I. MODELS
   A. Definition: A representation of some phenomenon of the real world
      i. Goal: facilitate an understanding of its workings.
      ii. A model is a simplified and generalized version of real events,
          from which the incidental detail, or 'noise' has been removed.
      iii. It has some of the characteristics of the world but not all of them
   B. Utility of Models -- a model becomes a characterization of a process and,
      as such, its value and usefulness derive from the predictions one can make
      from it and its role in guiding and developing theory and research.

II. Game Theory—as a model

   Game theory is about perfectly rational players interested only in winning.
   When you credit your opponent with both rationality and the desire to win,
   and play so as to encourage the best outcome for yourself, then the game
   is open to the analysis of game theory.

   A. Definition
      1. Interdependence of choice
         a. outcome requires both choices
         b. outcome is the intersection of the two choices
      2. Jointly produced outcomes
         a. player chooses either a row or a column
         b. player does not choose an single cell

   B. Concept of a Game
      1. Players (rational)
         a. Assess outcomes
         b. Calculate paths to outcomes
         c. Choose actions that lead to most preferred outcomes
      2. Alternatives
      3. Outcomes
      4. Values (ordinal, cardinal)
      5. Communication – can the players talk to one another before
         moving
      6. Strategies – a decision rule that specifies what they will do in any
         contingency; a predetermined program of play that indicates what
         actions to taken in response to every possible strategy the other
         player might try.
C. Types of Games
   1. Two person vs. N person
   2. Zero Sum vs Non Zero Sum
   3. Simultaneous vs Sequential

D. Game Forms
   1. Normal – games in matrix form
   2. Extensive – games as trees
      a. Represent sequential games
      b. Show the order in which actions take place

E. Strategy--Decision Rule
   1. Pure
   2. Mixed

F. Equilibrium Outcome
   Definition: an outcome is an equilibrium if no player can unilaterally improve his or her own payoff.

II. Zero Sum Games – A Brief Introduction
   • Total winnings or payoffs are fixed
   • In order for someone to win, the other must lose
   • No cooperation is possible
   • Cake cutting problem
      o Cutter decides the row
         ▪ He will get the minimum piece in the chosen row
         ▪ He looks for the maximum minimum
      o Chooser decides the column
         ▪ He will look to get the biggest piece in the chosen column
         ▪ He looks for the minimum column maximum
      o When minimax = maximin, saddle point
   • Not all games have a saddle point
   • Matching pennies
      o Choice depends upon what the other person does
      o Best strategy is choose randomly
      o Play each choice \( \frac{1}{2} \) of the time
   • Millionaire jackpot matching pennies
   • Minimax Theorem
      o Every finite, two person, zero sum game has a rational solution in the form of pure or mixed strategies
III. Non Zero Sum Games
   A. Generic Notation
      1. Alternatives
         ! Cooperate -- C
         ! Defect -- D
      2. Payoffs
         ! B -- best
         ! S -- second best
         ! T -- third best
         ! W -- worst
   B. Outcomes--in terms of Public Goods
      1. Mutual Cooperation
         ! Good is produced
         ! Costs are shared
         ! Players both benefit
      2. One Player cooperates/One Player Defects
         ! Good may or may not be produced
         ! One player bears the entire cost
         ! Players benefit equally
      3. Mutual Defection
         ! Good is not produced
         ! There are no costs
         • There are no benefits
Player 1 or Row Player

C

D

Player 2 or Column Player

C

D

Player 1 or Row Player

C

D

Player 2 or Column Player

P^1_{CC}, P^2_{CC}

P^1_{CD}, P^2_{CD}

P^1_{DC}, P^2_{DC}

P^1_{DD}, P^2_{DD}
**Pay-off = Benefit - Cost**

Assess the costs associated with each of the four basic outcomes (player 1)
- **CC** – -c/2 – assume share costs evenly
- **CD** – -c – assume one person pays all of the cost
- **DC** – 0 -- no cost
- **DD** – 0 – no cost

Determine the nature of the “good” being provided or pursued
- Is it jointly supplied? That is, does my consumption of the good reduce the available supply?
- Is it feasible to exclude someone who does not cooperate in the production from consuming the good once it is produced?
- Does the production of the good require cooperation in either effort or shared resources?

Assess the benefits associated with each of the four basic outcomes
- **CC** – does mutual cooperation produce the good?
- **CD** – can one person produce the good?
- **DC** – can one person produce the good?
- **DD** – 0

**Key Questions:**
- Is the good jointly produced?
  - no positive benefit is associated with CD and/or DC; good can only be produced by both people cooperating [CASE 3]
  - Positive benefit is associated with CD and/or DC; good can only be produced by both people cooperating
    - Is the good public consumption good?
      - A positive benefit is associated with CD and/or DC
      - What is the relationship between b and c
        - b-c < 0 and b-c/2 > 0 [CASE 1]
        - b - c > 0 [CASE 2]
        - b – c/2 < 0 [CASE 4]
Case # 1

Player 2 or Column Player

Player 1 or Row Player

\[
\begin{array}{cc}
C & D \\
\hline
C & P_{CC}^1 & P_{CD}^1 \\
D & P_{DC}^1 & P_{DD}^1 \\
\end{array}
\]

Player 2 or Column Player

<table>
<thead>
<tr>
<th>Outcome</th>
<th>Player #1</th>
<th>Player #2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>benefit</td>
<td>cost</td>
</tr>
<tr>
<td>CC</td>
<td>-c/2</td>
<td>-c/2</td>
</tr>
<tr>
<td>CD</td>
<td>-c</td>
<td>0</td>
</tr>
<tr>
<td>DC</td>
<td>0</td>
<td>-c</td>
</tr>
<tr>
<td>DD</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Player 2 or Column Player

\[
\begin{array}{cc}
C & D \\
\hline
C & S & W \\
D & B & T \\
\end{array}
\]
<table>
<thead>
<tr>
<th>Outcome</th>
<th>Player #1 benefit</th>
<th>Player #1 cost</th>
<th>Player #1 rank</th>
<th>Player #2 benefit</th>
<th>Player #2 cost</th>
<th>Player #2 rank</th>
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</thead>
<tbody>
<tr>
<td>CC</td>
<td>-c/2</td>
<td></td>
<td></td>
<td>-c/2</td>
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<td></td>
</tr>
<tr>
<td>CD</td>
<td>-c</td>
<td></td>
<td></td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>DC</td>
<td>0</td>
<td></td>
<td></td>
<td>-c</td>
<td></td>
<td></td>
</tr>
<tr>
<td>DD</td>
<td>0</td>
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Player 2 or Column Player

\[
\begin{array}{cc}
C & D \\
C & P^{2}_{CC} & P^{2}_{CD} \\
D & P^{2}_{DC} & P^{2}_{DD} \\
\end{array}
\]

Player 2 or Column Player

\[
\begin{array}{cc}
C & D \\
C & S & B \\
D & W & T \\
\end{array}
\]
Model of Case #1 Choice Situation

<table>
<thead>
<tr>
<th></th>
<th>C</th>
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<tbody>
<tr>
<td>C</td>
<td>S,S</td>
<td>W,B</td>
</tr>
<tr>
<td>D</td>
<td>B,W</td>
<td>T,T</td>
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## Case #2

<table>
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<tr>
<th>Outcome</th>
<th>Player #1</th>
<th>Player #2</th>
</tr>
</thead>
<tbody>
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<td></td>
<td>benefit</td>
<td>cost</td>
</tr>
<tr>
<td>CC</td>
<td>-c/2</td>
<td></td>
</tr>
<tr>
<td>CD</td>
<td>-c</td>
<td></td>
</tr>
<tr>
<td>DC</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>DD</td>
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**Table:**

<table>
<thead>
<tr>
<th></th>
<th>C</th>
<th>D</th>
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<tbody>
<tr>
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<td></td>
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</table>
Case #3

<table>
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<th>Player #2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>benefit</td>
<td>cost</td>
</tr>
<tr>
<td>CC</td>
<td>-c/2</td>
<td></td>
</tr>
<tr>
<td>CD</td>
<td>-c</td>
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</tr>
<tr>
<td>DC</td>
<td>0</td>
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</tr>
<tr>
<td>DD</td>
<td>0</td>
<td>0</td>
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</tbody>
</table>

LL

\[ \text{C} \quad \text{D} \]

\[ \text{C} \quad \text{D} \]

\[ \text{C} \quad \text{D} \]
### Case #4

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<tr>
<th>Outcome</th>
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<tbody>
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<td>benefit</td>
<td>cost</td>
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<tr>
<td>CC</td>
<td>-c/2</td>
<td></td>
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<tr>
<td>CD</td>
<td>-c</td>
<td></td>
</tr>
<tr>
<td>DC</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>DD</td>
<td>0 0</td>
<td>0 0</td>
</tr>
</tbody>
</table>

The payoff matrix is:

```
  C  D
C
D
```
CASE 1 – PRISONER’S DILEMMA  
CASE 2 – CHICKEN  
CASE 3 – STAG HUNT OR ASSURANCE  
CASE 4 – DEADLOCK  

Consider Prisoner’s Dilemma as Base  
• Chicken exchanges T and W  
• Stag Hunt exchanges S and B  
• Deadlock exchanges S and T
Prisoner’s Dilemma

- Two partners in crime are being interrogated separately
- State lacks evidence to convict them of the crime they committed but can convict both on a lesser charge with a sentence of 1 year
- Prosecutor wants a conviction on the more serious charge
  - He tells each that if you confess but your partner does not, I will use your testimony to convict him – you will go free and he will get 10 years
  - If you both confess you both go away for three years
- Will the prisoner’s cooperate with one another or confess?

<table>
<thead>
<tr>
<th>Your Strategy</th>
<th>Stay Mum</th>
<th>Cop Plea</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stay Mum</td>
<td>-1, -1</td>
<td>-10, 0</td>
</tr>
<tr>
<td>Cop Plea</td>
<td>0, -10</td>
<td>-3, -3</td>
</tr>
</tbody>
</table>

Note the differences between this matrix and the one presented by Wright in Appendix I

Questions:

1. Provide an argument that identifies the rational strategy choice for Player 1 and Player 2.
2. Assess whether the rational strategy is also a dominant strategy.
3. What is the equilibrium outcome of the game? What does it mean for an outcome to be an equilibrium?
4. Does the game have a dilemma (i.e., a usually undesirable or unpleasant choice)?
III. Chicken
• Played by choosing a long straight road with a white line down the middle
• Start two cars from opposite ends of the road on the white line
• Whoever swerves first is the “chicken”

Questions:
1. Provide an argument that identifies the rational strategy choice for Player 1 and Player 2.
2. Assess whether the rational strategy is also a dominant strategy.
3. What is the equilibrium outcome of the game? What does it mean for an outcome to be an equilibrium?
4. Does the game have a dilemma (i.e., a usually undesirable or unpleasant choice)?
Stag Hunt

- Hunting the stag requires everyone’s help – they must form a circle and everyone must stay at their post
- If a hare happened to pass within reach of one of the individuals, that person could leave their post, catch the hare, and have their own meal

<table>
<thead>
<tr>
<th></th>
<th>Hunt Stag</th>
<th>Chase Hare</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hunt Stag</td>
<td>3, 3</td>
<td>0, 2</td>
</tr>
<tr>
<td>Chase Hare</td>
<td>2, 0</td>
<td>1, 1</td>
</tr>
</tbody>
</table>

Questions:

1. Provide an argument that identifies the rational strategy choice for Player 1 and Player 2.

2. Assess whether the rational strategy is also a dominant strategy.

3. What is the equilibrium outcome of the game? What does it mean for an outcome to be an equilibrium?

4. Does the game have a dilemma (i.e., a usually undesirable or unpleasant choice)?