I. The Data

A. Here are examples of arguments that are valid:

   P1. If I am in my office, my lights are on
   P2. I am in my office
       ------------------
   C. My lights are on

   P1. He is either in class or he is at home
   P2. He is not in class
       ------------------
   C. He is at home

   P1. All of the students will understand validity
   P2. You are one of the students
       ------------------
   C. You will understand validity

B. Here are examples of arguments that are not valid:

   P1. If I am in my office, my lights are on
   P2. I am in Cancun
       ------------------
   C. My lights are off (*But wait: the janitor is in my office cleaning it right now*)

   P1. He is either in class or he is at home
   P2. He is sleeping
       ------------------
   C. He is at home (*But wait: he always sleeps in class*)

   P1. All of the students will understand validity
   P2. She is not one of the students
       ------------------
   C. She doesn’t understand validity  (*But wait: she is a logic professor!*
II. The Definition

A. Two main forms:

1. An argument A is valid if and only if whenever its premises are all true, its conclusion is also true.

2. An argument A is valid if and only if whenever its conclusion is false, at least one of its premises is false.

B. These are equivalent:

1. Case 1
   a. (1a) implies: An argument is valid if and only if it is not possible that all of the premises are true and the conclusion false.

      (1a) above implies that you will never find a situation in which a valid argument has true premises and a false conclusion, which implies that it is not possible for the premises of a valid argument to be true and the conclusion false.

   b. It is not possible that all of the premises are true and the conclusion false

      It is not possible that the conclusion is false and all of the premises true.

      Here we just reorder two conjuncts around the ‘and’, which results in an equivalent statement.

   c. An argument is valid if and only if it is not possible that the conclusion is false and all of the premises true implies: (1b)

      The first part of this implies that if the argument is valid, then if the conclusion of a valid argument is false, at least one of the premises is false as well, which is (1b).

   d. Thus, (1a) implies (1b)

2. Case 2 – this is left as an exercise. It works in precisely the same way, in reverse, establishing that (1b) implies (1a).

3. Conclusion: these two forms of the definition say the same thing.
III. Possibility vs. Actuality

A. Validity is essentially connected with the notions of possibility and necessity:

1. An argument is valid if and only if it is not possible for the conclusion to be false if all the premises are true.

2. An argument is valid if and only if it is necessary for the conclusion to be true if all the premises are true.

3. As we argued above, these are two different ways of saying the same thing.

B. Thus, in order to understand validity, you need to think in terms of what is possible and what is necessary. With respect to a particular argument, this can take the following forms:

1. Positive form (i.e., seeking validity): if I make the conclusion false, does that necessitate that at least one of the premises false? (Alternatively, if I make all the premises true, does that necessitate that the conclusion true?) Here the background assumption is one of rationality – would it be irrational for me to deny the conclusion without also denying at least one of the premises? If so, then the argument is valid.

2. Negative form (i.e., seeking invalidity): is it possible for the premises to be true and the conclusion false? That is, can I think of any situation that is possible (given the constraints of the situation) in which I can make the premises true and the conclusion false? If so, the argument is not valid.

C. In each of these cases, I am thinking about what is possible or what is necessary. I am not focused on what is actually the case with the argument, i.e., what the actual truth values of the steps in the argument are. If I were to do that, I could find myself in any one of the following situations: (convince yourself that all of the possible truth value distributions are represented here)

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1. So if I have a three step argument involving indicative sentences, it would actually be the case that I am in one of these 8 situations, i.e., either the premises are both true and the conclusion is true; or the premises are both true and the conclusion is false; or the first premise is true, the second false, and the conclusion true; or so on.
2. What we need to do is ask ourselves what we can learn from these cases, given that what is actually up with an argument we are considering is going to be represented by one of them. Let’s consider each of the cases, and ask: does the truth value assignment in this case tell us whether it is necessary that the conclusion is true if the premises are (or alternatively but really the same: is it necessary that at least one of the premises is false if the conclusion is)?

a. **Case 1**: every step is true, but this doesn’t tell us that the conclusion must be true if the premises are; it might simply be a coincidence that all of the steps in the argument are true – what is true in this one case does not give us any indication of what is true in *all* cases. (Consider: I am wearing a shirt; I am wearing pants; therefore, I am wearing shoes. This is an invalid argument, but even so, all of these may be true, as when I am in class; the truth of the conclusion, however, is not forced by the truth of the premises as can be seen when I am getting dressed in the morning and am walking around barefoot.)

b. **Case 2**: here we have the one case where the *actual* distributions of truth values tells us something about the validity of the argument! In this case, we know that the argument is invalid because it establishes that the truth of all the premises does not force the conclusion to be true – this is the counterexample! Alternatively, it reveals that we can make the conclusion false without making at least one of the premises false. So the argument is *not valid*.

c. **Case 3**: P1 is true, P2 is false, and C is true. The argument could be valid – we just don’t know from the information we’re given. The definition we have considered above doesn’t really apply here, in any of its various forms: not all of the premises are true, so we can’t read off of this whether the conclusion would have to be true in that case. And the conclusion is not false, so we can’t read off of this whether at least one of the premises would have to be false in that case. So it might be valid based on this information, or it might not be. *<shrugs>*

d. **Case 4**: P1 is true, P2 is false, and C is false. Validity requires that if the conclusion is false, at least one of the premises is also false, so given this assignment of truth values to the steps, this argument could be valid. But it could also be invalid, given that it might not be the case that whenever the premises are true, the conclusion must be true – what is true in this one case does not give us any indication of what is true in *all* cases.

e. **Case 5**: See Case 3.

f. **Case 6**: See Case 4.

g. **Case 7**: This is just like Case 3 – we can’t tell if the truth of all the premises forces the conclusion to be true, since the premises aren’t true, and we can’t tell if the falsity of the conclusion forces one of the premises to be false, since the conclusion is true. So again, we just don’t know one way or the other.

h. **Case 8**: all the steps are false. As with Case 4, validity requires that if the conclusion is false, at least one of the premises is also false, so in this case the argument could still be valid. But it might just be coincidence that they are all
false. (Consider the invalid argument example in Case 1 in a situation where none of the steps are true.)

3. So what do we now know? We can summarize it in another table, where “?” means “Not sure – might be, might not be.”

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IV. Testing for Validity by Breaking Arguments

A. If you are given an argument and asked to test for validity, there are various ways you can do this:

1. Demonstrate that whenever the premises are all true, the conclusion is true. The best way to do this involves developing a proof within some kind of proof system. (Later in the semester, we will show how to do this with Venn diagrams and with truth tables.) But you can do this for our purposes right now by arguing that there are no counterexamples available to the argument, i.e., that there is no way to make the premises all true and the conclusion false.

2. Demonstrate that there is a counterexample by specifying a case in which the premises are all true and the conclusion is false. It could be fanciful – it just must be possible within the fixed constraints of the problem.

B. If you are asked to determine whether an argument is valid and you don’t have a proof system handy, like on the worksheet, the best way to respond is by trying to break the argument.

1. That is, try to come up with a case in which the premises are all true and the conclusion is false.

2. Here you need to think about situations that are possible within the fixed constraints of the problem – sometimes it isn’t obvious what these are, but they should be recoverable from the steps of the argument you are given.

3. If you can find one of these possible cases, then you judge the argument invalid; otherwise, it is valid.
4. **Example:** “Since Jimmy Carter was president, he must have won an election” (from the worksheet)

   a. Jimmy Carter was president, and he did win an election. But that doesn’t establish that he *must* have won an election to be president. The argument is making that further claim, and that is what we need to assess here.

   b. Using the “try to break it” strategy, we need to try to find a possible case in which Jimmy Carter was president but he didn’t win an election.

   c. Consider the following: let’s say that Mo Udall had won the 1976 election, with George Wallace as his VP; then Wallace resigned in 1977, at which point Udall appointed peanut farmer Jimmy Carter (who in this scenario had never run for elected office) to the VP job (as Nixon did with Ford after Agnew resigned); then let’s say that Udall died from a tragic three-hole punch accident, at which point Carter became president.

   d. In this scenario, the *fixed* parts have to do with the US presidency and the rules for becoming a president; the *flexible* parts have to do with facts about Jimmy Carter (as well as other facts). Given these constraints, we can imagine a possible scenario in which the premise is true – Jimmy Carter was president – but the conclusion is false – he didn’t win an election.

   e. Thus, the argument is *invalid*, and since it isn’t valid, it is also *unsound*. 