This handout draws on Sider, *Logic for Philosophy*, ch. 6.

I. *Introducing Modal Propositional Logic (MPL)*

A. **What a logic is:**

1. A model of acceptable patterns of inference

2. A model of our core logical notions, viz., logical truth and logical consequence.

B. **A brief history of modal logic**

1. The focus is on the deductive behavior of necessity and possibility. (Why ‘modal’?)

2. **Historical influences:** Aristotle, Islamic philosophers (e.g., Avicenna (TML), Al-Ghazali), and Scholastics (e.g., Ockham, Duns Scotus).

3. **Contemporary contributors:** C. I. Lewis (e.g., the “strict” conditional), Saul Kripke (Kripke semantics), Ruth Barcan Marcus (axiomatic systems for quantified modal logic [QML]).

4. **Importance to philosophy**

C. **What does MPL model?**

1. **Modal notions of necessity and possibility**

   a. These are influenced by natural language terms and commonsense concepts. (What is the debt to these? Should this relationship be respected as the logic is developed? Why?)

   b. Possibility: ‘can’, ‘might’, ‘may’, ‘could be’, ‘might have been’, ‘can be’, …
c. **Necessity**: ‘must’, ‘needs to’, ‘necessarily’, ‘ought’

2. Modality comes in *strengths*
   a. **Examples**:
      - ‘**Can**’: “I can do that”, “We can succeed”, “Can we survive after the sun explodes?”
      - ‘**Possible**’: “It’s possible that we could power the world with fusion energy”, “It’s possible for something to travel faster than the speed of light”, “It is not possible to change the past”
      - ‘**Might**’: “It might be nice weather for the INPC”, “Ahhh, what might have been”
      - ‘**May**’: “You may be allowed to resubmit the PS for Chapter 4”
   b. Each of these is associated with a notion of necessity, given that *necessarily* is equivalent to *it is not possible that not*.
   c. These include physical, metaphysical, temporal, deontic, and epistemic possibility/necessity.
   d. Some grades of necessity are *alethic*, in that they imply truth. But not all, e.g., deontic necessity.

3. **Intuitive semantics**
   a. **Possible worlds**: complete and possible scenarios.
   b. Intuitively, we can cash out modal claims relative to possible worlds by evaluating claims within possible worlds.
   c. “It is possible that \( \phi \)” \( \rightarrow \) “\( \phi \) is true in at least one possible world”
   d. “It is necessary that \( \phi \)” \( \rightarrow \) “\( \phi \) is true in all possible worlds.”
e. **Question:** what are we to make of the useful notion of *possible worlds* here? Should we take this to be merely heuristic, or do we make metaphysical commitments by using it?

D. **Grammar of MPL**

1. The primitive vocabulary of MPL is identical to PL with one addition, the \( \Box \), which is introduced as a primitive, one-place propositional connective.

2. The \( \Box \) stands for *necessity*, and it forms wffs in the standard way, viz., if \( \phi \) is a wff, \( \Box \phi \) is also a wff.

3. We can use this to get two nonprimitive modal connectives:
   a. \( \Box \phi \) (“Possibly \( \phi \)” is short for \( \sim \sim \phi \)
   b. \( \phi \rightarrow \psi \) (the strict conditional) is short for \( (\phi \rightarrow \psi) \).

E. **Symbolization in MPL**

1. Consider a few examples of modal sentences, including those involving the strict conditional.

2. Consider: “If O’Rourke is a philosophy professor, then he must be a professor.”

F. **Semantics for MPL**

1. **Two Challenges:**
   a. **Philosophical:** What are the logical truths?
   b. **Technical:** Neither necessity nor possibility are truth-functional, which means that we cannot use truth tables to give their semantics. Bummer.

2. **Kripke models**
   a. We will give a *possible worlds* semantics for MPL, following Kripke.
b. **Definition of a model** (139): An MPL-model is an ordered triple, \( \langle W, R, I \rangle \), where:

- \( W \) is a non-empty set of objects – possible worlds
- \( R \) is a binary relation over \( W \) – the accessibility relation
- \( I \) is a two-place function that assigns 0 or 1 to each sentence letter, relative to each world – the interpretation function

c. **Definition of the Valuation Function** (139-140). This tells us how the interpretation function works to assign 0 and 1 to MPL-wffs. The trick here is to see these assignments as relative to possible worlds.

d. **Definition of ‘Model’ for Modal Systems** (140): An “S-model”, for any system S, is defined as an MPL-model \( \langle W, R \rangle \) whose accessibility relation \( R \) has the formal feature given for system S in this chart:

- K – no requirement
- D – serial in \( W \)
- T – reflexive in \( W \)
- B – reflexive in \( W \) and symmetric
- \( S_4 \) – reflexive in \( W \) and transitive
- \( S_5 \) – an equivalence relation in \( W \)
This gives us various grades of necessity, based on properties of the accessibility relation.

e. **Definition of Validity in an MPL-model** (141): An MPL-wff \( \phi \) is valid in MPL-model \( M = (W, R, I) \) iff for every \( w \in W \), \( V_M(\phi, w) = 1 \).

f. **Definition of Validity and Semantic Consequence** (141):
   
   - **Validity**: An MPL-wff is valid in system S (where S is one of K, D, T, B, S\(_4\), or S\(_5\)) iff it is valid in every S-model.
   
   - **Semantic Consequence**: MPL-wff \( \phi \) is a semantic consequence in system S of a set of MPL-wffs \( \Gamma \) iff for every S-model \( (W, R, I) \) and each \( w \in W \) if \( V_M(\gamma, w) = 1 \) for each \( \gamma \in \Gamma \), then \( V_M(\phi, w) = 1 \).

G. **Proving Validity in MPL**

1. In proving validity in MPL, you are asked to show that different formulas are valid in different systems. This often turns on different ways in which the accessibility relation is constrained.

2. The systems are nested, as follows: \( K \rightarrow D \rightarrow T \rightarrow (B \lor S_4) \rightarrow S_5 \)

   a. Thus, if you establish a validity for \( K \), then you have established it as valid for all systems.

   b. Another way to look at this: there are many models of K, and fewer and fewer models as you move to the right; thus, there are fewer and fewer potential countermodels (i.e., models of the system that are available to reveal the invalidity of a formula).