I. Conjunction and Negation in English and Propositional Logic

In general, conjunction is a kind of additive combination that is often represented in English by ‘and’, ‘but’, ‘moreover’, and others; negation, represented by ‘not’, ‘no’, ‘un-’, and so forth, functions to reverse or undo claims and operations. Propositional logic is interested in the impact conjunction and negation have on inferences involving complex propositions that feature conjunctions or negations. In propositional logic, conjunction (‘&’) and negation (‘~’) are propositional connectives (or propositional functions) that are used to create more complex propositions out of simpler propositions.

You can use propositional logic to model the logical character of propositions expressed by natural language sentences (e.g., English sentences), but you have to be careful. Many appearance of conjunction and negation in English correspond to & and ~ in propositional logic, but not all do. There are such things as non-propositional conjunction and negation – these are conjunctions and negations in English that cannot be represented by & or ~ in the symbolic representations of propositional logic.

II. Propositional and Non-Propositional Conjunction – Relevant to PS#3, Ch. 6.V

Examples of propositional conjunction in English (with the symbolic representations) are as follows:

1. Liela is in Detroit and Kathy is in Detroit $p \land q$
2. Liela is hungry and thirsty $p \land q$
3. Liela and Kathy are in Detroit $p \land q$

(1) is fully unpacked in English, but (2) and (3) are both compressed. You can unpack them as follows:

2’. Liela is hungry and Liela is thirsty
3’. Liela is in Detroit and Kathy is in Detroit

By unpacking them, the propositional nature of the conjunction stands out.

But not all conjunctions are propositional. Consider these examples:

4. The judge married Michael and Liela
5. Liela and Kathy tied for the lead

Both of these are compressed, and unpacking them yields:

4’. The judge married Michael and the judge married Liela.

5’. Liela tied for the lead and Kathy tied for the lead.\(^1\)

In both (4’) and (5’), there is a strong suggestion that the two conjuncts (i.e., the two propositions that are conjoined) are not related – i.e., that Michael and Liela did not marry each other in (4’) and that Liela and Kathy did not tie each other in (5’); however, this is in direct conflict with the natural readings of (4) and (5), where Michael and Liela married one another and Kathy tied Liela for the lead. There is something about married and tied in (4) and (5) that binds the two people named together into a relationship, and that relationship isn’t preserved in (4’) and (5’). Thus, representing either (4) or (5) as \(p \& q\) would be a mistake; rather, they should just be represented by \(p\).

III. Propositional and Non-Propositional Negation – Relevant to PS#3, Ch. 6.IX

Examples of propositional negation in English (with their symbolic representations) are as follows:

6. It is not the case that I am thirsty \(\sim p\)

7. I am not thirsty \(\sim p\)

8. It isn’t Saturday \(\sim p\)

In both (6) and (7), \(p\) replaces the sentence ‘I am thirsty’, whereas in (8) it replaces the sentence ‘It is Saturday’. Negation can operate at different places in a sentence, but sentences in which it functions propositionally can be unpacked to reveal its propositional character. So (7) and (8) are similar to (2) and (3) above in that they can be unpacked to demonstrate the propositional character of the negation. They would yield:

7’. It is not the case that I am thirsty

8’. It is not the case that it is Saturday

The operator, *it is not the case that*, can be seen as corresponding to the \(\sim\). Thus, if you are going to represent a negation propositionally – i.e., if you are going to represent it as \(\sim p\) – then you need to be able to pull the negation out of the sentence in which it is located and rewrite it as *it is not the case that* \(p\). If you can do this, you are working with a propositional negation.

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\(^1\) Consider if (5) had been “Liela and Kathy tied each other for the lead”. Unpacked, that would yield “Liela tied each other for the lead and Kathy tied each other for the lead”, which isn’t grammatical. There is arguably no way to unpack the *and* in this new version that preserves the meaning of the original.
However, it is not the case that you can do this with all negations. Consider these examples:

9. It might not rain tomorrow

10. I believe that it isn’t cold outside

11. It is necessary that I not forget my car keys

To see if these are propositional negation, you need to take the negations out of where they are in the sentences and then tack them on the front in the form of *it is not the case that*. In each of these examples, that would yield the following:

9’. It is not the case that it might rain tomorrow

10’. It is not the case that I believe it is cold outside

11’. It is not the case that it is necessary that I forget my car keys

If (9), (10), and (11) were propositional negation – i.e., if they could be written $\sim p$ – then the unpacking evident in (9’), (10’), and (11’) would preserve their meanings. But do they? Let’s consider them in turn.

In (9’), we are foreclosing the possibility of rain tomorrow – by saying that it is not the case that it *might* rain, you are ruling out even the chance of rain. But clearly the chance of rain is something you want to leave open with (9). Thus, (9) and (9’) do not mean the same thing, and so the *not* in (9) is non-propositional negation.

In (10’), I am denying that I believe that it is cold outside, and there are two ways to make this true. One way is that I believe that it is *not* cold outside, which is equivalent to (10), but the other way that I can make (10’) true is by having no beliefs about the temperature outside at all. By failing to have any temperature beliefs, I would make (10’) true but not (10), establishing them as different in meaning. Thus, the *n’t* in (10) is non-propositional negation.

Like (10’), there are options left open by (11’) that are not consistent with (11). Specifically, (11’) seems to leave open that it is possible that I forget my car keys, but that is precisely what is being ruled out by (11). Thus, they have different meanings and so the *not* in (11) is non-propositional.

Therefore, it would be a mistake to render (9), (10), or (11) as $\sim p$; rather, they should all be rendered as $p$. 