Rationale

Across the United States, many students enter their first year of college unprepared to take college level mathematics: at public 4-year institutions, 16% of entering freshmen took developmental (remedial) mathematics in 2000 (U.S. DOE, 2003, p. 18). These students are less likely to enter or persist in STEM majors, and even less likely to graduate from college than students who are prepared for, and succeed in, college mathematics in their first year (Adelman, 2006). According to the recent AAU (2012) draft discussion document announcing their STEM initiative, about 25% of freshmen across the country intend to go into a STEM field, but only 15 -17% of graduates complete a STEM major (AAU, pp. 2-3). Most students who drop STEM majors do so during the first two years of college, often because of trouble in their first year mathematics courses (AAU, 2012, p. 4).

These issues are at the top of the national agenda as evidenced by recent reports from several important organizations:

• The PCAST report (2012) sets a goal of adding one million STEM graduates over the next decade. Among the recommendations of the report are to:
  o Catalyze widespread adoption of empirically validated teaching practices [in undergraduate STEM classes];
  o Launch a national experiment in postsecondary mathematics education to address the mathematics preparation gap. (p. ii-iii)

• Recent updates of “The Gathering Storm” report from the National Academy of Sciences (NAS, 2007, 2010) call for improving K-12 education with 10,000 new, well-prepared mathematics and science teachers. A second recommendation includes increasing the number of undergraduate STEM majors by improving retention in the majors.

• The AAU initiative (2011) aims to change undergraduate STEM education by improving curricula and pedagogy in STEM courses, especially during the first two years.

At least two issues are apparent here: First is the “preparation gap”, the inadequate mathematical preparation of incoming college students. Institutions of higher education commonly offer developmental mathematics courses to address this problem, placing students in pre-college level algebra in an effort to get them ready to succeed in higher-level mathematics and science courses. The root cause of this problem, though, lies not in the postsecondary institution, but in the preparation of students before they reach college, in their elementary, middle and high schools. Many point to the inadequate preparation of mathematics teachers as an important factor, and call for better preparation of these teachers (Conference Board of the Mathematical Sciences, 2012; Schmidt et al., 2011). Although post-secondary institutions cannot directly influence what happens in high schools and earlier, they can work to improve what new K-12 teachers bring to K-12 mathematics classrooms.

The second problem is the retention of students in STEM majors once they enter college. The two are closely related, for mathematics serves as a gateway into advanced courses and majors (Adelman, 2006). Researchers and policy makers agree that one of the major problems in initial mathematics courses is inadequacy of pedagogy (e.g., AAU, 2011; Fairweather, 2008). Even though disciplinary-based researchers who study mathematics and science education have extensive evidence about how to teach introductory mathematics and science college courses in ways that “work”, large scale adoption of effective teaching methods has been elusive. We know very little about how to scale such practices. For example, research across disciplines has repeatedly shown that students need to be engaged with the content, not just listen to explanations (Fairweather, 2008). Faculty have been reluctant to change their teaching practices even in the face of strong empirical evidence about what works because the rewards for doing so are limited and it is hard work that requires substantial investment of time and effort (AAU, 2011).

These are nationwide problems are reflected in the enrollment patterns and STEM retention at Michigan State. Approximately 13% of freshman (about 1000 students each year) at MSU are placed in a
developmental mathematics course (MTH1825, Intermediate Algebra) that does not count toward graduation. These students must successfully complete MTH1825 before they can move on to college algebra and then calculus. Since most STEM majors require some amount of calculus, developmental mathematics students are delayed by at least a year in proceeding with their intended major. Additionally, about 30% of those who enroll in MTH1825 either drop before they receive a grade or complete the course with a grade below C and must repeat the course. That 30% of students – approximately 300 freshmen – are at least a year behind if they intend to enter a STEM major.

In addition, students enrolled in MTH 1825, and the subset enrolled in enrichment sections that will be sites for this research, are disproportionately African-American and Hispanic, as compared to the rest of the University population. In 2011-12, the University undergraduate population was 6.5% African-American and 3.3% Hispanic. In Fall 2011, in MTH1825, 29% of students identified as African-American and 7.6% as Hispanic. In the enrichment sections, 56% identified as African-American, 12% as Hispanic. This population is typically underrepresented in STEM majors, an outcome that is of great concern across the U.S.

Addressing these problems at MSU and nationwide requires a combination of programs, pedagogical models, and materials, along with evidence that will enable and encourage faculty to adopt new approaches given the rewards and incentives that discourage them from doing so. As the AAU report makes clear, the reward structure (tenure, promotion, and salary) is based primarily on research production with much less attention to teaching (AAU, 2011).

Framework for the research

Research on effective mathematics teaching and learning shows that, although students learn in different ways, the goal of mathematical proficiency is best reached through methods that engage students in doing, talking, and thinking about mathematics. The National Research Council’s report Adding it Up, authored by Kilpatrick and colleagues (2001), identified and defined five strands to explain what it means to be proficient in mathematics. The strands are defined as follows:

- **Conceptual understanding** is the “comprehension of mathematical concepts, operations, and relations”
- **Procedural fluency** is “knowledge of procedures, knowledge of when and how to use them appropriately, and skill in performing them flexibly, accurately, and efficiently.”
- **Strategic competence** is the “ability to formulate, represent, and solve mathematical problems”
- **Adaptive reasoning** is the “capacity for logical thought, reflection, explanation, and justification”
- **Productive disposition** is the “habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy.” (Kilpatrick, et al., p.116, 121)

To promote mathematical proficiency, a focus on all five strands needs to occur during instruction, as the strands depend on each other and need to be interwoven. In the report, Kilpatrick and colleagues make clear through examples from research that it is not adequate to focus solely on knowledge of procedures – being able to do symbolic problems and know and use algorithms – or solely on conceptual knowledge – understanding the concepts and ideas. As part of mathematical proficiency, procedural fluency goes beyond knowledge of procedures and includes knowing when and how to use them and more, as indicated above. Effective mathematics teaching attends to all five strands, aiming to help students learn to solve problems, apply mathematics in new contexts, and see mathematics as a coherent and powerful discipline.

Research on post-secondary education makes clear, however, that current teaching of mathematics is most often focused on algorithms and knowledge of procedures delivered in lecture mode, an approach that is unsuccessful for many students, even those who go on in STEM majors (e.g., Thompson, 2007). Successful teaching of post-secondary mathematics has been described and documented (e.g., Kalchman & Bransford, 1999; National Academy of Engineering, 2010; Treisman,
1992), but evidence of success has not resulted in widespread change in how mathematics is taught by most faculty at most institutions.

An effective method of teaching for mathematical proficiency entails engaging students in problem solving and mathematical discourse, working in small groups and whole class discussions to explain mathematical ideas and justify mathematical solutions (Fuson, et al., 1999; Kilpatrick et al., 2001). This kind of teaching can be seen in “flipped” classrooms (Bergman & Sams, 2012), reformed calculus classrooms (Treisman, 1992) and other innovations that have proven successful (e.g., Moses, 1995). Research on classroom discourse in mathematics and on teaching through problem solving and discourse provides guidance about how to plan for and manage classes that maintain a rigorous mathematical focus while encouraging students to participate (Chapin et al., 2003; Stein et al., 2008).

This kind of teaching, however, has been difficult to bring to scale because it is hard, time consuming and takes several iterations of enactment and reflection to develop proficiency. In K-12 education, there is evidence that doing such teaching even for segments of a class – a mathematical unit of days or weeks, for example – can have long-term effects for students (Burns, 1995; Cohen & Hill, 2000). Thus, a possible strategy for postsecondary institutions is to development small units that support faculty in changing their teaching for specific topics or ideas – “replacement units” like those created for K-12 mathematics classrooms.

What will we do?

This study puts these elements together through an intervention in a developmental mathematics course that uses prospective secondary mathematics teachers (PSTs) to implement research based teaching methods using innovative materials. The research addresses two distinct aspects of the problems faced by universities as outlined: (1) students arrive at the university unprepared; and (2) their first mathematics course is often unsuccessful. The first problem – unprepared students – is not something a university can intervene in directly. The method proposed here is to improve the preparation of future teachers by using the developmental mathematics course as a site for their teaching. The outcome will be better secondary teachers in the future and thus better prepared incoming students. The second problem – unsuccessful developmental mathematics courses – is addressed directly in this proposal through an intervention with expected improvements as soon as the first semester of adoption. The intervention includes use of new materials organized into units that could be adopted by others; research-based teaching methods planned and implemented by PSTs under the supervision of faculty from mathematics and mathematics education; and a model for improving both teacher preparation and developmental mathematics outcomes by having PSTs teach in the developmental course.

Research Questions

The primary research questions for this project, expanded in the designated sections below with sub questions and measures, are:

1. (Developmental Mathematics Intervention): In a developmental mathematics course, what is the impact on students’ mathematical proficiency of an intervention using teaching methods and materials designed to help them develop mathematical proficiency? The research will test whether developmental math students show substantive gains in mathematical proficiency as a result of this intervention.

2. (Teacher Preparation Intervention): To what extent, and in what ways, is a developmental mathematics course an appropriate site for supervised teaching experiences for prospective secondary mathematics teachers? The research will evaluate whether PSTs learn appropriate methods and strategies for teaching a diverse population of students, and whether the developmental math course is an effective and appropriate site for their learning.

Research Design

In the two sections below, the two interwoven interventions – in the developmental mathematics course and in the teacher preparation program – are described with rationale, data collection, and data analysis specified for each.
Developmental Mathematics Intervention

One major aim of this project is to provide students in developmental mathematics an opportunity to become mathematically proficient through face-to-face classes involving materials and teaching methods that engage students with mathematics through problem solving and discussion. The overall structure of the research design, including the intervention, the data that will be collected and the outcomes, is shown in Figure 4 on page 8.

Research Focus 1

The research questions are:

(Developmental Mathematics Intervention): In a developmental mathematics course, what is the impact on students’ mathematical proficiency of an intervention using teaching methods and materials designed to help them develop mathematical proficiency?

a) What do intervention students learn and how does their learning compare to other 1825 students (as measured by standard course assessments, focusing primarily on knowledge of procedures)?

b) What changes are observed in students’ mathematical proficiency, including conceptual understanding, procedural fluency, strategic competence, adaptive reasoning and productive dispositions toward mathematics?

Our hypotheses are:

Hypothesis 1. Students in the intervention sections will perform better on course assessments of knowledge of procedures than students in the non-intervention sections.

Hypothesis 2. Students in the intervention sections will perform better on assessments of strategic competence, conceptual understanding, and adaptive reasoning than students in the non-intervention sections.

Hypothesis 3. Students in the intervention sections will increase their productive dispositions toward mathematics.

The Context

MTH 1825 is a non-credit developmental mathematics course that students must pass before progressing to mathematics courses that count toward fulfilling undergraduate degree requirements. Since Fall 2010, the MTH 1825 course has been offered as a fully online course, supplemented by a textbook, Miller et al. (2010). For regular sections of MTH1825, there is no face-to-face instruction.

Students (n ≈ 1000) are placed in MTH 1825 based primarily on an MSU placement test. The Office of Student Support at Michigan State University identifies some enrollees in MTH 1825 as needing extra support to complete successfully their general education requirements. These students (n ≈ 200) are required to enroll in MTH 100E, a mathematics enrichment one-credit course, graded pass/fail, a face-to-face component that students must attend twice a week for approximately two hours at each session. These students are enrolled in MTH 1825 online and must complete all requirements in addition to attending the enrichment section. The intervention will be implemented in two of the approximately 10 sections of MTH 100E (see Figure 1). Staffed by an undergraduate teaching assistant (TA), MTH 100E is intended to provide extra support for students to complete MTH 1825 successfully. The curriculum for MTH 100E consists of notes and worksheets provided to the TA by the faculty coordinator. The TA is expected to answer students’ questions, go over problems on the worksheets provided, use the notes to write clarifying content on the board, and generally provide help as needed. The delivery of the course does not reflect what is known about successful mathematics teaching and learning, but instead relies on the effect of a smaller class with access to a knowledgeable tutor.

All students who enroll in MTH1825 are required to purchase a license for the ALEKS software program that serves as the curriculum for the course. ALEKS (Assessment and Learning in Knowledge Spaces) “is an artificial intelligence-based system for individualized assessment and learning in
mathematics and statistics courses” (from the ALEKS web site, http://www.aleks.com/). Students use ALEKS to get an overview of procedures, practice using those procedures to solve problems, get feedback about whether their answer is correct, and either use online help to learn the mathematics they need, try new problems similar to ones they missed, or move on to a new topic. The premise of the system is that students are given new topics and problems only when they have mastered prior topics needed to advance, as evidenced by accurate performance on practice problems. It uses artificial intelligence techniques to assess student progress and determine readiness to go on to the next topic or level. Instructors of the ALEKS sections (approximately 200 students in each of 5 sections) get online reports about each student showing their progress across topics, and are responsible for grading exams and answering student questions via email, but there is no face-to-face interaction between students and the instructors. Research about persistence in STEM courses and majors suggests that personal involvement with faculty is an important factor in students’ decisions to stay in a STEM field (Seymour & Hewitt, 1996).

Students complete modules at their own pace and take four common exams. Problems in the online ALEKS course are not multiple choice; they require students to enter mathematical responses and answers using tools provided in the system. The tools allow students to write algebraic equations and create number lines and graphs. Although some of the problems are contextualized, the system does not evaluate how students arrived at a solution, but rather whether the solution is right or wrong, and whether the form of their answer is acceptable. The system demands precise responses, and students have an initial learning curve to be able to use the system. The course syllabus indicates milestones that must be met in ALEKS and aligns them with the textbook.

Although grades in 1825 suggest that outcomes for students have improved with ALEKS compared to previous years, grades reveal very little about what students learn, and there are still about 30% of students who drop or receive a grade lower than C. The ALEKS online curriculum has been shown to develop students’ knowledge of procedures adequately (e.g., Hagerty, 2005), but for mathematical proficiency and later success in STEM fields, students need other kinds of knowledge, including conceptual understanding, strategic competence, adaptive reasoning and productive disposition (Kilpatrick et al., 2001).

The Intervention in Developmental Mathematics Sections

This project entails an intervention in two sections of the enrichment course, MTH100E, that accompanies the developmental mathematics course, by using materials designed to engage students in developing conceptual understanding, adaptive reasoning, and strategic competence, taught by prospective secondary mathematics teachers (PSTs) who are mathematics majors. Overall, the project includes two innovative features, designed to benefit both students in developmental mathematics and PSTs.

1. New materials for 100E consisting of activities that entail mathematical problem solving, reasoning, explanation, and justification will be developed by PI Sikorskii and co-PIs Bieda and McCrory. These materials will focus on developing strategic competence and adaptive reasoning as well as conceptual understanding.

2. Innovative, research-based teaching methods will be incorporated into the intervention sections with instruction provided by PSTs from the teacher preparation program under the
supervision of their teacher education instructor. The details of their preparation to teach MTH1825 and how the project has designed an intervention in their methods course are explained later in the proposal.

1. **Materials.** The materials developed for 100E will involve tasks designed to help develop students’ conceptual understanding, strategic competence, and adaptive reasoning based upon topics identified as challenging for MTH1825 students. The focal topics will be rational numbers and proportional reasoning, as well as linear and quadratic functions. The materials will consist of tasks designed to be taught in a single two-hour session that includes small group and whole class discussion. Developed based on current research about learning mathematics, each task will allow multiple entry points so that students at different levels of mathematical proficiency can participate in mathematical work on the problem. The tasks will addresses important mathematics, provide multiple solution paths, use mathematically authentic contexts, and require more than application of an algorithm, aiming for cognitive complexity. Figure 2 is an example of a task appropriate for MTH100E. It involves a real-life context, important mathematics, (two-dimensional measurement, proportional reasoning, and creating a function to represent a linear relationship), and is well suited to group work. Students of all ability levels can start by analyzing the representation of the nested carts and determining a pattern for decomposing the length into segments of each cart.

2. **Research based teaching methods.** Instructors for the intervention sections will use methods known to be successful for engaging students with mathematics in ways that lead to increased mathematical proficiency (Fuson et al., 1999; Hiebert & Grouws, 2007; National Mathematics Advisory Panel, 2008). These teaching methods will be described in the instructor’s materials and will help to build students’ productive disposition.

Research-based teaching strategies include recognizing and building on students’ prior knowledge, anticipating and responding to student thinking, selecting and sequencing students responses to achieve specific mathematical goals, pressing for justification and explanation, and maintaining a high level of cognitive demand (Kilpatrick, et al., 2001). Such teaching includes opportunities for students to solve problems in small groups, to participate in whole class discussions of mathematical ideas and problems, and to explain their thinking in such discussions. These methods support students’ development of mathematical proficiency, including all five strands.

**Data Collection for RQ1**

1. **ALEKS and 1825 course data.** Data from the ALEKS online system will be collected to gauge students’ development of *procedural fluency*. These data include a pre-assessment, four quizzes, a final assessment, and time spent on the system. The pre-assessment consists of 25-30 questions, uniquely tailored to an individual student, to determine a student’s ability to answer problems involving topics of intermediate algebra. From their responses, a “knowledge pie” (Figure 3) is constructed to represent students’ mastery of intermediate algebra procedures. This pre-assessment serves as a baseline measure of students’ procedural fluency, and their “knowledge pie” is updated each time they master a topic. Measures of their knowledge throughout the course will be obtained from responses to four quizzes and a final assessment, all of which have been developed by MSU mathematics instructors specifically for the MTH 1825 course. All students in MTH 1825 take the same quizzes and final assessment, providing a comparison across all MTH 1825 students. In addition, the placement pretest, developed by MSU mathematics faculty and used as the basis for placing students in 1825, will provide a second baseline measure for comparisons across the 1825 population.

2. **Project pre/post test.** To assess *conceptual understanding*, *strategic competence*, and *adaptive reasoning*, project personnel will design and administer a pre/post-assessment to all MTH 100E students.
This assessment will include questions that require students to reason from problem situations, generate mathematical generalizations, and justify their mathematical thinking. These assessments will be developed collaboratively between MSU mathematics and mathematics education faculty during Spring 2013, and will be piloted and revised during Spring 2013 prior to data collection in Fall 2013. Tasks on the assessments will be similar to those that form the basis for the new 100E materials, using tasks such as the one shown in Figure 2 from the Balanced Assessment Project (2000) whenever appropriate. Scoring will be done via problem specific rubrics developed along with the problems.

3. **Exit slips.** 100E students will complete exit slips after each class session. They consist of question(s) posed by their instructor (a PST) about the days’ content and the student’s engagement that students respond to in the last 5 minutes of class. This information will be used not only to investigate students’ experiences but also for formative assessment of MTH100E students’ **conceptual understanding** and **adaptive reasoning** in the preparation of lesson plans. Exit slips will be prepared by the PSTs as part of their lesson plans.

4. **Video & observations.** To understand students’ participation and engagement with mathematical ideas during the lessons, video will be collected in each 100E class session taught by the PSTs. The video will capture the PST’s instructional moves during the lessons in 100E, as well as interactions with students during whole class discussions and small group work. The instructor of TE407, PSTs in TE407, and project personnel will observe each session during which PSTs are teaching. Mathematics faculty will also observe some sessions. These videos will help us capture students’ engagement, providing a measure of MTH 100E students’ **productive disposition** along with evidence about **adaptive reasoning**.

5. **Interviews.** Interviews will be conducted with 100E students to assess their experiences in the course, the development of their productive dispositions in mathematics, and to investigate what aspects of the instruction they thought worked for them as learners and why. In a recent dissertation study that examined students in MTH 100E at Michigan State, Larnell (2010) developed and used a questionnaire that will form the basis for our instrument. Other researchers have studied African-American students mathematical experiences (Martin, 2000); and mathematical identity more generally (Boaler & Greeno, 2000). Calling on the expertise of Dr. Herbel-Eisenmann (senior project personnel, c.f., Herbel-Eisenmann & Otten, 2011), the project will develop an instrument based on these resources to ascertain how important students think mathematics is, how they view themselves with respect to mathematics, what they see as constraints and opportunities in their classroom experiences, and the resulting motivations and strategies students develop to succeed in the course. The interviews will be focused primarily on students’ **productive disposition** toward mathematics.

6. **Survey.** A survey will be given at the end of the course to appraise students’ conceptions (including attitudes and beliefs) about mathematics, as another measure of their **productive dispositions** toward learning mathematics. This survey will also be administered to the comparison classes. The instrument will be the **Conceptions of Mathematics Inventory** (Crawford, et al., 1994, 1998), an instrument that has been validated, widely used, and shown to correlate with student achievement.

**Data analysis for RQ1**

Data from ALEKS, the initial placement exam, and the MTH1825 course graded assignments will be compared to data from the larger populations in MTH100E and MTH1825 to determine whether and how the intervention students differ from other 1825 and 100E students at the start of the semester and at the end in their **procedural fluency**. The use of multiple data sources should allow us to make reasonable comparisons between the intervention sample (n=50) and the comparison groups in MTH100E (n=150) and the general pool (n=800). Available baseline data (age, race, high school GPA, ACT score, placement test score) will be used to identify and control for potential selection biases numerically or through identification of matched comparison groups.

The project assessment will be developed using assessments similar to those in the Balanced Assessment Project. It will be used to determine intervention students’ **mathematical proficiency** at the start and end of the semester. The assessment will focus on **conceptual understanding**, **strategic competence**, and **adaptive reasoning** and will be administered to all enrichment students, including those...
not in the intervention sections. Scoring rubrics will indicate levels of accomplishment for each problem. For example, a problem aimed at assessing adaptive reasoning will specify levels such as “no evidence of adaptive reasoning” to “high level of adaptive reasoning” with specific examples of indicators for each level. The rubrics will allow for consistent grading across all students enabling a quantitative and qualitative analysis of differences between the intervention and non-intervention students.

To further articulate aspects of students’ mathematical dispositions, the Conceptions of Mathematics Survey (Crawford et al., 1994) data will be analyzed using Crawford et al.’s constructs that categorize conceptions of mathematics in five categories (e.g., “3. Math is a complex logical system; a way of thinking”, p. 335) and preferred ways of learning mathematics in five categories (e.g., “2. Learning by doing lots of examples with an intention of gaining a relational understanding of the theory and concepts” p. 337). This will provide detailed insights into students’ productive disposition and will make it possible to look across students for trends within the intervention sections.

Qualitative data from student interviews and video will be used to develop an understanding of other impacts of the intervention, particularly productive dispositions toward mathematics. We will code students’ responses to analyze how students see themselves in relationship to mathematics, their reaction to particular problem solving experiences and their use of mathematical discourse practices developed in the courses. In order to increase the trustworthiness of the categories and interpretations, interviews will be triangulated with observations and video of class periods.

**Figure 4: Research Design**

<table>
<thead>
<tr>
<th>INTERVENTION</th>
<th>ALEKS System</th>
<th>MEASURES</th>
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<td>Placement test</td>
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<td><em>Important mathematics</em></td>
<td>High level mathematical discourse in small groups and whole class</td>
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<td><em>High level tasks</em></td>
<td>High level mathematical discourse in small groups and whole class</td>
<td>1825 quizzes and exams</td>
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<tr>
<td><em>Multiple entry levels</em></td>
<td>High level mathematical discourse in small groups and whole class</td>
<td></td>
</tr>
<tr>
<td><em>Multiple solution paths</em></td>
<td>High level mathematical discourse in small groups and whole class</td>
<td></td>
</tr>
</tbody>
</table>

| Features of teaching | Teaching by PSTs prepared for teaching this population | Project designed pre/post test |
| *Anticipate student thinking* | Using mathematical language | Video & live classroom observations using RTOP |
| *Select and sequence student responses to achieve mathematical goals* | *Talking and writing in mathematically precise ways* | Interviews |
| *Press for justification and explanation* | *Supporting solutions with explanations and justifications* | |
| *Maintain high level of cognitive demand* | |

| Features of discourse | Teaching by PSTs prepared for teaching this population | CMI: Conceptions of Mathematics Inventory |
| *Using mathematical language* | High level mathematical discourse in small groups and whole class | |
| *Talking and writing in mathematically precise ways* | High level mathematical discourse in small groups and whole class | |
| *Supporting solutions with explanations and justifications* | High level mathematical discourse in small groups and whole class | |

**Hypotheses:**
1. Students in the intervention sections will perform better on course assessments than other students in enrichment sections
2. Students in the intervention sections will perform better on assessments of strategic competency, conceptual understanding, and adaptive reasoning than students in the non-intervention sections.
3. Students in the intervention sections will increase their productive dispositions toward mathematics
Drawing on systemic functional linguistics, the videotaped observations will be analyzed for the developing themes and coherence (Herbel-Eisenmann & Otten, 2011). In order to do this analysis, mappings are created to examine the ways in which mathematical terms are put in relationship to one another and lexical chain tables are created to look at the coherence of ideas over a period of time. For example, the ways in which the words “area” and “triangle” are used in relation to each other are mathematically constrained – it makes no sense to say “the triangle of an area”, for example. An analysis of classroom discourse can reveal whether students are using terms correctly and how use changes over time. Doing such an analysis provides insights into students’ conceptual understanding and strategic competence, and how the discourse contributes to development over time.

Teacher Preparation Intervention

The second part of this project, on which success of the first part is dependent, is an intervention in the teacher preparation program. This intervention addresses a problem in teacher preparation – finding appropriate, high quality sites for “practice” teaching. Taken together, the two interventions provide a benefit to both prospective secondary mathematics teachers and developmental mathematics students.

An issue in the preparation of teachers, even more problematic in recent years because of pressure on teachers due to high stakes testing, is finding appropriate and educative sites for PSTs to observe teaching and gain experience by teaching on their own. The need for authentic sites for PSTs to practice teaching is necessary to develop what Shulman (1986) terms strategic knowledge. Grossman and colleagues (2009) name three important kinds of experiences in teacher preparation that can support PSTs in developing strategic knowledge, namely representations, decompositions, and approximations of practice. This intervention focuses on the third: approximations of practice “are more or less proximal to the practices of a profession” (p. 2055), such as teaching a short lesson to peers or co-teaching a lesson with a mentor teacher.

To address this problem, the intervention will use developmental mathematics as a site for PSTs to engage in authentic, supervised teaching experiences, decreasing the need to find placements in secondary schools and increasing the likelihood that their experience will be uniformly high quality and educative.

Research Focus 2

To what extent, and in what ways, is a developmental mathematics course an appropriate site for supervised teaching experiences for prospective secondary mathematics teachers?

a) To what extent do the PSTs develop lesson plans appropriate for developmental mathematics students to help foster mathematical proficiency (i.e., ability to formulate, represent, and solve mathematical problems and build capacity for logical thought, reflection, explanation, and justification)?

b) To what extent do PSTs teach in the intervention sections in ways consistent with the methods and strategies to promote mathematical proficiency, (i.e., recognizing and building on students’ prior knowledge, anticipating and responding to student thinking, selecting and sequencing students’ responses to achieve specific mathematical goals, pressing for justification and explanation, and maintaining a high level of cognitive demand during task enactment)?

c) In what ways do the PSTs improve the ability to observe and reflect on teaching to promote mathematical proficiency?

d) How do PSTs evaluate their experiences teaching in MTH 1825 and how do they compare those experiences to their work in a secondary classroom during the second semester?

Hypothesis

1. PSTs will exhibit satisfactory learning from their experiences teaching and observing in developmental mathematics as evidenced by the course assessments.

2. PSTs will appraise the MTH 1825 experience as useful and important for their learning to teach mathematics.
There is no comparison group for this intervention because all of the senior PSTs are enrolled in the methods course. The research questions address whether MTH100E functions effectively as an “approximation of practice” and is an appropriate site for supervised teaching experiences to prepare PSTs for teaching in actual secondary mathematics classrooms. This entails assessing PST learning of teaching methods and strategies, with special attention to whether and how the developmental math setting contributes to or distracts from their ability to engage in educative teaching experiences.

The Context

At MSU, prospective secondary mathematics teachers complete a five-year program. The 5th year is an internship year during which PSTs are placed in a single classroom for the entire year, working with a mentor teacher to gain increasing responsibility for teaching. The 4th (senior) year includes two mathematics methods (pedagogy) courses with supervised experiences teaching to their peers (microteaching lab) in the fall semester and beginning in late fall and into the spring, observation and teaching for 4 hours per week with a mentor teacher in a secondary classroom. The microteaching lab and field experience both function as approximations of practice; however they share complimentary weaknesses. Although the microteaching lab occurs in a highly controlled setting supervised by mathematics education faculty, PSTs teach to their peers: mathematics majors who already know the mathematics being taught. On the other hand, working with a mentor teacher in a secondary classroom involves teaching to authentic learners; however, the quality of PSTs experience depends upon the expertise and practices of the mentor teacher, and it is not always possible to place PSTs in classrooms with high quality teaching. This research uses MTH 100E as a site for PSTs to engage in observation and teaching in lieu of the microteaching lab. It provides PSTs with an authentic experience teaching a diverse population of students who continue to struggle with secondary level mathematics, a closer approximation of practice than the microteaching lab. Under the supervision of experienced faculty members, PSTs will plan and teach lessons, and observe both the faculty and their peers teaching in MTH 100E sections.

At all levels of mathematical learning, engaging with mathematics that is of high cognitive demand, using problems that require both procedural and conceptual understanding and that connect to multiple mathematical ideas, is key to developing mathematical proficiency (Bransford, et al., 1999; Henningsen & Stein, 1997). For PSTs, methods and strategies for planning and enacting high cognitive demand tasks, including techniques for orchestrating productive mathematical discussions (Stein, Engle, Smith & Hughes, 2008) are a focus of methods courses. There is growing evidence that teachers who can engage students in discourse that reveals students’ mathematical understandings and presses students to explain their reasoning are better positioned to build students’ conceptual understanding than teachers who rely primarily on a transmission mode of communication (e.g., Cobb, Boufi, McClain, & Whitenack, 1997; Cobb, Wood, Yackel, & Perlwitz, 1992; Hufferd-Ackles, Fuson, & Sherin, 2004; Kazemi & Stipek, 2001; Sfard, 2001; Silver, 1990; Staples, 2007; Stein & Lane, 1996). Furthermore, there is evidence that when teachers explicitly help students engage in mathematical argumentation, justification, and conceptual explanations in increasingly complex ways, they can help students improve their adaptive reasoning and strategic competence (Chapin & O’Connor, 2004; Chapin, O’Connor, & Anderson, 2003).

One goal of this project is to assess whether and how PSTs successfully use 100E as a site for building these skills with a population that is struggling with mathematics.

As a result of this project, PSTs will be better prepared to enact these practices in secondary schools that align with all 5 strands of mathematical proficiency, resulting in more students coming to the university prepared in mathematics. Adelman (2006) reports that the best
predictor of undergraduate student success and persistence in STEM majors is the “academic content and performance the student brings forward from secondary school” which he measures as a composite of the intensity and quality of the secondary school curriculum (p. 5). Although a post-secondary institution cannot easily intervene in what students bring from high school, they can work to prepare mathematics teachers more effectively to improve outcomes for high school students in the future.

**The Intervention in Teacher Preparation**

PSTs in the TE407 methods class will be assigned to one of two sections of MTH 100E for one day per week (Figure 5). An instructor will be assigned to each group, one the regular instructor for the TE407 seminar and the other a teaching and research assistant on the project. For the first three weeks of class, PSTs will observe in their assigned section of MTH100E while their instructor teaches. During these three weeks, the TE methods class, which meets for an addition 3 hours per week, will help PSTs prepare for teaching in 100E. Beginning in week 4, PSTs will teach the 100E sections in pairs, with one PST taking the lead. Other PSTs in the 100E section will observe their colleagues. Each PST will have one 100E class session as lead teacher, one as co-teacher, and approximately 8 sessions as observer.

**Data Collection for RQ2**

Three areas of PSTs’ ability to teach mathematics for mathematical proficiency will be assessed for this project: a) whether PSTs identify an appropriate mathematical goal that aims to develop the five strands of mathematical proficiency, anticipate student thinking, identify common misconceptions and designate specific and general questions and prompts that support student learning; b) whether the PSTs plan for and teach through mathematical discourse that supports mathematical reasoning, maintains cognitive demand of lesson tasks, and adopts research-based practices for orchestrating productive mathematical discussions (Stein, Engle, Smith & Hughes, 2008); and c) whether PSTs improve in their ability to observe and reflect on teaching.

1. Lesson plans & audio of planning sessions. Lesson plans will be collected from each PST for the lessons they teach and co-teach. The PST submits a draft lesson plan, gets feedback from the TE407 instructor, and then meets with the instructor and co-teacher to finalize the plan. The final planning meetings will be audiotaped, another data source about the PSTs planning process.

2. Video & observations. For evidence about the quality and characteristics of instruction provided by the PSTs during the course, observers from the project will attend each class and will use the **Reform Teaching Observation Protocol (RTOP)** from Arizona State University (Sawada et al., 2000a; Thompson et al., 2007) to record their observations. The measure has high reliability, and scores on this measure correlated strongly with student outcomes in mathematics and science classrooms (Henry et al., 2007). The protocol measures the extent to which the teacher uses discourse to develop mathematical community, engage all learners, connect mathematical ideas within the lesson topic and with other mathematical topics, and maintain the intellectual rigor of the tasks. In addition, video will be collected in every session in which a PST is the teacher.

3. Observation journals & reflection. Fellow PSTs will be observing their peers teach in 100E. Their observations will be used to assess the development of observers’ skills in noticing and understanding teaching. The PSTs will use the **Reform Teaching Observation Protocol (RTOP)** (Sawada et al., 2000a; Thompson et al., 2007) to record their observations in sessions of 100E in which they are not the co-instructor. A training guide for the instrument is available (Sawada et al., 2000b) and will be used at the start of the semester in TE407 to prepare PSTs for observing. By the end of the semester, each PST will have a set of observations of their fellow classmates, referred to here as their observation journal. As part of the course assignments for TE407, each PST will also write a reflection about their own teaching that will be used as a data source.

4. Exit interview. PSTs will be interviewed at the end of the semester and again at the end of the course to appraise their view of the effectiveness of using MTH 1825 as a site for their teaching. In the second semester, they teach and observe in a secondary classroom and we will ask about their learning in both settings.
Data analysis for RQ2

For TE407 PSTs, project personnel will evaluate lesson plans and videotaped lessons using a rubric developed from categories based on the Thinking Through a Lesson protocol (Smith et al., 2008) and further refined using the Project 2061 rubric (AAAS, 2002). Based on the Thinking Through a Lesson protocol, the lesson plans and enacted lessons will be evaluated for clear contextualization and purpose; utilization of high cognitive demand tasks designed to develop procedural fluency, conceptual understanding, strategic competence, and adaptive reasoning; accurate and appropriate representation of mathematics; consideration of students’ prior knowledge and misconceptions; planning of discussions to focus on content and engage students in ways that improve their productive disposition; coherence of the lesson and connections across mathematical ideas; formative assessments; and plans for differentiation. These are important features for facilitating mathematical proficiency in students. In addition to these categories, the analysis will also consider the cognitive demand of the tasks (Boston & Wolf, 2006) and how it develops from the first draft of the plan, to the planning meeting, to the final plan.

Video and RTOP observation notes (Sawada, 2000a) will be analyzed to appraise PSTs success in implementing their lessons consistent with their goals and plans and using strategies that are effective in teaching for mathematical proficiency. The lesson plan protocol, Thinking Through a Lesson, provides a rubric for coding the video qualitatively, while the RTOP instrument yields a quantitative measure of adherence to teaching in ways consistent with research. Video will be used as evidence for PSTs attention to students’ productive disposition and PSTs use of strategies that encourage adaptive reasoning. These data will enable project researchers to develop individual teaching cases and to look across the methods class for themes in the enactment of lessons.

To better understand how PSTs engage students in adaptive reasoning, we will draw on the work of Forman and colleagues (1998) to assess collective argumentation during lessons in 100E by mapping out the structure of arguments in the discussions of activities. This analysis identifies, among other aspects of mathematical argumentation, the emergence of claims about mathematical ideas and deductive reasoning to support the claims, as well as who is responsible for contributing to the mathematical argument. These two analyses together will help us to understand the quality of the mathematics classroom discourse, not only the discourse moves the PSTs make to facilitate discussion or the amount of communication students contribute.

Finally, PSTs observation journals and reflections will be analyzed to provide evidence of what they are attending to and seeing, and whether and how their use of the RTOP protocol and the aspects of teaching they attend to changes over time. The project will use protocols about teacher responding developed by Sherin and colleagues (2010) and by Milewski (2012) that discriminate among different kinds of teacher moves such as revoicing, evaluating, and providing wait time, all shown to contribute to teaching that helps students develop mathematical proficiency.

Project Timeline

The project will implement a pilot in Fall 2012, supported by the Colleges of Natural Science and Education through the CREATE for STEM Institute. The pilot will include preliminary development of materials, assessment instruments, and modification of TE407 to prepare for and teach sections of MTH 100E. Assessment data will be collected in 100E and 407 as described, and analysis of the data will be used to modify the design as needed. Full implementation of the project will begin in Summer 2013. Project personnel will modify and finalize materials based on feedback from the pilot. Figure 6 provides the overview of the project timeline.

<table>
<thead>
<tr>
<th>Fall 2013</th>
<th>Spring 2014</th>
<th>Summer 2014</th>
<th>Fall 2014</th>
<th>Spring 2015</th>
<th>Summer 2015</th>
<th>Fall 2015</th>
<th>Spring 2015</th>
</tr>
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<tbody>
<tr>
<td>Pilot</td>
<td>Data Analysis</td>
<td>Modifications</td>
<td>First Cohort</td>
<td>Data Analysis</td>
<td>Modifications</td>
<td>Second Cohort</td>
<td>Final analysis and reports</td>
</tr>
</tbody>
</table>

Figure 6: Project Timeline
In fall 2013, the intervention will be fully implemented with TE407 PSTs teaching once a week in each of two sections of 100E. A TE407 instructor (a mathematics educator) will be present for all PST teaching. A complete set of data will be collected in Fall 2013 in both 100E and TE407, including assessments, video, and other data as indicated in the data section. Data analysis will be carried out in spring and summer 2014. Modifications to the model, materials, and assessments will be complete in summer 2014 and the second implementation will be carried out in Fall 2014. Full implementation is illustrated in Figure 7.

Figure 7: Interventions: Integrating TE407 PSTs as instructors of MTH100E sections

**Evaluation Plan**

The plan for evaluation is to have the CREATE for STEM Institute director, Professor Joe Krajcik, and the Mathematics Department Chair, Professor Yang Wang, complete annual reviews of the project during its two year duration, preceded by a review at the end of our pilot project in 2013. The project team will meet at least two times a year with Drs. Krajcik and Wang to review materials, outcomes, and data analysis. They will submit a written evaluation at least once a year, beginning in the pilot year, providing feedback on strengths and weaknesses, and recommendations for changes or improvements.

Both Dr. Krajcik and Dr. Wang are familiar with MTH 1825, MTH 100E and TE 407. Dr. Krajcik has extensive experience in intervention research over his many years as a researcher in science education. As director of the CREATE for STEM Institute, Krajcik is involved in a number of cross-disciplinary projects at Michigan State and is himself interested in finding models that work to improve STEM education at MSU. Dr. Wang was instrumental in moving 1825 online and is committed to learning more about how to improve outcomes for students. Both will be objective evaluators because of their independent and passionate commitments to finding things that work to improve STEM education.

**Future Directions for Research**

As a result of this project, the research can proceed in at least two directions. First, the intervention could be extended to additional enrichment sections by designing educational experiences for the TAs assigned as instructors, helping them learn to use the materials and lesson plans developed in TE407. Success in such an effort would mean that the intervention could be scaled up to a different level, not entirely dependent on PSTs and the TE407 lab. Second, because every student in 1825 is struggling with mathematics, arriving in their first year underprepared for college-level mathematics, the project could expand to the larger 1825 population, adding a face-to-face component for students not in the enrichment (100E) sections. This extension could test two hypotheses: (1) that the intervention will be effective with the larger population, increasing scores and grades and also improving their knowledge of
other strands of mathematical proficiency; (2) that students in an online class are more successful if the class includes a face-to-face component.

Another possibility is to expand this research to another institution, working with faculty to adopt the model of using PSTs as teachers in developmental mathematics courses to the benefit of both PSTs and struggling mathematics students.

Finally, a follow-up study, to track the progress of students in the two cohorts proposed here, could be implemented to understand the longer-term impact of the intervention as the students complete their university programs.

**Dissemination Plan**

The results of this research will be disseminated to a broad array of audiences. The findings will be submitted for presentation at the annual meetings of the American Educational Research Association, National Council of Teachers of Mathematics, Research on Undergraduate Mathematics Education SIG of the Mathematical Association of America (MAA), and the joint meeting of the AMA (American Mathematical Association) and MAA. Journal articles written for a variety of venues will make the results accessible to mathematics and mathematics education faculty, post-secondary administrators, and faculty and administrators in other STEM fields. Articles will be submitted to several journals with high visibility, such as the *Journal for Research in Mathematics Education*, *American Educational Research Journal*, *Educational Evaluation and Policy Analysis*, *Journal of Engineering Education*, *Notices of the American Mathematical Association*, and *Educational Leadership*. One-page briefs will summarize main results, with at least one brief focusing on the findings from RQ1, for an audience of undergraduate education administrators and mathematics department faculty interested in improving instruction in development mathematics courses. Another brief will focus on the findings from RQ2, for an audience of secondary mathematics teacher educators, and potentially another brief will share the findings with high school administrators and educators to address concerns with the mathematical preparation of students intending to obtain a 4-year college degree. All conference papers, articles in progress, and the 1-page briefs will be readily available through project website. The website will also be linked to other relevant websites (e.g., the CREATE for STEM Institute website and the Program in Mathematics Education website at MSU). The Dean’s office in the College of Education at MSU also has a communications staff that distributes information about research in the College via emails, briefs, and podcasts to alumni, other universities, and other media outlets.

**Limitations**

This intervention will be carried out in enrichment sections of the developmental math course and will include only about 50 students. This small n means that the data are unlikely to be powerful enough to draw strong causal conclusions. The research design includes qualitative data as well that will enable a stronger case for explaining outcomes.

The 100E sections are made up of students who have been identified as needing special help for success in mathematics. These sections have a larger percentage of students of color than the 1825 population and the university as a whole. This is both a feature of the research and a limitation: we will learn about whether and how this intervention works for this population, and be able to compare results to similar students not in the intervention, and to the larger population of 1825 students. The limitation, however, is that the enrichment sections encompass only about 20% of the students enrolled in MTH1825, and they are not representative of the larger population. Consequently, this research will not provide evidence about the effects of the intervention in the larger population.

**Project Personnel**

**Dr. Kristen Bieda** (Co-PI) is a faculty member in the Department of Teacher Education and the Program in Mathematics Education. She has a M.S. in Mathematics as well as secondary (grades 9-12) teacher certification from Missouri State University, and a PhD in Curriculum and Instruction from University of Wisconsin - Madison. Her research focuses on the development of students’ abilities to justify and prove in secondary mathematics classrooms, and ways to support teachers in utilizing curricular materials and productive discourse to engage students in collective argumentation. She also serves as a subject area leader for the secondary mathematics teacher preparation program at MSU.
Dr. Beth Herbel-Eisenmann (senior project personnel) is a faculty member in the Department of Teacher Education. She graduated summa cum laude with a BA in mathematics education from Mayville State University, with distinction with an MA in mathematics from Northern Arizona University, and with a PhD in Curriculum, Teaching and Policy from Michigan State University. Her funded research has focused primarily on secondary mathematics classroom discourse and on collaborating with and studying secondary mathematics teachers’ action research on mathematics classroom discourse. In 2010, she was the recipient of the Early Career Award for the Association of Mathematics Teacher Educators. She is currently PI on a large-scale study of early algebra completion in secondary schools.

Dr. Raven McCrory (Co-PI) is a faculty member in the Department of Teacher Education and the Program in Mathematics Education. She holds a BA in mathematics from Smith College, and an MA in mathematics from the University of Massachusetts at Amherst. She has taught mathematics at the elementary, secondary, and college levels, initiating the AP Calculus course in an urban high school in 1973. She holds a PhD from the University of Michigan in Educational Studies. Dr. McCrory’s research has focused on the undergraduate mathematical preparation of future elementary teachers. She has completed extensive training in statistical methods including multilevel modeling and item response theory.

Dr. Pavel Sikorskii (PI) is a faculty member in the Department of Mathematics and the Executive Associate Undergraduate Director in the Mathematics Department at MSU. He has a strong track record in undergraduate education including curriculum development and outreach. He will be responsible for the overall integrity of the program and will oversee all aspects of the project. Sikorskii will supervise and address personnel issues, will manage the overall budget, and will oversee preparation of project reports. He will participate in curriculum development, and will co-supervise all teaching and mentoring activities.

Project Investigator results from Prior NSF Funding

Pavel Sikorskii served as senior personnel on a CCLI grant "Collaborative research on improving mathematics courses for elementary teachers" (DUE 0737000, 2008-12, $98,085). The project focused on improving mathematics courses for elementary teachers and conducted jointly by Co-PIs Tom Parker at MSU and Scott Baldridge at Louisiana State University. The project created and piloted three products designed to help mathematics departments implement, create, and improve courses for elementary teachers: A manual that provides lesson plans and guidance to instructors and departments; web resources for instructors, including an expandable bank of carefully-selected quiz and exam questions; and practice exams for students. All resources are available on the web, http://iremt.math.msu.edu/.

Raven McCrory was PI on a CAREER grant (NSF# 0447611, $661,437) from 2006-2012. The project investigated mathematics courses for prospective elementary teachers. Results from the project suggest that PSTs learn a great deal of mathematics in mathematics courses required for certification (effect size of 0.7). Using hierarchical linear modeling, McCrory found predictors that explain differences in PST outcomes across instructors. Significant predictors include whether a textbook specifically written for the population of preservice teachers is used and to what extent the instructor teaches in a way that engages students directly with mathematics, rather than, for example, lecturing. Publications from the project include Park, Güçler & McCrory (in review, 2012), McCrory & Cannata (2011), Lo & McCrory (2010), McCrory, Zhang, Francis & Young (2009), Lo, McCrory & Young (2009), Güçler, Park, & McCrory (2009), Francis, Yu, Francis, & McCrory (2009), and McCrory (2006). McCrory was also co-PI on the Knowledge of Algebra for Teaching (KAT) project, (NSF# DR-0106709, $1,154,109). The KAT project developed and validated an instrument for measuring teachers’ knowledge of algebra. The instrument is widely used across the U.S. A paper describing the framework for the instrument development is in press as of May 2012 (McCrory, Floden, et al. 2012 in press). Technical reports from the project are available on the web site, and papers based on the technical reports are in preparation.
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