Making Sentences True: The Logic of Propositional Sentences

Philosophy 130
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Truth tables can be used to reveal the logical properties of sentences composed out of propositions and sentential connectives. We aren’t going to address these in class, but they are important and interesting in their own right. These concepts **satisfiability, logical truth, and tautology.** What follows is an attempt to shed some light on these concepts through the use of truth tables.

I. Consider the sentence: (a) "My cup is on the table." This is a true sentence—my coffee cup over there is on the table. However, the sentence will soon be false when I go and put the cup in the sink. This is a sentence that can be true but can also be false, depending on the way the world is. That is, the **world** makes this sentence true, if it is true.

II. Consider another sentence: (b) "Creosote stinks or creosote doesn't stink." We can represent this symbolically as

\[ (b') A \lor \neg A \]

where ‘A’ stands for "Creosote stinks" and ‘\( \lor \)’ marks the disjunction. This is also a true sentence; in fact, this sentence can't fail to be true. But note that the reason this sentence is true has nothing to do with the world: this sentence is true simply because of the structure of the sentence; that is, simply because of the connectives employed and the restrictions placed on what we can substitute in for the sentence symbols (i.e., in this case, we must replace both sentence symbols with the same sentence). The **structure of the sentence** makes this sentence true.

III. Consider a third sentence: (c) "Frank Zappa is identical with Frank Zappa, or I’m a monkey’s uncle." The symbolization of this sentence is as follows:

\[ (c') A \lor B \]

where ‘A’ stands for "Frank Zappa is identical with Frank Zappa" and ‘B’ stands for "I’m a monkey’s uncle." If we look at (c), we know that it is true and that it can't fail to be true, but this is not what we would learn if we looked at (c'). Thus, the sentence is different from (b): you know from looking at (b') that (b) must be true, but one glance
at (c') suggests that (c) could very easily be false. Therefore, (c) can't be true for the reason (b) is. Is it true for the reason (a) is? No, since the truth of (c) will be true in all possible worlds where you could evaluate it; that is, it is not the specific world in which (c) is evaluated that makes it true. Sentence (c) is true because of sentence A: A is a necessarily true sentence, and so the only circumstance where a sentence of this form could be false (namely, the circumstance represented by row 4 of the truth table for \textit{or}) is ruled out as impossible. Once again, this sentence is made true by its \textbf{structure}, but it is a different type of structure than was involved with (b). The structure in this case is the term/predicate structure at the level of the atomic sentences that constitute the complex sentence, whereas the structure involved with (b) was at the level of the connectives.

IV. Let's summarize. A sentence can be true for two general reasons: (1) the world made it true (and could have made it false), or (2) the sentence's own structure made it true (and it can't be false). Now all complex sentences have structure at two levels: (i) at the level of the atomic sentences, in terms of the predicates and terms involved, and (ii) at the level of the connectives, where we need to look at the specific connectives and the distribution of the component atomic sentences (although not their term/predicate structure). Call (i) the sentence's \textit{atomic structure} and call (ii) the sentence's \textit{complex structure}. A sentence could be made true by its atomic structure, as with (c) above, or by its complex structure, as with (b) above.

V. \textbf{THE BIG FINISH}: If a sentence is made true by the world, then it is \textit{satisfiable} but not logically true. If a sentence is made true by its atomic structure, then it is \textit{logically true} but not a tautology. If a sentence is made true by its complex structure, then it is a \textit{tautology}. These notions are important because of what they tell us about the truth conditions (or \textit{meanings}) of sentences: if a sentence is made true by the world, then its truth (or its falsity) tells us something about the world; if it is made true by its structure alone, then it tells us nothing about the world. Zilch. Zippo. Such a sentence would be of very little interest to a scientist, for example.