Do all problems (30 points total). Place your answers in the spaces provided. You must show your work clearly in order to receive credit. No calculators allowed.

1. (6 pts) Give an algebraic proof for the following fact: The product of two odd numbers is an odd number.

Proof. Let $A = 2k + 1$ and $B = 2l + 1$ be two odd numbers. Then the product of $A$ and $B$ is

$$A \times B = (2k+1)(2l+1)$$

$$= 2k \cdot 2l + 2k + 2l + 1$$

$$= 2(2kl + k + l) + 1.$$ 

But this is twice a whole number plus one. So $A \times B$ is odd. □

2. (4 pts) Complete the following sentences:

(a) $A$ is divisible by $k$ if $A = k \ell$ for some whole number $\ell$.

(b) One other way to say "$A$ is divisible by $k$" is...

$A$ is a multiple of $k$

or $k$ is a factor of $A$

or $k$ divides $A$

3. (8 pts)

(a) Circle the numbers below that divide the number 117, 645.

2 3 4 5 8 9 10 11

(b) Circle the numbers which divide $n = 129, 030$.

2 3 4 5 8 9 10 11
4. (8 pts) The letters \( a \) and \( b \) are digits (i.e., 0, 1, ..., 9). Find all choices of \( a \) and \( b \) so that the number \( 41a,233,0b0 \) is divisible by 8, but not divisible by 3. [Hint: Start by considering divisibility by 8.]

To be divisible by 8, \( 0b0 \) must be. So \( b = 0, 4, \) or \( 8 \).

Suppose \( b = 0 \). Then \( 411a + 2 + 3 + 3 \) mustn't be divisible by 3.

Equivalently, \( 411a + 2 \) mustn't be. So \( a = 0, 3, 4, 6, 7, \) or \( 9 \).

Suppose \( b = 4 \). Then \( 411a + 2 + 3 + 3 + 4 \) mustn't be divisible by 3.

Equivalently, \( 411a + 4 \) mustn't be. So \( a = 0, 2, 3, 5, 6, 8, \) or \( 9 \).

Suppose \( b = 8 \). Then \( 411a + 2 + 3 + 3 + 8 \) mustn't be divisible by 3.

Equivalently, \( 411a + 12 \) mustn't be. So \( a = 1, 2, 4, 5, 7, \) or \( 8 \).

So the choices for \( (a, b) \) are \((0,0), (0,1), (0,3), (0,4), (0,6), (0,7), (0,9)\)
\((4,0), (4,2), (4,3), (4,5), (4,6), (4,8), (4,9), (8,1), (8,2), (8,4), (8,5), (8,7), (8,8)\).

5. (4 pts) Finish the statement of the Fundamental Theorem of Arithmetic: Every whole number \( N > 1 \)...

\[
\text{can be written as the product of primes.}
\]

\[
\text{or has a (unique) prime factorization.}
\]

\[
\text{or can be written}
\]
\[
N = p_1 p_2 p_3 \cdots p_n
\]

\[
\text{for some primes } p_1, p_2, p_3, \ldots, p_n.
\]