal, and invest $x_3 = +1$. The reason is that the market maker sets the third period price knowing that the third trader will buy independent of his signal. Thus he pools over the trader’s next period’s signal being High or Low with equal probability. If the third informed gets a Low signal, the price she faces when she buys exceeds her expectation of the terminal value. If she buys, she makes an expected loss and is better off not herding. Consequently, herding cannot be supported in equilibrium for any set of parameter values.
II Herding as Equilibrium Behavior

A Herding Equilibrium

We now modify the model and information structure to obtain a herding equilibrium that can be supported by prices established in Table 1. In addition to the information in Sections I.A and I.B., we make the assumption that before she trades, each informed trader has at least a minimum amount of inventory of I(> 0) units of the security. While the market maker is aware that the informed trader’s inventory exceeds some minimum level, he need not know the exact value of I.14

One can immediately see the potential effect this modification has on herding behavior. Even though the third informed trader makes a loss on trading if she herds, she stands to gain on her inventory of securities since the expected terminal value of the security is higher under herding. In other words, she knows that by herding she will create an externality and as long as she has a sufficient stake in the externality, she will choose to do so. For a sufficiently large inventory, herding will be profitable for the third informed trader. Indeed, we can establish the following result.

**Proposition 3** In addition to the model above, if informed traders have inventories $I > I^*$, where

$$I^* = \frac{3}{d(2q-1)} \left[ \frac{q^2}{q^2+(1-q)^2} - q \right] + \left[ \frac{1}{q^2+(1-q)^2} - 1 \right]$$

there exists a herding equilibrium $\forall q \in (0.5, 1]$.

Proof: See Appendix A.

The criticality of the feedback effect of prices in our model is evident from Proposition 3. In the absence of any externality (i.e. if $d$ is zero), the third informed trader
does not make any gains on her inventory that can offset her trading loss. Alternatively, this can be seen from the expression for $I^*$, the minimum required inventory becomes very large as $d$ goes to zero. Hence, for herding behavior to exist in a rational expectations framework, we require that a) there is a feedback effect of prices into the underlying value of the security, and b) traders need to make non-trading gains from this feedback effect; in other words, they must be interested in ‘making the externality occur’.

B Robustness of the Herding Equilibrium

A natural question to ask at this juncture is whether herding is possible when the signals are totally uninformative. After all, if there are gains to herding (in the form of the externality that is created), totally uninformed traders might decide to start herding by trading $x_i = +1$. In terms of our model, is it possible that herding survives as an equilibrium when $q=0.5$? Corollary 1 provides the answer to this question, along with simple comparative statics on the minimum inventory bound $I^*$.

**Corollary 1**  a. *With totally uninformative signals, i.e. with $q = 0.5$, a herding equilibrium is not possible.*

b. *For given $d$, $I^*$ decreases in $q$, $\forall q > 0.5$.*

c. *$\forall q \in (0.5, 1], I^*$ decreases in $d$, $\forall d > 0$*

Proof:

a. Substitute $q=0.5$ into Condition (1) in Appendix A. Clearly the inequality is not strictly satisfied. Stated another way, this means that it is not possible for traders with totally uninformative signals to start and maintain herding in equilibrium.
b. This part is illustrated in Fig.1.a. The figure shows that herding is more likely when the quality of the informed traders’ information is higher. For higher quality information, later informed are more likely to trust the information of those who have already traded and are more comfortable disregarding their own signal. For the same reason, the critical level of inventory needed to support herding is negatively related to information quality. With higher information quality, the information loss from herding is less. Thus prices remain close to expected prices conditional on no information loss. Since trading losses from herding are less, smaller inventory is needed to recover the losses.

c. See Fig.1.b. for an illustration of this part of the corollary. The intuition here is that holding q constant, as d increases, the potential gains to herding increase, and hence smaller levels of inventory are sufficient to start herding. Not surprisingly, herding is more likely when the feedback effect is larger.

III A No-Herding Equilibrium

To facilitate comparison with the herding case, it is useful to establish conditions under which there exists a no-herding equilibrium. Note that the additional investment still occurs after a \{+2, +2, +2\} pattern of trades. However, without herding, the probability of the additional investment occurring is less as the third informed trades \(x_3 = +1\) only if her signal is H. Recall that with herding, she trades \(x_3 = +1\) independent of her signal. There is also a counter effect. Since in the no-herding equilibrium the additional investment occurs conditional on three H signals, the probability of the terminal state to be High is greater than in the herding equilibrium. Thus, from a social welfare angle, it is not clear in which equilibrium society is better off. We next attempt to answer this question.
**Definition 2** A no-herding equilibrium is defined as one where every informed trader follows her own signal.

Table 3 presents the prices established under the beliefs of no-herding.\textsuperscript{15} See Table 4 for all combinations of signals and trades by the traders in a no-herding equilibrium. Once again, as for the herding case, given the prices, we need to check whether the informed traders find it profitable to adopt the conjectured equilibrium strategies rather than deviate (and herd). The following result can be established.

**Proposition 4** For the above model, if informed traders have inventories $I < I_3$, where

$$I_3 = \left[ \frac{2q^2}{d(2q-1)(q^2+(1-q)^2)} + \frac{q^3}{d(2q-1)(q^2+(1-q)^2)} - \frac{3q}{d(2q-1)} + \frac{q^3(1-q)^3}{(2q-1)(q^2+(1-q)^2)} - 1 \right]$$

there exists a no-herding equilibrium.

Proof: See Appendix B.

Intuitively, Proposition 4 says that for each informed trader to follow her own signal in equilibrium, their individual inventories should be small enough. If they are large, then informed traders will be tempted to go against their L signals and trade in the positive direction in the hope of creating this feedback effect, and making profits on their inventories. But this deviation on their part causes the equilibrium to break down.

**IV Multiple Equilibria**

Comparing $I^*$ (which is the lower bound on inventory in the herding equilibrium) and $I_3$ (the upper bound on inventory in the no-herding case), we can see that for the relevant support of $q$ that we consider i.e. for $q \in (0.5, 1]$, $I_3$ is always greater than $I^*$. 

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While this is not immediately obvious from the expressions for the inventory bounds, it can be seen from Fig. 2 that there is always an intermediate range of inventory for which both equilibria are possible. Though we cannot tell which equilibrium will occur, we are afforded the opportunity of deriving some welfare implications.

The region for which multiple equilibria exist while widest for low $q$ (see Fig. 2), increases fastest for the intermediate $q$ s. The reason is that the information loss from herding, $P(\theta = H \mid s_1 = H, s_2 = H, s_3 = H) - P(\theta = H \mid s_1 = H, s_2 = H, s_3 = H or L)$, is greatest at the intermediate quality of information. For low $q$, information loss from herding is small because the signal is not particularly informative. For high $q$, the marginal contribution of the third signal is low since there is little noise in the first two signals. Thus, the slopes of the lower and upper inventory bounds are steepest at these values.

There are some results about conditional volatility of the price paths. It turns out that on the extreme price paths, the ones with the highest volatility, herding serves to dampen volatility, thus making price movements less extreme. See Fig. 3.a and 3.b. The reason lies in the way market makers set prices under the two equilibria. The figures show that, not only is $p_0$ higher with herding, but so are $p_1$ and $p_2$. However, $p_3$ is higher without herding. $p_0$ is higher because the probability of the increase in investment is higher with herding. $p_1$ and $p_2$ are higher both because of the higher probability of investment and because that probability approaches one faster (after two rounds of trading). $p_3$ is lower because the market maker cannot infer the third trader’s signal under herding since the latter buys independent of her signal. So he averages over her expected value of firm with an L and an H signal. The price is lower than under no-herding which is unambiguously conditioned on three H signals.

Since these extreme paths have ‘bubble-like’ characteristics, herding can be argued to be moderating the harm done by ‘bubbles’. This pattern is more obvious in Fig. 3.b. Under an unconventional definition of bubbles, that they are due to bad
realizations and not necessarily bad expectations, one can see that when the bubble
bursts, the drop in price is less under herding. From Fig 4 one can observe that
the conditional volatility of the extreme price path is uniformly less for the herding
equilibrium, independent of the quality of information.

It turns out that the time $t=0$ price is higher for the herding equilibrium than for
the no-herding equilibrium. Compare the values of time 0 price in Tables 1 and 3 of
the text. This can also be seen from Fig. 5.a. The reason is that herding gets the
new project accepted with a higher probability, thus increasing expected fundamental
value. However, since some information is lost in herding, the probability that the
outcome will be good given a pattern of trades is reduced, which decreases funda-
mental value. In this region, though, the first effect appears to dominate, resulting
in a higher value with herding. However, the expected volatility of prices (with the
expectation taken over all 27 possible paths) is higher under herding, as can be seen
from Fig. 5.b. Also, since the increase in both quantities is dependent on the amount
of feedback effect $d$, it is not clear under which equilibrium society is better off.

V  Conclusion

In this paper we establish price manipulation and herding with rational expectations
without imposing strong restrictions like incomplete markets or restricted participa-
tion. When price paths influence a firm’s investment decisions, just giving informed
investors positive inventories results in herding and price manipulation. In this en-
vironment, herding by institutions or funds (‘animal spirits’) increases fundamental
value and is unlikely to be destabilizing. Also, volatility of extreme price paths
is dampened because rational price makers cannot perfectly infer investors’ signals
from their trades and thus pool over them. Thus investors are hurt less when bad
outcomes occur.
To our knowledge, this is the first time that asset prices have been explicitly solved for in a market microstructure model in the presence of feedback. The advantage of framework is that the basic model that we have outlined here can be extended in several directions in order to analyze a rich set of open questions in finance. For instance, one could use the basic structure of our model to analyze the impact of risk aversion in order to understand the relationship between risk aversion, firm fundamentals, and asset price volatility.

In our model, rational herding does not translate into momentum profits. As in other rational expectation models, our prices have no predictive power over future prices. The market maker conditions both on the feedback effect and the possibility of herding when establishing prices. Whether this model can be extended to result in momentum profits is left to future research.
Price at time $t=0$

\[ p_0 = \frac{1}{2} + \frac{d(2q-1)}{16} \]

**Round 1 of Trading**

\[ p^1_{-2} = (1 - q) \]
\[ p^1_0 = \frac{1}{2} \]
\[ p^1_2 = q + \frac{d(2q-1)}{4} \]

**Round 2 of Trading**

\[ p^2_a = p^2_{-2,-2} = \frac{(1-q)^2}{q^2+(1-q)^2} \]
\[ p^2_b = p^2_{-2,0} = p^2_{0,-2} = (1 - q) \]
\[ p^2_c = p^2_{2,-2} = p^2_{2,2} = p^2_{0,0} = \frac{1}{2} \]
\[ p^2_d = p^2_{-2,2} = p^2_{2,0} = q \]
\[ p^2_e = p^2_{2,2} = \frac{q^2}{q^2+(1-q)^2} + \frac{d}{2} \frac{2q-1}{q^2+(1-q)^2} \]

**Round 3 of Trading**

\[ p^3_{a,-2} = \frac{(1-q)^3}{q^3+(1-q)^3} \]
\[ p^3_{a,0} = \frac{(1-q)^2}{q^2+(1-q)^2} \]
\[ p^3_{a,2} = (1 - q) \]
\[ p^3_{b,-2} = \frac{(1-q)^2}{q^2+(1-q)^2} \]
\[ p^3_{b,0} = (1 - q) \]
\[ p^3_{b,2} = \frac{1}{2} \]
\[ p^3_{c,-2} = (1 - q) \]
\[ p^3_{c,0} = \frac{1}{2} \]
\[ p^3_{c,2} = q \]
\[ p^3_{d,-2} = \frac{1}{2} \]
\[ p^3_{d,0} = q \]
\[ p^3_{d,2} = \frac{q^2}{q^2+(1-q)^2} \]
\[ p^3_{e,-2} = q \]
\[ p^3_{e,0} = \frac{q^2}{q^2+(1-q)^2} \]
\[ p^3_{e,2} = \frac{q^2}{q^2+(1-q)^2} + \frac{d(2q-1)}{q^2+(1-q)^2} \]

Table 1: Prices under Herding
<table>
<thead>
<tr>
<th>$s_1$</th>
<th>$x_1$</th>
<th>$y_1$</th>
<th>$s_2$</th>
<th>$x_2$</th>
<th>$y_2$</th>
<th>$s_3$</th>
<th>$x_3$</th>
<th>$y_3$</th>
<th>Value $v$ in State</th>
</tr>
</thead>
<tbody>
<tr>
<td>L</td>
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Table 2: Beliefs and Outcomes in Herding Equilibrium
Price at time $t=0$

$$p_{n0} = \frac{1}{2} + \frac{d(q^3-(1-q)^3)}{2}$$

Round 1 of Trading

$$p_{n1} = (1-q)$$

$$p_{01} = \frac{1}{2}$$

$$p_{21} = q + \frac{d(q^3-(1-q)^3)}{4}$$

Round 2 of Trading

$$p_{n2} = p_{n2}^{n2} = \frac{(1-q)^2}{q^2+(1-q)^2}$$

$$p_{b2} = p_{n2}^{n2} = p_{a2}^{n2} = (1-q)$$

$$p_{c2} = p_{n2}^{n2} = p_{a2}^{n2} = p_{b2}^{n2} = p_{02}^{n2} = \frac{1}{2}$$

$$p_{d2} = p_{n2}^{n2} = p_{a2}^{n2} = q$$

$$p_{e2} = p_{n2}^{n2} = \frac{q^2}{q^2+(1-q)^2} \cdot \left[ \frac{(1-q)^2}{q^2+(1-q)^2} \right] + \frac{d}{2} \frac{q^2-(1-q)^3}{q^2+(1-q)^2}$$

Round 3 of Trading

$$p_{n3} = \frac{(1-q)^3}{q^3+(1-q)^3}$$

$$p_{a3} = \frac{(1-q)^2}{q^3+(1-q)^3}$$

$$p_{a2}^{n3} = (1-q)$$

$$p_{b3} = \frac{(1-q)^2}{q^3+(1-q)^3}$$

$$p_{b0}^{n3} = (1-q)$$

$$p_{b2}^{n3} = \frac{1}{2}$$

$$p_{c3} = \frac{1}{2}$$

$$p_{c0}^{n3} = q$$

$$p_{c2}^{n3} = q$$

$$p_{d3} = q$$

$$p_{d0}^{n3} = q$$

$$p_{d2}^{n3} = \frac{q^2}{q^2+(1-q)^2}$$

$$p_{e3} = q$$

$$p_{e0}^{n3} = \frac{q^2}{q^2+(1-q)^2}$$

$$p_{e2}^{n3} = \frac{q^3}{q^3+(1-q)^3} + \frac{d}{2} \frac{q^3-(1-q)^3}{q^3+(1-q)^3}$$

Table 3: Prices under No-Herding
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<td></td>
<td></td>
<td>L</td>
<td>-1</td>
<td>-2</td>
<td>1</td>
<td>0</td>
<td></td>
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<td></td>
<td>L</td>
<td>-1</td>
<td>0</td>
<td>1</td>
<td>0</td>
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<tr>
<td>H</td>
<td>+1</td>
<td>+2</td>
<td>L</td>
<td>-1</td>
<td>0</td>
<td>H</td>
<td>+1</td>
<td>+2</td>
<td>1</td>
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<td>H</td>
<td>+1</td>
<td>0</td>
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<td>L</td>
<td>-1</td>
<td>-2</td>
<td>1</td>
<td>0</td>
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<td></td>
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<td>L</td>
<td>-1</td>
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<tr>
<td>H</td>
<td>+1</td>
<td>+2</td>
<td>H</td>
<td>+1</td>
<td>0</td>
<td>H</td>
<td>+1</td>
<td>+2</td>
<td>1+d</td>
<td>-d</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>H</td>
<td>+1</td>
<td>0</td>
<td>1</td>
<td>0</td>
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<td></td>
<td>L</td>
<td>-1</td>
<td>-2</td>
<td>1</td>
<td>0</td>
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<td></td>
<td></td>
<td>L</td>
<td>-1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

Table 4: Beliefs and Outcomes in No-Herding Equilibrium
$I^*$ is the minimum bound on inventory for a herding equilibrium to exist. It can be seen that holding the value of $d$ constant, as $q$ increases and signals become more informative, smaller levels of inventories are needed to start herding.

$I^*$ is the minimum bound on inventory for a herding equilibrium to exist. It can be seen that holding the value of $q$ constant, as $d$ increases and the potential gains to herding increase, smaller levels of inventories are needed to start herding.
Fig 2. Range of inventory for multiple equilibria (d=0.1)

$I_3$ is the maximum bound on inventory for a no-herding equilibrium to exist. $I^*$ is the minimum bound on inventory for a herding equilibrium to exist. It can be seen from the above figure that over the support of $q$ that is relevant i.e. for $q \in (0.5, 1]$, there is always an intermediate range of inventory for which multiple equilibria exist.
Fig 3.a. Price path along the trade sequence \(+2,+2,+2\) with H realization \((q=0.75, d=0.4)\)

Fig 3.b. Price path along the trade sequence \(+2,+2,+2\) with L realization \((q=0.75, d=0.4)\)
Fig 4. Volatility (time-series standard deviation) of prices along the price path $\{p_0, p_1, p_2, p_3\}$

(d=0.4)
The unconditional expected security value is the price that would prevail at time $t=0$ in each equilibrium.

We first calculate the variance of the four prices ($p^0, p^1, p^2,$ and $p^3$) for each possible path, and then calculate a (probability) weighted average of such variances over all the 27 paths. The implicit assumption underlying this calculation is a frequentist interpretation of the probabilities of occurrence of the 27 paths.
Appendix A

Prices under Herding: Expressions in Table 1

Price Setting: Round 1 of Trading

\[ p_{1,2} = P(\theta = H \mid s_1 = L) = (1 - q) \]

\[ p_0^1 = \frac{1}{2} [P(\theta = H \mid s_1 = H) + P(\theta = H \mid s_1 = L)] = \frac{1}{2} \]

\[ p_2^1 = P(s_2 = L \& \theta = H \mid s_1 = H) + P(s_2 = H \& y_2 = 0 \& \theta = H \mid s_1 = H) \]

\[ + (1 + d)P(s_2 = H \& y_2 = 2 \& s_3 = H \& y_3 = 2 \& \theta = L \mid s_1 = H) \]

\[ - dP(s_2 = H \& y_2 = 2 \& s_3 = H \& y_3 = 2 \& \theta = L \mid s_1 = H) \]

\[ + (1 + d)P(s_2 = H \& y_2 = 2 \& s_3 = L \& y_3 = 2 \& \theta = H \mid s_1 = H) \]

\[ - dP(s_2 = H \& y_2 = 2 \& s_3 = L \& y_3 = 2 \& \theta = L \mid s_1 = H) \]

\[ + P(s_2 = H \& y_2 = 2 \& s_3 = L \& y_3 = 0 \& \theta = H \mid s_1 = H) \]

\[ + P(s_2 = H \& y_2 = 2 \& s_3 = L \& y_3 = 0 \& \theta = H \mid s_1 = H) \]

\[ = q + \frac{d}{4}(2q - 1) \]

From the above three prices, it is easy to derive the price at time 0 that would prevail in this equilibrium.

\[ p^0 = p_{1,2}^1 P(y_1 = -2) + p_0^1 P(y_1 = 0) + p_2^1 P(y_1 = 2) \]

\[ = \frac{1}{7}(1 - q) + \frac{1}{7}(\frac{1}{2}) + \frac{1}{7}[q + \frac{d}{4}(2q - 1)] \]

\[ = \frac{1}{2} + \frac{d}{16}(2q - 1) \]

Price Setting: Round 2 of Trading

\[ p_{2,2,-2}^2 = P(\theta = H \mid s_1 = L \& s_2 = L) = \frac{(1-q)^2}{q^2 + (1-q)^2} \]

\[ p_{2,0}^2 = P(\theta = H \mid s_1 = L) = (1 - q) \]

\[ p_{2,2}^2 = P(\theta = H \mid s_1 = L \& s_2 = H) = \frac{1}{2} \]
Price Setting: Round 3 of Trading

\[ p_{0,-2}^2 = P(\theta = H \mid s_2 = L) = (1 - q) \]
\[ p_{0,0}^2 = P(\theta = H) = \frac{1}{2} \]
\[ p_{0,2}^2 = P(\theta = H \mid s_2 = H) = q \]
\[ p_{2,-2}^2 = P(\theta = H \mid s_1 = H \& s_2 = L) = \frac{1}{2} \]
\[ p_{2,0}^2 = P(\theta = H \mid s_1 = H) = q \]
\[ p_{2,2}^2 = (1 + d)P(s_3 = H \& y_3 = 2 \& \theta = H \mid s_1 = H \& s_2 = H) \]
\[ -dP(s_3 = H \& y_3 = 2 \& \theta = L \mid s_1 = H \& s_2 = H) \]
\[ +P(s_3 = H \& y_3 = 0 \& \theta = H \mid s_1 = H \& s_2 = H) \]
\[ +(1 + d)P(s_3 = L \& y_3 = 2 \& \theta = H \mid s_1 = H \& s_2 = H) \]
\[ -dP(s_3 = L \& y_3 = 2 \& \theta = L \mid s_1 = H \& s_2 = H) \]
\[ +P(s_3 = L \& y_3 = 0 \& \theta = H \mid s_1 = H \& s_2 = H) \]
\[ = \frac{q^2}{q^2 + (1-q)^2} + \frac{d - 2q - 1}{2q^2 + (1-q)^2} \]
\[ p_{c,2}^3 = p_{2,2,2}^3 = p_{0,0,2}^3 = P(\theta = H \mid s_3 = H) = q \]

\[ p_{d,-2}^3 = p_{0,2,-2}^3 = p_{2,0,-2}^3 = P(\theta = H \mid s_1 = H \& s_3 = L) = \frac{1}{2} \]

\[ p_{d,0}^3 = p_{0,2,0}^3 = p_{2,0,0}^3 = P(\theta = H \mid s_1 = H) = q \]

\[ p_{d,2}^3 = p_{0,2,2}^3 = p_{2,0,2}^3 = P(\theta = H \mid s_1 = H \& s_3 = H) = \frac{q^2}{q^2 + (1-q)^2} \]

\[ p_{e,0}^3 = p_{2,2,0}^3 = P(\theta = H \mid s_1 = H \& s_2 = H) = \frac{q^2}{q^2 + (1-q)^2} \]

\[ p_{e,2}^3 = p_{2,2,2}^3 = (1 + d)P(\theta = H \mid s_1 = H \& s_2 = H \& s_3 = \text{HorL}) \]

\[ -dP(\theta = L \mid s_1 = H \& s_2 = H \& s_3 = \text{HorL}) \]

\[ = \frac{q^2}{q^2 + (1-q)^2} + d\frac{2q^2 - (1-q)^2}{q^2 + (1-q)^2} \]

\[ = \frac{q^2}{q^2 + (1-q)^2} + d\frac{2q - 1}{q^2 + (1-q)^2} \]

Under the proposed herding equilibrium, \( p_{c,-2}^3 = p_{2,2,-2}^3 \) is off the equilibrium path. Here we use the intuitive criterion (Cho and Kreps (1987)) to interpret this sequence of events i.e. the market maker assumes that the third informed trader made a mistake in moving off-equilibrium by not herding, but that such a mistake is more likely if she had an L signal rather an H signal.

Therefore, \( p_{c,-2}^3 = P(\theta = H \mid s_1 = H \& s_2 = H \& s_3 = \text{HorL}) = q \)

This completes the derivation of the prices in Table 1.

Proof of Proposition 1: It does not benefit the first two informed traders to ever herd while it does benefit the third informed trader to herd when she can conclusively observe that the first two traders have invested \( x_i = +1 \).

This proof is almost identical to the proof of Proposition 1 in Khanna (1998). However, to situate this proof in the context of this paper, and for the sake of completeness, we provide the proof here in full. We first show that the first and second informed
traders strictly decrease the probability of being correct ex-post, by ignoring their own private signal, and then proceed to show that it could benefit the third informed trader to herd in some cases.

Table A.1. below sets out the (posterior) probabilities of being correct and wrong for each informed trader, with the available choice of trades.

<table>
<thead>
<tr>
<th>IT i’s trade</th>
<th>$x_i = +1$</th>
<th>$x_i = -1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pr(correct)</td>
<td>$P(\theta = H \mid s_j = H, j \neq i, s_i)$</td>
<td>$P(\theta = L \mid s_j = H, j \neq i, s_i)$</td>
</tr>
<tr>
<td>Pr(wrong)</td>
<td>$P(\theta = L \mid s_j = H, j \neq i, s_i)$</td>
<td>$P(\theta = H \mid s_j = H, j \neq i, s_i)$</td>
</tr>
</tbody>
</table>

Table A.1. Generic table of posterior probabilities

**Informed Trader 1**: It can be easily verified that, for both $s_1 = H$ and $s_1 = L$, IT 1 strictly decreases her probability of being correct by ignoring her own signal. For instance, when $s_1 = L$, following her signal and choosing $x_1 = -1$ implies a probability of being correct of $P(\theta = L \mid s_1 = L) = q$, while ignoring her signal and choosing $x_1 = +1$ implies a probability of being correct of $P(\theta = H \mid s_1 = L) = 1-q$. The case $s_1 = L$ can be dealt with in similar fashion.

**Informed Trader 2**: Table A.2. will facilitate the calculation of the posterior probabilities in this case.

<table>
<thead>
<tr>
<th>Action/signal of IT 1</th>
<th>IT 2’s signal</th>
<th>Prob. that the signals come from state $\theta = H$</th>
<th>state $\theta = L$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_1 = +1/s_1 = H$</td>
<td>H</td>
<td>$q^2$</td>
<td>$(1-q)^2$</td>
</tr>
<tr>
<td></td>
<td>L</td>
<td>$q(1-q)$</td>
<td>$q(1-q)$</td>
</tr>
<tr>
<td>$x_1 = -1/s_1 = L$</td>
<td>H</td>
<td>$q(1-q)$</td>
<td>$q(1-q)$</td>
</tr>
<tr>
<td></td>
<td>L</td>
<td>$(1-q)^2$</td>
<td>$q^2$</td>
</tr>
</tbody>
</table>

Table A.2. Decisions and probabilities: the first two informed traders

Check the first row. At this point, we know that the first informed had a signal H, and had chosen $x_1 = +1$. (IT 2 of course, knows this for sure because she observes $y_1 = +2$). IT 2 has a H signal. Thus, if she goes with her signal and chooses $x_2 = +1,$
then her probability of being correct is \( \Pr(\theta = H \mid s_1 = H, s_2 = H) \), which, by an application of Bayes’ rule, can be calculated as \( \frac{P(s_1 = H, s_2 = H \mid \theta = H)P(\theta = H)}{P(s_1 = H, s_2 = H)} = 0.5q^2/P(s_1 = H, s_2 = H) \). If she ignores her signal (and chooses \( x_2 = -1 \)), then her probability of being correct is \( \Pr(\theta = L \mid s_1 = H, s_2 = H) \), which can be calculated as \( 0.5(1 - q)^2/P(s_1 = H, s_2 = H) \). Since \( q > 0.5 \), she is better off following her signal. It can be seen that for the second and third rows, she is indifferent, while for the fourth row, her probability of being correct is again higher, if she chooses \( x_2 = -1 \) as per her L signal.

**Informed Trader 3**: Table A.3. will facilitate the calculation of the posterior probabilities in this final case. Here we focus on the case where it is unambiguously known that \( x_1 = +1 \) and \( x_2 = +1 \).

<table>
<thead>
<tr>
<th>Action and signal</th>
<th>Signal</th>
<th>Prob. signals come from</th>
</tr>
</thead>
<tbody>
<tr>
<td>IT 1 ( x_1 = +1 )/IT 2 ( x_2 = +1 )/IT 3 ( s_1 = H ) ( s_2 = H )</td>
<td>IT 3 ( s_3 = H )</td>
<td>( q^3 )</td>
</tr>
<tr>
<td>( q^2(1 - q) )</td>
<td>( q(1 - q)^2 )</td>
<td></td>
</tr>
</tbody>
</table>

Table A.3. Decisions and probabilities: all three informed traders

Check the first row of the above table. The third informed has a H signal. If she follows her own signal, and chooses \( x_3 = +1 \), then the probability of her being correct is \( \Pr(\theta = H \mid s_1 = H, s_2 = H, s_3 = H) \), which, by an application of Bayes’ rule, can be calculated as \( \frac{P(s_1 = H, s_2 = H, s_3 = H \mid \theta = H)P(\theta = H)/P(s_1 = H, s_2 = H, s_3 = H)}{0.5q^3/P(s_1 = H, s_2 = H, s_3 = H)} \). If she ignores her signal (and chooses \( x_3 = -1 \)), then her probability of being correct is \( \Pr(\theta = L \mid s_1 = H, s_2 = H, s_3 = H) \), which can be calculated as \( 0.5(1 - q)^3/P(s_1 = H, s_2 = H, s_3 = H) \). Since \( q > 0.5 \), she is better off following her signal.
The interesting case is in the second row of the above table. The third informed has a L signal. If she follows her own signal, and chooses \( x_3 = -1 \), then the probability of her being correct is \( \Pr(\theta = L \mid s_1 = H, s_2 = H, s_3 = L) \), which, by an application of Bayes’ rule, can be calculated as \( P(s_1 = H, s_2 = H, s_3 = L \mid \theta = L)P(\theta = L)/P(s_1 = H, s_2 = H, s_3 = L) = 0.5q(1 - q)^2/P(s_1 = H, s_2 = H, s_3 = L). \) On the other hand, if she ignores her signal and herds, choosing \( x_3 = +1 \), her probability of being correct is \( \Pr(\theta = H \mid s_1 = H, s_2 = H, s_3 = L) \), which can be calculated as \( 0.5q^2(1 - q)/P(s_1 = H, s_2 = H, s_3 = L) \). Since \( q > 0.5 \), she can clearly improve her probability of being correct ex-post by herding and is better off following the two earlier traders in this case.

**Proofs of Propositions 2 and 3: A herding equilibrium does not exist unless \( I > I^* \)**

Under the prices established above, we have to verify whether the conjectured behavior is indeed optimal for each of the informed traders. Specifically, we verify the equilibrium behavior for each informed trader assuming equilibrium behavior on part of the other two.

**Informed Trader 1:**

IT 1’s expected profits are given by

\[
E[\pi_1 \mid s_1, x_1 = +1] = E[v \mid s_1, x_1 = +1] - \frac{1}{2}[p'_0 + p'_2] \quad \text{and}
\]

\[
E[\pi_1 \mid s_1, x_1 = -1] = \frac{1}{2}[p'_0 + p'_2] - E[v \mid s_1, x_1 = -1]
\]

We need to develop expressions for \( E[v \mid s_1, x_1 = +1] \) and \( E[v \mid s_1, x_1 = -1] \)

\[
E[v \mid s_1, x_1 = +1] = \frac{1}{2}[P(\theta = H \mid s_1)]
\]

\[
+ \frac{1}{2}\{P(s_2 = L \& \theta = H \mid s_1)
\]

\[
+ P(s_2 = H \& y_2 = 0 \& \theta = H \mid s_1)
\]

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\[
+ P[(s_2 = H \& y_2 = 2 \& s_3 = H \& y_3 = 0) \text{ and } \theta = H \mid s_1]
+ P[(s_2 = H \& y_2 = 2 \& s_3 = L \& y_3 = 0) \text{ and } \theta = H \mid s_1]
+ (1 + d)P[(s_2 = H \& y_2 = 2 \& s_3 = H \& y_3 = 2) \text{ and } \theta = H \mid s_1]
- d.P[(s_2 = H \& y_2 = 2 \& s_3 = H \& y_3 = 2) \text{ and } \theta = L \mid s_1]
+(1 + d)P[(s_2 = H \& y_2 = 2 \& s_3 = L \& y_3 = 2) \text{ and } \theta = H \mid s_1]
- d.P[(s_2 = H \& y_2 = 2 \& s_3 = L \& y_3 = 2) \text{ and } \theta = L \mid s_1]
\]

So, \( E[v \mid s_1 = H, x_1 = +1] = q + \frac{d}{2}(2q - 1) \) and
\[
E[v \mid s_1 = L, x_1 = +1] = (1 - q)
\]

Setting \( d=0 \) in the above two expressions, we get the following :
\[
E[v \mid s_1 = H, x_1 = -1] = q
E[v \mid s_1 = L, x_1 = -1] = (1 - q)
\]

Trading Strategy:
\[
E[\pi_1 \mid s_1 = H, x_1 = +1] = q + \frac{d}{2}(2q - 1) - \frac{1}{2}[p_0^1 + p_2^1], \text{ and}
E[\pi_1 \mid s_1 = H, x_1 = -1] = \frac{1}{2}[p_0^1 + p_2^1] - q
\]

When \( s_1 = H, x_1 = +1 \) is preferred to \( x_1 = -1 \) if, and only if
\[
q + \frac{d}{2}(2q - 1) - \frac{1}{2}[\frac{1}{2} + q + \frac{d}{4}(2q - 1)] > \frac{1}{2}[\frac{1}{2} + (1 - q)] - q
\]
\[
\iff q > \frac{1}{2}
\]

which is true under the assumptions of the model. Thus, \( x_1 = +1 \) is optimal when \( s_1 = H \).
\[
E[\pi_1 \mid s_1 = L, x_1 = +1] = (1 - q) - \frac{1}{2}[p_0^1 + p_2^1], \text{ and}
E[\pi_1 \mid s_1 = L, x_1 = -1] = \frac{1}{2}[p_0^1 + p_2^1] - (1 - q)
\]

When \( s_1 = L, x_1 = -1 \) is preferred to \( x_1 = +1 \) if, and only if
\[ \frac{1}{2}[q + (1 - q)] - (1 - q) > (1 - q) - \frac{1}{2}[q + q + \frac{d}{q}(2q - 1)] \]
\[ \Leftrightarrow (2q - 1) > \frac{d}{8}(1 - 2q) \]

which is true \( \forall q > \frac{1}{2} \). Thus, when \( s_1 = L \), the first informed trader makes a trading loss if she trades \( x_1 = +1 \) rather than \( x_1 = -1 \). Also, she cannot gain on her existing inventory of securities by choosing \( x_1 = +1 \) over \( x_1 = -1 \), as
\[ E[v \mid s_1 = L, x_1 = +1] = E[v \mid s_1 = L, x_1 = -1] = (1 - q). \]
Thus, \( x_1 = -1 \) is optimal when \( s_1 = L \).

**Informed Trader 2:**

Here we have to consider all three cases that might have occurred in the first round of trading.

**Case 1 :** \( p^1 = p^1_{-2} (\Rightarrow s_1 = L) \)

IT 2’s expected trading profits are given by
\[ E[\pi_2 \mid p^1 = p^1_{-2}, s_2, x_2 = +1] = E[v \mid p^1 = p^1_{-2}, s_2, x_2 = +1] - \frac{1}{2}[p^2_{-2,0} + p^2_{-2,2}] \]
and
\[ E[\pi_2 \mid p^1 = p^1_{-2}, s_2, x_2 = -1] = \frac{1}{2}[p^2_{-2,0} + p^2_{-2,-2}] - E[v \mid p^1 = p^1_{-2}, s_2, x_2 = -1] \]
Here we know for sure that there cannot be any externality created by prices.

So, \( E[v \mid p^1 = p^1_{-2}, s_2 = H, x_2 = +1] = E[v \mid p^1 = p^1_{-2}, s_2 = H, x_2 = -1] \)
\[ = P(\theta = H \mid s_1 = L, s_2 = H) = \frac{1}{2}, \text{ and} \]
\[ E[v \mid p^1 = p^1_{-2}, s_2 = L, x_2 = +1] = E[v \mid p^1 = p^1_{-2}, s_2 = L, x_2 = -1] \]
\[ = P(\theta = H \mid s_1 = L, s_2 = L) = \frac{(1-q)^2}{q^2 + (1-q)^2} \]
Trading Strategy:

When \( s_2 = H, x_2 = +1 \) is preferred to \( x_2 = -1 \) if, and only if
\[
\frac{1}{2} - \frac{1}{2} \left[ (1 - q) + \frac{1}{2} \right] > \frac{1}{2} \left[ (1 - q) + \frac{(1-q)^2}{q^2 + (1-q)^2} \right] - \frac{1}{2}
\]
\[\Leftrightarrow \frac{3}{2} > 2(1-q) + \frac{(1-q)^2}{q^2 + (1-q)^2} \]
which is true \( \forall q > \frac{1}{2} \). Thus, \( x_2 = +1 \) is optimal when \( s_2 = H \).

When \( s_2 = L, x_2 = -1 \) is preferred to \( x_2 = +1 \) if, and only if
\[
\frac{1}{2} \left[ (1 - q) + \frac{(1-q)^2}{q^2 + (1-q)^2} \right] - \frac{(1-q)^2}{q^2 + (1-q)^2} > \frac{(1-q)^2}{q^2 + (1-q)^2} - \frac{1}{2} \left[ \frac{1}{2} + (1 - q) \right]
\]
which is true \( \forall q > \frac{1}{2} \). Thus, when \( s_2 = L \), the IT 2 makes a trading loss if she trades \( x_2 = +1 \) rather than \( x_2 = -1 \). Also, she cannot gain on her existing inventory of securities by choosing \( x_2 = +1 \) over \( x_2 = -1 \), as
\[
E[v \mid p^1 = p^1_{-2}, s_2 = L, x_2 = +1] = E[v \mid p^1 = p^1_{-2}, s_2 = L, x_2 = -1].
\]
Thus, \( x_2 = -1 \) is optimal when \( s_2 = L \).

Case 2 : \( p^1 = p^1_0 \) (\( \Rightarrow s_1 = H \) or \( L \))

IT 2’s expected trading profits are given by
\[
E[\pi_2 \mid p^1 = p^1_0, s_2, x_2 = +1] = E[v \mid p^1 = p^1_0, s_2, x_2 = +1] - \frac{1}{2} [p^2_{0,0} + p^2_{0,2}]
\]
and
\[
E[\pi_2 \mid p^1 = p^1_0, s_2, x_2 = -1] = \frac{1}{2} [p^2_{0,0} + p^2_{0,-2}] - E[v \mid p^1 = p^1_0, s_2, x_2 = -1]
\]
Here too we know for sure that there cannot be any externality created by prices.

So, \( E[v \mid p^1 = p^1_0, s_2 = H, x_2 = +1] = E[v \mid p^1 = p^1_0, s_2 = H, x_2 = -1] \)
\[= P(\theta = H \mid s_2 = H) = q, \text{ and} \]
\[
E[v \mid p^1 = p^1_0, s_2 = L, x_2 = +1] = E[v \mid p^1 = p^1_0, s_2 = L, x_2 = -1]
\]
\[= P(\theta = H \mid s_2 = L) = (1 - q) \]
Trading Strategy:

When \( s_2 = H, x_2 = +1 \) is preferred to \( x_2 = -1 \) if, and only if
\[
q - \frac{1}{2} \left[ \frac{1}{2} + q \right] > \frac{1}{2} \left[ \frac{1}{2} + (1 - q) \right] - q
\]
\[\iff q > \frac{1}{2}\]

which is true under the assumptions of the model. Thus, \( x_2 = +1 \) is optimal when \( s_2 = H \).

When \( s_2 = L, x_2 = -1 \) is preferred to \( x_2 = +1 \) if, and only if
\[
\frac{1}{2} \left[ \frac{1}{2} + (1 - q) \right] - (1 - q) > (1 - q) - \frac{1}{2} \left[ \frac{1}{2} + q \right]
\]
\[\iff q > \frac{1}{2}\]

which is true under the assumptions of the model. Thus, when \( s_2 = L \), the IT 2 makes a trading loss if she trades \( x_2 = +1 \) rather than \( x_2 = -1 \). Also, she cannot gain on her existing inventory of securities by choosing \( x_2 = +1 \) over \( x_2 = -1 \), as
\[
E[v \mid p^1 = p^1_0, s_2 = L, x_2 = +1] = E[v \mid p^1 = p^1_0, s_2 = L, x_2 = -1].
\]
Thus, \( x_2 = -1 \) is optimal when \( s_2 = L \).

Case 3 : \( p^1 = p^1_2 (\Rightarrow s_1 = H) \)

IT 2’s expected trading profits are given by
\[
E[\pi_2 \mid p^1 = p^1_2, s_2, x_2 = +1] = E[v \mid p^1 = p^1_2, s_2, x_2 = +1] - \frac{1}{2} [p^2_{2,0} + p^2_{2,-2}]
\]
and
\[
E[\pi_2 \mid p^1 = p^1_2, s_2, x_2 = -1] = \frac{1}{2} [p^2_{2,0} + p^2_{2,-2}] - E[v \mid p^1 = p^1_2, s_2, x_2 = -1]
\]
\[
E[v \mid p^1 = p^1_2, s_2, x_2 = +1] = \frac{1}{2} [P(\theta = H \mid s_1 = H \& s_2)]
\]
\[
+ \frac{1}{2} \{(1 + d) P(s_3 = H \& y_3 = 2 \& \theta = H \mid s_1 = H \& s_2)
\]
\[\]
\[- d P(s_3 = H \& y_3 = 2 \& \theta = L \mid s_1 = H \& s_2)
\]
\[\]
\[+(1 + d) P(s_3 = L \& y_3 = 2 \& \theta = H \mid s_1 = H \& s_2)
\]
\[36\]
\[-d.P(s_3 = L \& y_3 = 2 \& \theta = L \mid s_1 = H \& s_2)\]
\[+P(s_3 = H \& y_3 = 0 \& \theta = H \mid s_1 = H \& s_2)\]
\[+P(s_3 = L \& y_3 = 0 \& \theta = H \mid s_1 = H \& s_2)\}\]
So, \[E[v \mid p^1 = p^1_2, s_2 = H, x_2 = +1] = \frac{q^2}{q^2 + (1-q)^2} + \frac{d}{4} \frac{2q-1}{q^2 + (1-q)^2}\]
and, \[E[v \mid p^1 = p^1_2, s_2 = L, x_2 = +1] = \frac{1}{2}\]
Setting \(d=0\) in the above two expressions, we get the following:
\[E[v \mid p^1 = p^1_2, s_2 = H, x_2 = -1] = \frac{q^2}{q^2 + (1-q)^2}\]
\[E[v \mid p^1 = p^1_2, s_2 = L, x_2 = -1] = \frac{1}{2}\]

Trading Strategy:

When \(s_2 = H, x_2 = +1\) is preferred to \(x_2 = -1\) if, and only if
\[\frac{q^2}{q^2 + (1-q)^2} + \frac{d}{4} \frac{2q-1}{q^2 + (1-q)^2} - \frac{1}{2} \left[ q + \frac{q^2}{q^2 + (1-q)^2} + \frac{d}{2} \frac{2q-1}{q^2 + (1-q)^2} \right] > \frac{1}{2} \left[ q + \frac{1}{2} \right] - \frac{q^2}{q^2 + (1-q)^2}\]
\[\iff \frac{3}{2} \frac{q^2}{q^2 + (1-q)^2} > q + \frac{1}{4}\]
which is true \(\forall q > \frac{1}{2}\). Thus, \(x_2 = +1\) is optimal when \(s_2 = H\).

When \(s_2 = L, x_2 = -1\) is preferred to \(x_2 = +1\) if, and only if
\[\frac{1}{2} \left[ q + \frac{1}{2} \right] - \frac{1}{2} > \frac{1}{2} - \frac{1}{2} \left[ q + \frac{q^2}{q^2 + (1-q)^2} + \frac{d}{2} \frac{2q-1}{q^2 + (1-q)^2} \right] \]
\[\iff \left[ 2q + \frac{q^2}{q^2 + (1-q)^2} - \frac{3}{2} \right] > \frac{d}{2} \frac{(1-2q)}{q^2 + (1-q)^2}\]
which is true \(\forall q > \frac{1}{2}\) and \(d > 0\). Thus, when \(s_2 = L\), the IT 2 makes a trading loss if she trades \(x_2 = +1\) rather than \(x_2 = -1\). Also, she cannot gain on her existing inventory of securities by choosing \(x_2 = +1\) over \(x_2 = -1\), as \(E[v \mid p^1 = p^1_2, s_2 = L, x_2 = +1] = E[v \mid p^1 = p^1_2, s_2 = L, x_2 = -1]\). Thus, \(x_2 = -1\) is optimal when \(s_2 = L\).
Informed Trader 3:

Here we have to consider five cases, i.e., all distinct prices that could have been established in the second round of trading. All the possible cases are listed below:

a. \( p^2 = p_a^2 \equiv p_{a,2}^{-2} \)

b. \( p^2 = p_b^2 \equiv p_{b,0}^{-2} = p_{0,2}^2 \)

c. \( p^2 = p_c^2 \equiv p_{2,2}^{-2} = p_{2,-2}^2 = p_{0,0}^2 \)

d. \( p^2 = p_d^2 \equiv p_{2,0}^2 = p_{0,2}^2 \)

e. \( p^2 = p_e^2 \equiv p_{2,2}^2 \)

In Cases 1 through 4 above, IT3’s trade does not impact the value as the externality in value is likely to be created only in Case 5. It can be verified that in all these cases, IT3 trades according to her own signal i.e. she trades \( x_3 = +1 \) when \( s_3 = H \) and \( x_3 = -1 \) when \( s_3 = L \). We take up the more interesting Case 5 below:

\textit{Case 5 :} \( p^2 = p_e^2 = p_{2,2}^2 (\Rightarrow s_1 = H, s_2 = H) \)

IT 3’s expected trading profits are given by

\begin{align*}
E[\pi_3 \mid p^2 = p_e^2, s_3, x_3 = +1] &= E[v \mid p^2 = p_e^2, s_3, x_3 = +1] - \frac{1}{2}[p_{e,0}^3 + p_{e,2}^3] \\
E[\pi_3 \mid p^2 = p_e^2, s_3, x_3 = -1] &= \frac{1}{2}[p_{e,0}^3 + p_{e,-2}^3] - E[v \mid p^2 = p_e^2, s_3, x_3 = -1] \\
E[v \mid p^2 = p_e^2, s_3, x_3 = +1] &= \frac{1}{2}\{(1 + d)P(\theta = H \mid s_1 = H, s_2 = H, s_3) \\
&\quad - d.P(\theta = L \mid s_1 = H, s_2 = H, s_3)\} + P(\theta = H \mid s_1 = H, s_2 = H, s_3) \}
\end{align*}

So, \( E[v \mid p^2 = p_e^2, s_3 = H, x_3 = +1] = \frac{q^3}{q^3 + (1-q)^3} + \frac{d}{2}\frac{q^3 - (1-q)^3}{q^3 + (1-q)^3} \)

and \( E[v \mid p^2 = p_e^2, s_3 = L, x_3 = +1] = q + \frac{d}{2}(2q - 1) \)

When IT3 invests \( x_3 = -1 \), she knows that the externality \( d \) will not be created.
So, \( E[v \mid p^2 = p_e^2, s_3 = H, x_3 = -1] = \frac{q^3}{q^3 + (1 - q)^3} \)
\( E[v \mid p^2 = p_e^2, s_3 = L, x_3 = -1] = q \)

Trading Strategy:

When \( s_3 = H, x_3 = +1 \) is preferred to \( x_3 = -1 \) if, and only if
\[
\frac{2q^3}{q^3 + (1 - q)^3} + \frac{d [q^3 - (1 - q)^3]}{2q^3 + (1 - q)^3} > \frac{3}{2} \frac{q^2}{q^2 + (1 - q)^2} + \frac{d}{2} \frac{(2q - 1)}{q^2 + (1 - q)^2} + \frac{q}{2}
\]
which is true \( \forall q > \frac{1}{2} \) and \( d > 0 \). Thus, \( x_3 = +1 \) is optimal when \( s_3 = H \). When \( s_3 = L, x_3 = +1 \) is preferred to \( x_3 = -1 \) (i.e. IT 3 herds) if, and only if
\[
\frac{3q}{2} - \frac{3}{2} \frac{q^2}{q^2 + (1 - q)^2} > \frac{d}{2} (2q - 1) \left[ \frac{1}{q^2 + (1 - q)^2} - 1 \right]
\]
which fails for \( q > \frac{1}{2} \) and \( d > 0 \). Thus, when \( s_3 = L, IT 3 \) makes a trading loss if she trades \( x_3 = +1 \) rather than \( x_3 = -1 \) (i.e. if she herds). The quantum of this trading loss is:
\[
\frac{3}{2} \left[ \frac{q^2}{q^2 + (1 - q)^2} - q \right] + \frac{d}{2} (2q - 1) \left[ \frac{1}{q^2 + (1 - q)^2} - 1 \right]
\]
It is clear that under the information and model structure outlined in Sections I.A and I.B of the paper (i.e. in the absence of inventory), herding is not an equilibrium. However, when we allow informed traders to have inventories in the security (as in Section II), IT 3 can gain on her inventory by herding since
\[
E[v \mid p^2 = p_e^2, s_3 = L, x_3 = +1] = q + \frac{d}{2} (2q - 1) > E[v \mid p^2 = p_e^2, s_3 = L, x_3 = -1] = q \forall d > 0
\]
By herding, IT3 can gain an extra \( d(2q-1)/2 \) per each security in her inventory. Thus,
for herding to be supported in equilibrium, we need

\[ I \frac{d}{2}(2q - 1) > \frac{3}{2} \left[ \frac{q^2}{q^2 + (1 - q)^2} - q \right] + \frac{d}{2}(2q - 1) \left[ \frac{1}{q^2 + (1 - q)^2} - 1 \right] \tag{1} \]

which reduces to (since \( q > \frac{1}{2} \) is assumed)

\[ I > I^* = \frac{3}{d(2q - 1)} \left[ \frac{q^2}{q^2 + (1 - q)^2} - q \right] + \left[ \frac{1}{q^2 + (1 - q)^2} - 1 \right] \]
Appendix B

Prices under No-Herding: Expressions in Table 3

Price Setting: Round 1 of Trading

\[ p_{n_2}^n = P(\theta = H \mid s_1 = L) = (1 - q) \]

\[ p_{n_1}^n = 1/2[P(\theta = H \mid s_1 = H) + P(\theta = H \mid s_1 = L)] = \frac{1}{2} \]

\[ p_2^n = P(s_2 = L \& \theta = H \mid s_1 = H) + P(s_2 = H \& y_2 = 0 \& \theta = H \mid s_1 = H) + P(s_2 = H \& y_2 = 2 \& s_3 = L \& \theta = H \mid s_1 = H) + P(s_2 = H \& y_2 = 2 \& s_3 = H \& y_3 = 0 \& \theta = H \mid s_1 = H) + (1 + d)P(s_2 = H \& y_2 = 2 \& s_3 = H \& y_3 = 2 \& \theta = H \mid s_1 = H) - dP(s_2 = H \& y_2 = 2 \& s_3 = H \& y_3 = 2 \& \theta = L \mid s_1 = H) = q + \frac{d}{4}[q^3 - (1 - q)^3] \]

From the above three prices, it is easy to derive the price at time 0 that would prevail in this equilibrium.

\[ p^0 = p_{n_2}^n P(y_1 = -2) + p_{n_1}^n P(y_1 = 0) + p_2^n P(y_1 = 2) = \frac{1}{4}(1 - q) + \frac{1}{2}\left(\frac{1}{2}\right) + \frac{1}{4}\left[q + \frac{d}{4}(q^3 - (1 - q)^3)\right] = \frac{1}{2} + \frac{d}{16}[q^3 - (1 - q)^3] \]

Price Setting: Round 2 of Trading

Prices \( p_{a_2}^n \) through \( p_{d_2}^n \) can be derived exactly like \( p_{a_2}^n \) through \( p_{d_2}^n \) of the herding case (see Appendix A). The interesting case is \( p_{e_2}^n = p_{2_2}^n \), which is derived below.

\[ p_{e_2}^n = p_{2_2}^n = P(s_3 = L \& \theta = H \mid s_1 = H \& s_2 = H) + P(s_3 = H \& y_3 = 0 \& \theta = H \mid s_1 = H \& s_2 = H) + (1 + d)P(s_3 = H \& y_3 = 2 \& \theta = H \mid s_1 = H \& s_2 = H) \]
\[-dP(s_3 = H & y_3 = 2 & \theta = L \mid s_1 = H & s_2 = H)\]
\[= \frac{q^3}{q^2 + (1-q)^2} + \frac{d}{2} \left[ \frac{q^3 - (1-q)^3}{q^2 + (1-q)^2} \right] \]

**Price Setting: Round 3 of Trading**

Prices \(p^{n^3}_{a,2}\) through \(p^{n^3}_{a,-2}\) can be derived exactly like \(p^{n^3}_{a,-2}\) through \(p^{n^3}_{a,-2}\) in the herding case (see Appendix A). The interesting cases are \(p^{n^3}_{c,0}\) and \(p^{n^3}_{c,2}\), which are derived below.

\[p^{n^3}_{c,0} = p^{n^3}_{2,2,0} = P(\theta = H \mid s_1 = H & s_2 = H & s_3 = H or L) = \frac{q^2}{q^2 + (1-q)^2}\]
\[p^{n^3}_{c,2} = p^{n^3}_{2,2,2} = (1 + d)P(\theta = H \mid s_1 = H & s_2 = H & s_3 = H)\]
\[-dP(\theta = L \mid s_1 = H & s_2 = H & s_3 = H)\]
\[= \frac{q^3}{q^2 + (1-q)^2} + \frac{d}{2} \left[ \frac{q^3 - (1-q)^3}{q^2 + (1-q)^2} \right] \]

This completes the derivation of the prices in Table 3.

**Proof of Proposition 4: A no-herding equilibrium exists as long as \(I < I_3\)**

Under the prices established above, we have to verify whether the conjectured behavior is indeed optimal for each of the informed traders. Specifically, we verify the equilibrium behavior for each informed trader assuming equilibrium behavior on part of the other two.

**Informed Trader 1**:

IT 1’s expected profits are given by
\[E[\pi_1 \mid s_1, x_1 = +1] = E[v \mid s_1, x_1 = +1] - \frac{1}{2}[p^{n^1}_{0} + p^{n^1}_{2}] \text{ and} \]
\[E[\pi_1 \mid s_1, x_1 = -1] = \frac{1}{2}[p^{n^1}_{0} + p^{n^1}_{2}] - E[v \mid s_1, x_1 = -1] \]

We need to develop expressions for \(E[v \mid s_1, x_1 = +1]\) and \(E[v \mid s_1, x_1 = -1]\)
\[ E[v \mid s_1, x_1 = +1] = \frac{1}{2} [P(\theta = H \mid s_1)] \]
\[ + \frac{1}{2} \{P(s_2 = L \& \theta = H \mid s_1) \]
\[ + P(s_2 = H \& y_2 = 0 \& \theta = H \mid s_1) \]
\[ + P(s_2 = H \& y_2 = 2 \& s_3 = L \& \theta = H \mid s_1) \]
\[ + P((s_2 = H \& y_2 = 2 \& s_3 = H \& y_3 = 0) \text{ and } \theta = H \mid s_1) \]
\[ + (1 + d) P((s_2 = H \& y_2 = 2 \& s_3 = H \& y_3 = 2) \text{ and } \theta = H \mid s_1) \]
\[ - d P((s_2 = H \& y_2 = 2 \& s_3 = H \& y_3 = 2) \text{ and } \theta = L \mid s_1) \} \]

So, \( E[v \mid s_1 = H, x_1 = +1] = q + \frac{d}{8}[q^3 - (1 - q)^3] \) and
\[ E[v \mid s_1 = L, x_1 = +1] = (1 - q) + \frac{d}{8}[3q^2 - 2q^3 - q] \]

Setting \( d = 0 \) in the above two expressions, we get the following:
\[ E[v \mid s_1 = H, x_1 = -1] = q \]
\[ E[v \mid s_1 = L, x_1 = -1] = (1 - q) \]

Trading Strategy:
\[ E[\pi_1 \mid s_1 = H, x_1 = +1] = q + \frac{d}{8}[q^3 - (1 - q)^3] - \frac{1}{2}[p_0^{n_1} + p_2^{n_1}], \text{ and} \]
\[ E[\pi_1 \mid s_1 = H, x_1 = -1] = \frac{1}{2}[p_0^{n_1} + p_2^{n_1}] - q \]

When \( s_1 = H, x_1 = +1 \) is preferred to \( x_1 = -1 \) if, and only if
\[ q + \frac{d}{8}[q^3 - (1 - q)^3] - \frac{1}{2} \left[ \frac{1}{2} + q + \frac{d}{4}[q^3 - (1 - q)^3] \right] > \frac{1}{2} \frac{1}{2} + (1 - q) - q \]
\[ \Leftrightarrow q > \frac{1}{2} \]
which is true under the assumptions of the model. Thus, \( x_1 = +1 \) is optimal when \( s_1 = H \).
\[ E[\pi_1 \mid s_1 = L, x_1 = +1] = (1 - q) + \frac{d}{8}[3q^2 - 2q^3 - q] - \frac{1}{2}[p_0^{n_1} + p_2^{n_1}], \text{ and} \]
\[ E[\pi_1 \mid s_1 = L, x_1 = -1] = \frac{1}{2}[p_0^{n_1} + p_1^{n_2}] - (1 - q) \]

When \( s_1 = L, x_1 = -1 \) is preferred to \( x_1 = +1 \) if, and only if

\[
\frac{1}{2}[\frac{1}{2} + (1 - q)] - (1 - q) > (1 - q) + \frac{d}{8}[3q^2 - 2q^3 - q] - \frac{1}{2} \left[ \frac{1}{2} + q + \frac{d}{4}(q^3 - (1 - q)^3) \right]
\]

\( \iff (2q - 1) > \frac{d}{8}(6q^2 - 4q^3 - 4q + 1) \)

which is true \( \forall q > \frac{1}{2} \) and \( d > 0 \). Thus, when \( s_1 = L \), trading \( x_1 = -1 \) instead of trading \( x_1 = +1 \) yields IT 1 an expected profit of

\[
(2q - 1) - \frac{d}{8}(6q^2 - 4q^3 - 4q + 1)
\]

However, trading \( x_1 = +1 \) rather than \( x_1 = -1 \) increments the expected value of each unit of inventory by \( \frac{d}{8}[3q^2 - 2q^3 - q] \). (This is the difference between \( E[v \mid s_1 = L, x_1 = +1] \) and \( E[v \mid s_1 = L, x_1 = -1] \)). Thus, we require the inventory of the first informed trader to be such that her potential expected value addition to inventory from deviating from the equilibrium behavior (and trading \( x_1 = +1 \)) is outweighed by her trading profits from following the equilibrium strategy. i.e. we need that:

\[
I < \left[ \frac{8(2q - 1)}{d(3q^2 - 2q^3 - q)} - \frac{(6q^2 - 4q^3 - 4q + 1)}{(3q^2 - 2q^3 - q)} \right]
\]

**Informed Trader 2:**

Here we have to consider all three cases that might have occurred in the first round of trading. Cases 1 and 2 where \( p^{n_1} = p^{n_2} \) and \( p^{n_1} = p^{n_1}_0 \) can be dealt with exactly as in the herding case (see Appendix A). We deal with the more interesting Case 3 here.
Case 3: \( p^{n1} = p_2^{n1} \Rightarrow s_1 = H \)

IT 2’s expected trading profits are given by

\[
E[\pi_2 \mid p^{n1} = p_2^{n1}, s_2, x_2 = +1] = E[v \mid p^{n1} = p_2^{n1}, s_2, x_2 = +1] - \frac{1}{2}[p_2^{n2}_0 + p_2^{n2}]
\]

and

\[
E[\pi_2 \mid p^{n1} = p_2^{n1}, s_2, x_2 = -1] = \frac{1}{2}[p_2^{n2}_0 + p_2^{n2} - 2] - E[v \mid p^{n1} = p_2^{n1}, s_2, x_2 = -1]
\]

\[
E[v \mid p^{n1} = p_2^{n1}, s_2, x_2 = +1] = \frac{1}{2}P(\theta = H \mid s_1 = H \& s_2)
\]

\[+\frac{1}{2}\{P(s_3 = L \& \theta = H \mid s_1 = H \& s_2)
\]

\[+P(s_3 = H \& y_3 = 0 \& \theta = H \mid s_1 = H \& s_2)
\]

\[+(1 + d)P(s_3 = H \& y_3 = 2 \& \theta = H \mid s_1 = H \& s_2)
\]

\[-dP(s_3 = H \& y_3 = 2 \& \theta = L \mid s_1 = H \& s_2)\}]

So, \( E[v \mid p^{n1} = p_2^{n1}, s_2 = H, x_2 = +1] = \frac{q^2}{q^2 + (1-q)^2} + \frac{d}{4} \frac{[q^3 - (1-q)^3]}{q^2 + (1-q)^2} \)

and, \( E[v \mid p^{n1} = p_2^{n1}, s_2 = L, x_2 = +1] = \frac{1}{2} + \frac{d}{2}(2q - 1) \)

Setting \( d=0 \) in the above two expressions, we get the following:

\( E[v \mid p^{n1} = p_2^{n1}, s_2 = H, x_2 = +1] = \frac{q^2}{q^2 + (1-q)^2} \)

\( E[v \mid p^{n1} = p_2^{n1}, s_2 = L, x_2 = +1] = \frac{1}{2} \)

Trading Strategy:

When \( s_2 = H, x_2 = +1 \) is preferred to \( x_2 = -1 \) if, and only if

\[
\frac{q^2}{q^2 + (1-q)^2} + \frac{d}{4} \frac{[q^3 - (1-q)^3]}{q^2 + (1-q)^2} - \frac{1}{2} \left[ q + \frac{q^2}{q^2 + (1-q)^2} + \frac{d}{2} \frac{[q^3 - (1-q)^3]}{q^2 + (1-q)^2} \right] = 0
\]

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\[
> \frac{1}{2} \left[ q + \frac{1}{2} \right] - \frac{q^2}{q^2 + (1-q)^2} \\
\Leftrightarrow \frac{3}{2} \left[ \frac{q^2}{q^2 + (1-q)^2} \right] > q + \frac{1}{4}
\]

which is true \( \forall q > \frac{1}{2} \). Thus, \( x_2 = +1 \) is optimal when \( s_2 = H \).

When \( s_2 = L, x_2 = -1 \) is preferred to \( x_2 = +1 \) if, and only if

\[
\frac{1}{2} \left[ q + \frac{1}{2} \right] - \frac{1}{2} > \frac{1}{2} + \frac{d}{8}(2q-1) - \frac{1}{2} \left[ q + \frac{q^2}{q^2 + (1-q)^2} + \frac{d[q^3 - (1-q)^3]}{2q^2 + (1-q)^2} \right] \\
\Leftrightarrow \left[ q + \frac{q^2}{2[q^2 + (1-q)^2]} - \frac{3}{4} \right] < \frac{d}{8} \left[ (2q-1) - \frac{2[q^3 - (1-q)^3]}{q^2 + (1-q)^2} \right]
\]

which is true \( \forall q > \frac{1}{2} \) and \( d > 0 \).

However, trading \( x_2 = +1 \) rather than \( x_2 = -1 \) increments the expected value of each unit of inventory by \( \frac{d}{8}(2q-1) \). (This is the difference between \( E[v \mid p^{n1} = p^{n1}_2, s_2 = L, x_2 = +1] \) and \( E[v \mid p^{n1} = p^{n1}_2, s_2 = L, x_2 = -1] \)). Thus, we require the inventory of the second informed trader to be such that her potential expected value addition to inventory from deviating from the equilibrium behavior (and trading \( x_2 = +1 \)) is outweighed by her trading profits from following the equilibrium strategy i.e. we need that :

\[
I \frac{d}{8}(2q-1) < \left( q - \frac{3}{4} \right) + \frac{q^2}{2[q^2 + (1-q)^2]} - \frac{d}{8} \left[ (2q-1) - \frac{2[q^3 - (1-q)^3]}{q^2 + (1-q)^2} \right]
\]

\[
\Leftrightarrow I < I_2 = \left[ \frac{8q - 6}{d(2q-1)} + \frac{4q^2}{d(2q-1)[q^2 + (1-q)^2]} + \frac{2[q^3 - (1-q)^3]}{(2q-1)[q^2 + (1-q)^2]} - 1 \right]
\]

**Informed Trader 3:**

Here, as with herding, we have to consider five cases, i.e. all distinct prices that could have been established in the second round of trading. All possible cases are listed
below:

a. \( p^{n^2} = p^o_{n^2} = p^o_{-2,-2} \)

b. \( p^{n^2} = p^o_{n^2} = p^o_{-2,0} = p^o_{0,-2} \)

c. \( p^{n^2} = p^c_{n^2} = p^c_{-2,2} = p^c_{2,-2} = p^c_{0,0} \)

d. \( p^{n^2} = p^d_{n^2} = p^d_{2,0} = p^d_{0,2} \)

e. \( p^{n^2} = p^e_{n^2} = p^e_{2,2} \)

Cases 1 through 4 above are identical to those in the herding equilibrium. In all these cases, IT 3’s trade does not impact the value as the externality in value can be created only in Case 5. It can be verified that in all these cases, IT3 trades according to her own signal i.e. she trades \( x_3 = +1 \) when \( s_3 = H \) and \( x_3 = -1 \) when \( s_3 = L \).

We take up the more interesting Case 5 below:

\textit{Case 5 :} \( p^{n^2} = p^e_{n^2} = p^e_{2,2} \) (\( \Rightarrow s_1 = H, s_2 = H \))

IT 3’s expected trading profits are given by

\[
E[\pi_3 | p^{n^2} = p^e_{n^2}, s_3, x_3 = +1] = E[v | p^{n^2} = p^e_{n^2}, s_3, x_3 = +1] - \frac{1}{2}[p^o^3 + p^e^2]
\]

and

\[
E[\pi_3 | p^{n^2} = p^e_{n^2}, s_3, x_3 = -1] = \frac{1}{2}[p^e^3 + p^o^2] - E[v | p^{n^2} = p^e_{n^2}, s_3, x_3 = -1]
\]

\[
E[v | p^{n^2} = p^e_{n^2}, s_3, x_3 = +1] = \frac{1}{2}P(\theta = H | s_1 = H, s_2 = H, s_3)
\]

\[
= \frac{1}{2}((1 + d)P(\theta = H | s_1 = H, s_2 = H, s_3))
\]

\[
- dP(\theta = L | s_1 = H, s_2 = H, s_3)
\]

So, \( E[v | p^{n^2} = p^e_{n^2}, s_3 = H, x_3 = +1] = \frac{q^1}{q^3 + (1-q)^2} + \frac{d \cdot (1-q)/(1-q)^2}{2} \)

and

\[
E[v | p^{n^2} = p^e_{n^2}, s_3 = L, x_3 = +1] = q + \frac{d}{2}(2q - 1)
\]
Setting $d=0$ in the above expressions, we get

\[ E[v \mid p^n = p^n, s_3 = H, x_3 = -1] = \frac{q^3}{q^3 + (1-q)^3} \]
\[ E[v \mid p^n = p^n, s_3 = L, x_3 = -1] = q \]

Trading Strategy:

When $s_3 = H, x_3 = +1$ is preferred to $x_3 = -1$ if, and only if

\[
\frac{q^3}{q^3 + (1-q)^3} + \frac{d[q^3 - (1-q)^3]}{2(q^3 + (1-q)^3)} \geq \frac{1}{2} \left[ \frac{q^2}{q^2 + (1-q)^2} + \frac{q^3}{q^3 + (1-q)^3} + \frac{d[q^3 - (1-q)^3]}{q^3 + (1-q)^3} \right]
\]

\[
> \frac{1}{2} \left[ q + \frac{q^2}{q^2 + (1-q)^2} \right] - \frac{q^3}{q^3 + (1-q)^3}
\]

\[
\Leftrightarrow \frac{3}{2} \left[ \frac{q^3}{q^3 + (1-q)^3} \right] > \left[ \frac{q^2}{q^2 + (1-q)^2} \right] + \frac{q}{2}
\]

which is true for $q > \frac{1}{2}$ and $d > 0$. Thus, $x_3 = +1$ is optimal when $s_3 = H$.

When $s_3 = L, x_3 = -1$ is preferred to $x_3 = +1$ if, and only if

\[
\frac{1}{2} \left[ q + \frac{q^2}{q^2 + (1-q)^2} \right] - q > q + \frac{d}{2(2q-1)} - \frac{1}{2} \left[ \frac{q^2}{q^2 + (1-q)^2} + \frac{q^3}{q^3 + (1-q)^3} + \frac{d[q^3 - (1-q)^3]}{q^3 + (1-q)^3} \right]
\]

\[
\Leftrightarrow \left[ \frac{q^2}{q^2 + (1-q)^2} + \frac{q^3}{2[q^3 + (1-q)^3]} \right] > \frac{d}{2(2q-1)} - \frac{d[q^3 - (1-q)^3]}{2q^3 + (1-q)^3}
\]

which is true for $q > \frac{1}{2}$ and $d > 0$. Therefore, trading $x_3 = -1$ instead of $x_3 = +1$ yields IT 3 a profit of:

\[
\frac{q^2}{q^2 + (1-q)^2} + \frac{q^3}{2[q^3 + (1-q)^3]} - \frac{3q}{2} - \frac{d}{2} \left( 2q - 1 \right) - \frac{d[q^3 - (1-q)^3]}{2q^3 + (1-q)^3}
\]
However, trading $x_3 = +1$ rather than $x_3 = -1$ increments the expected value of each unit of inventory by $\frac{d}{2}(2q - 1)$. Thus, we require the inventory of IT 3 to be such that her potential expected value addition to inventory from deviating from the equilibrium behavior (and trading $x_3 = +1$) is outweighed by her trading profit from following the equilibrium strategy i.e. we need that:

$$I \frac{d}{2}(2q - 1) < \frac{q^2}{q^2 + (1 - q)^2} + \frac{q^3}{2[q^3 + (1 - q)^3]} - \frac{3q}{2} \frac{d}{2}(2q - 1) + \frac{d[q^3 - (1 - q)^3]}{2q^3 + (1 - q)^3}$$

$$\Leftrightarrow I < I_3 \equiv \left[ \frac{2q^2}{d(2q-1)[q^2 + (1 - q)^2]} + \frac{q^3}{d(2q-1)[q^3 + (1 - q)^3]} - \frac{3q}{d(2q-1)} + \frac{|q^3 - (1 - q)^3|}{(2q-1)[q^3 + (1 - q)^3]} - 1 \right]$$

Since we have assumed symmetric informed traders (with identical inventories), a sufficient condition for a no-herding equilibrium to exist is that $I < \min(I_1, I_2, I_3)$. It can be verified that $I_3$ is the smallest of $I_1, I_2$ and $I_3$. This intuitively makes sense, as the third informed trader, knowing about the two H signals in the two earlier rounds of trading has the strongest temptation to herd, even though her own signal is an L signal. Thus, the constraint on her inventory is much more binding than those on the earlier two traders. It can be seen from Fig. B.1. that $I_1 - I_3$ and $I_2 - I_3$ are both positive over the support of $q$ that we consider in this model. Another interesting observation from Fig. B.1 is regarding the shape of $I_1 - I_3$, which is sloping upward as $q$ increases. The reason is that the first informed trader is increasingly more reluctant to start herding by trading against her own signal, as her signal becomes more informative.
Fig B.1. Comparison of inventory bounds for no-herding
References


Footnotes

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1For an insightful review of this literature see Hubbard (1998)

2See Fisher (1933), Bernanke (1983), and Eckstein and Sinai (1986) among others for description of these “accelerator-type” phenomena.

3Kiyotaki and Moore (1997) show a similar effect when asset values (as opposed to cash flows) are affected. A positive shock to land prices translates into increased borrowing capacity with land as collateral. The firm which owns land experiences a relaxation of its budget constraint and can make additional investments.

4Amazon.com made over 30 acquisitions between April 1998 and September 2000, when its stock price was between $70 and $207. No notable acquisition took place in the following 6 month period when its price plummeted. Nortel made about 20 acquisitions worth $33.7 billion between 1997 and late 2000. 95 percent were made through stock swaps. Again, the acquisitions dried up when its stock price plummeted. The same is true of companies like Cisco, Intel, Dell and Microsoft. All made numerous acquisitions during this period and mainly through stock swaps. The acquisition ac-
tivity slowed when their stock prices softened. Ironically, in 1999, Steven Case, CEO of AOL referred to this as the era of ‘free money’ for internet firms and wondered what would happen to the internet industry when resources became tighter.

Studies like Friend, Blume and Crockett (1970) and Lakonishok, Shleifer and Vishny (1992b) have reported that in an apparent attempt to take advantage of momentum profits, large traders follow similar or ‘herd-like’ trading strategies: buying when other participants are buying, and selling when others are selling. Also see Grinblatt, Titman and Wermers (1995) and Nofsinger and Sias (1999) among others.

For managers conditioning investments on signals provided by stock price changes see Khanna, Slezak and Bradley (1994), and Leland (1992). Also see Subrahmanyam and Titman (2001) for an interesting application of the feedback effect of prices on firm value. Unlike in their model, in our paper, the market maker is aware of the manipulator, and the strategies she would play in equilibrium. Hence, as we show later, he establishes prices taking the possibility of manipulation into account, ensuring that the manipulator makes negative trading profits when trading against her information. Thus, the feedback effect alone does not result in either manipulation or herding.

For tractability, we do not model this aspect explicitly. The underlying assumption is that market confidence in price levels is correlated with the length of time for which price stay high.

Unlike in Grossman and Stiglitz (1980) the informed trader makes a profit if she trades in the direction of her signal. This happens because of noise traders in these types of models.

Other papers have used similar arguments to support rational price manipulation or price bubbles. Allen and Gorton (1993) model agency problems between investors
and fund managers. Even with an optimal contract, the bad fund manager rationally buys overvalued shares. They do not mind making losing trades as it does not adversely affect their compensation. Kumar and Seppi (1992) model a manipulator who intentionally loses in the spot market to improve his/her previously taken position in the futures market.

10The main conclusions are not affected by which trading pattern signals better opportunities. The effect would be captured by any trading pattern that suggests that the probability of the High state is greater.

11Note that the externality ‘d’ occurs when there are several trades in a similar direction. Intuitively, we want to capture the feedback effect of prices, where higher prices lead to a broader opportunity set for the firm. However, as will be clear shortly, endogenously determined prices in our model move in discrete jumps, and hence modeling the feedback effect of prices in a straightforward way is not possible. Nevertheless, the conclusions from this model with this simple set-up are robust to alternative ways of modeling this resource allocation role of prices.

12This is similar to the approach in Khanna (1998). We later identify conditions that support this restriction on choice of candidate. It turns out that only the conditions on the third informed trader are binding.

13In Table 1, \( p_{ij} \) represents a price in round \( i \) of trading, following history \( j \) in previous rounds. \( i \) can take values 1, 2 or 3, while \( j \) can take -2, 0 or +2 (aggregate demand level) for each round of trading. For example, \( p_{0,-2}^2 \) means that this is a price established in the second round of trading following an aggregate demand of 0 in the first round of trading, and -2 in the second round of trading. Detailed derivations of the prices in Table 1 can be found in Appendix A.

14Most models of price manipulation or price bubbles assume no inventories. With-
out a feedback effect, inventory value is independent of the manipulation strategy and thus does not play a role.

\[ p_{ij} \] represents a price in round i of trading, following history j in previous rounds. i can take values 1, 2 or 3, while j can take -2, 0, +2 (aggregate demand level) for each round of trading. The n in the superscript refers to the fact that these prices are derived under the belief system of no-herding. Detailed derivations of these expressions can be found in Appendix B.