1) \( \Omega = x \, dy \wedge dz + y \, dz \wedge dx + z \, dx \wedge dy \), \( z \)-form in \( \mathbb{R}^3 \).

a. The map \( \sigma \) defined by \( \sigma(p, \varphi, \theta) = (x, y, z) = (p \sin \varphi \cos \theta, p \sin \varphi \sin \theta, p \cos \varphi) \) converts spherical coordinates into rectangular coordinates. We can now compute \( \Omega \) in spherical coordinates by

\[
\sigma^* \Omega = (p \sin \varphi \cos \theta) \, d(p \sin \varphi \cos \theta) \wedge d(p \cos \varphi) + (p \sin \varphi \sin \theta) \, d(p \cos \varphi) \wedge d(p \sin \varphi \cos \theta)
\]

Derivation:

\[
= (p \sin \varphi \cos \theta) [\sin \varphi \cos \theta \, dp \wedge d(p \sin \varphi \cos \theta) + p \cos \varphi \sin \theta \, dp \wedge d(p \sin \varphi \sin \theta) - p \sin \varphi \cos \theta \, dp \wedge d(p \cos \varphi)]
\]

Distribution:

\[
= [p \sin \varphi \cos \theta \, dp \wedge d(p \sin \varphi \cos \theta) + p \cos \varphi \sin \theta \, dp \wedge d(p \sin \varphi \sin \theta) - p \sin \varphi \cos \theta \, dp \wedge d(p \cos \varphi)]
\]

Collecting:

\[
= [p \sin \varphi \cos \theta \, dp \wedge d(p \sin \varphi \cos \theta) + p \cos \varphi \sin \theta \, dp \wedge d(p \sin \varphi \sin \theta) - p \sin \varphi \cos \theta \, dp \wedge d(p \cos \varphi)]
\]

Skew Cancellation:

\[
\sin^2 \varphi + \cos^2 \varphi = 1
\]

\[
= (p^3 \sin^2 \varphi \cos^2 \theta + p^3 \sin \varphi \cos \varphi \sin \theta + p^3 \sin \varphi \cos^2 \varphi \sin^2 \theta) \, d\varphi \wedge d\theta
\]

\[
= (p^3 \sin \varphi + p^3 \sin \varphi \cos^2 \varphi) \, d\varphi \wedge d\theta
\]

\[
= p^3 \sin \varphi \, d\phi \wedge d\theta.
\]
b. In rectangular coordinates
\[ d\Omega = \left( \frac{\partial(x)}{\partial x} + \frac{\partial(y)}{\partial y} + \frac{\partial(z)}{\partial z} \right) dx \wedge dy \wedge dz = 3 \, dx \wedge dy \wedge dz. \]

In spherical coordinates,
\[ d(\sigma^*\Omega) = \left( \frac{\partial(\rho^3 \sin^3 \phi)}{\partial \rho} \right) d\rho \wedge d\phi \wedge d\theta = 3 \rho^2 \sin \phi \, d\rho \wedge d\phi \wedge d\theta. \]

Comparing the two 3-forms, we see
\[ \sigma^*(d\Omega) = 3 \left[ \sin \phi \cos \theta \, d\rho \wedge d\phi \wedge d\theta \right] \]
\[ = 3 \left[ \sin \phi \cos \theta \, d\rho \wedge d\phi \wedge d\theta \right] \]
\[ = 3 \left( -\rho^2 \sin^3 \phi \cos^2 \theta \, d\rho \wedge d\phi \wedge d\theta \right) \]
\[ = 3 \left( \rho^2 \sin^3 \phi \cos^2 \theta \, d\rho \wedge d\phi \wedge d\theta \right) \]
\[ = 3 \rho^2 \left( \sin^3 \phi \cos \theta \, d\rho \wedge d\phi \wedge d\theta \right) \]
\[ = 3 \rho^2 \sin \phi \left( \cos^2 \theta \, d\rho \wedge d\phi \wedge d\theta \right) \]
\[ = 3 \rho^2 \sin \phi \left( \cos \theta \, d\rho \wedge d\phi \wedge d\theta \right) \]
\[ = 3 \rho^2 \sin \phi \, d\rho \wedge d\phi \wedge d\theta \]
\[ = d(\sigma^*\Omega). \]

They are the same.
C. In spherical coordinates we have \( i: S^2 \rightarrow \mathbb{R}^2 \) given by \( i(\varphi, \theta) = (1, \varphi, \theta) \). Thus to compute \( \Omega|_{S^2} \) we can pull back \( \sigma^*\Omega \) from part (a) via \( i \) instead of pulling back the original \( \Omega \) via a rectangular inclusion of \( S^2 \). We get

\[
\Omega|_{S^2} = i^*(\sigma^*\Omega) = \sin \varphi \, d\varphi \wedge d\theta.
\]

d. It is immediately clear that \( \Omega|_{S^2} \) is not zero everywhere except \( \varphi = 0, \pi \). Unfortunately the coordinates we are using are not well-defined there. To remedy this we will alter our coordinates.

The standard spherical coordinates have \( \varphi \) as the angle "down" from the \( z \)-axis (see figure). Let us permute \((x, y, z)\) into \((y, z, x)\) so that \( \varphi \) is now the angle "down" from the \( x \)-axis.

The result of this is that the trouble spots \( x = 0, y = 0, z = \pm 1 \) have now become \( x = 1, y = 0, z = 0 \), because it is still true that

\[\Omega|_{S^2} = \sin \varphi \, d\varphi \wedge d\theta.\]

This follows from

\[
x \, dy \wedge dz + y \, dz \wedge dx + z \, dx \wedge dy = y \, dx \wedge dy + z \, dy \wedge dz.
\]

The new case allows us to conclude that \( \Omega|_{S^2} \) is nonzero at \( x = 0, y = 0, z = \pm 1 \) and so it is nowhere zero.
2) \( g : \mathbb{R}^2 \to \mathbb{R}^2, (s, t) \mapsto (st, e^t) \) \\
\( \omega = x \, dy \)

\[ g^* \omega = (st) \, d(e^t) = st \cdot e^t \, dt. \]

\[ dw = \left( \frac{\partial (st)}{\partial x} - \frac{\partial (e^t)}{\partial y} \right) dx \wedge dy = dx \wedge dy. \]

\[ g^*(dw) = d(st) \wedge d(e^t) = (t \, ds + s \, dt) \wedge (e^t \, dt) = t \, e^t \, ds \wedge dt. \]

\[ d(g^* \omega) = \left( \frac{\partial (st)}{\partial s} \frac{\partial e^t}{\partial x} - \frac{\partial (st)}{\partial x} \frac{\partial e^t}{\partial s} \right) ds \wedge dt = te^t \, ds \wedge dt = g^*(dw). \]

b. \( g : \mathbb{R}^2 \to \mathbb{R}^3, (\theta, \phi) \mapsto (\cos (\phi + z) \cos \theta, \cos (\phi + z) \sin \theta, \sin \phi) \) - Embedded Torus

\( \omega = y \, dz \wedge dx \)

\[ g^* \omega = (\cos (\phi + z) \sin \theta) \, d(\sin \phi) \wedge d(\cos (\phi + z) \cos \theta) \]

\[ = (\cos (\phi + z) \sin \theta) (\cos \phi \, d\theta) \wedge (-\cos (\phi + z) \sin \theta \, d\phi - \sin \phi \cos \theta \, d\phi) \]

\[ = (\cos (\phi + z) \sin \theta) (\cos \phi \, d\theta) \wedge (\sin \phi \cos \theta \, d\phi) \]

\[ d\omega = \left( \frac{\partial (\cos \phi \, d\theta)}{\partial x} + \frac{\partial (\sin \phi \cos \theta \, d\phi)}{\partial y} \right) dx \wedge dy \wedge dz = dx \wedge dy \wedge dz. \]

\[ g^*(d\omega) = d((\cos (\phi + z) \cos \theta) \wedge d((\cos (\phi + z) \sin \theta) \wedge d(\sin \phi) \wedge d\phi) \]

\[ = 0 \] if it is a 3-form on \( \mathbb{R}^2 \). These are necessarily the 0-forms on two duals wedged.

\[ d(g^* \omega) = \frac{\partial (\cos (\phi + z) \cos \theta)}{\partial x} \, d\theta \wedge \theta \wedge \phi \, d\phi + \frac{\partial (\cos (\phi + z) \cos \theta \, d\phi)}{\partial y} \, d\phi \wedge \phi \wedge \theta \, d\phi \]

\[ = 0 \] = \( g^*(d\omega) \).

C. \( g : \{ (u, v) \in \mathbb{R}^2 \mid u^2 + v^2 < 1 \} \to \mathbb{R}^2 \setminus \{ 0 \}, (u, v) \mapsto (u, v, \sqrt{1 - u^2 - v^2}) \).

\( \omega = \frac{x \, dy \wedge dz + y \, dz \wedge dx + z \, dx \wedge dy}{(x^2 + y^2 + z^2)^{3/2}} \)

\[ g^* \omega = u \, dv \wedge d(\sqrt{1 - u^2 - v^2}) + v \, d(\sqrt{1 - u^2 - v^2}) \wedge du + \sqrt{1 - u^2 - v^2} \, du \wedge dv \]

\[ = u \, dv \wedge \left( \frac{\partial (\sqrt{1 - u^2 - v^2})}{\partial u} du - \frac{\partial (\sqrt{1 - u^2 - v^2})}{\partial v} dv \right) + v \left( \frac{\partial (\sqrt{1 - u^2 - v^2})}{\partial u} du - \frac{\partial (\sqrt{1 - u^2 - v^2})}{\partial v} dv \right) \wedge du + \sqrt{1 - u^2 - v^2} \, du \wedge dv \]

\[ = \frac{-u^2 \, du \wedge dv}{\sqrt{1 - u^2 - v^2}} - \frac{v^2 \, du \wedge dv}{\sqrt{1 - u^2 - v^2}} + \frac{1 - u^2 - v^2}{\sqrt{1 - u^2 - v^2}} \, du \wedge dv \]

\[ = \frac{du \wedge dv}{\sqrt{1 - u^2 - v^2}}. \]
\[ d\omega = \left( \frac{\partial z}{\partial x} \frac{x^2}{2} + \frac{\partial z}{\partial y} \frac{y^2}{2} + \frac{\partial z}{\partial z} \frac{z^2}{2} \right) dx \wedge dy \wedge dz \]
\[ = \left( \frac{1}{(x^2 + y^2 + z^2)^{3/2}} - \frac{3x^2}{(x^2 + y^2 + z^2)^{3/2}} + \frac{1}{(x^2 + y^2 + z^2)^{3/2}} - \frac{3y^2}{(x^2 + y^2 + z^2)^{3/2}} + \frac{1}{(x^2 + y^2 + z^2)^{3/2}} - \frac{3z^2}{(x^2 + y^2 + z^2)^{3/2}} \right) dx \wedge dy \wedge dz \]
\[ = 3 \left( \frac{1}{(x^2 + y^2 + z^2)^{3/2}} - \frac{x^2 + y^2 + z^2}{(x^2 + y^2 + z^2)^{3/2}} \right) dx \wedge dy \wedge dz \]
\[ = 3 \left( \frac{1}{(x^2 + y^2 + z^2)^{3/2}} - \frac{1}{(x^2 + y^2 + z^2)^{3/2}} \right) dx \wedge dy \wedge dz \]
\[ = 0. \quad \checkmark \]

\[ g^*(d\omega) = g^*(0) = 0 \] which makes sense b/c it is a 3-form on a subset of \( \mathbb{R}^3 \).

\[ d(g^*\omega) = \frac{\partial}{\partial u} \omega_{uv} \wedge dv + \frac{\partial}{\partial v} \omega_{uv} \wedge du \]
\[ \quad = 0 = g^*(d\omega). \quad \checkmark \]

3) Proposition A: \( dx + i_x d \) is a derivation on \( \Omega^2 M \).

Proof: Let \( \omega \in \Omega^2 M \) and \( \eta \in \Omega^2 M \). Note that \( d \) and \( i_x \) are skew-derivations.

Observe
\[ (dx + i_x d)(\omega \wedge \eta) = dx(\omega \wedge \eta) + i_x d(\omega \wedge \eta) \]
\[ = dx(\omega \wedge \eta) + i_x d(\omega \wedge \eta) \]
\[ = i_x dx(\omega \wedge \eta) + i_x (d\omega \wedge \eta) + i_x (\omega \wedge d\eta) \]
\[ = d_x \omega \wedge \eta + (\omega d \wedge \eta) + (-1)^k \omega d \wedge i_x \eta \] (reregarding)
\[ = dx_\omega \wedge \eta + \omega \wedge dx \eta + \omega \wedge i_x d \eta \]
\[ = dx_\omega \wedge \eta + \omega \wedge dx \eta + \omega \wedge i_x d \eta \]
\[ = (dx_\omega + i_x d) \wedge \eta + \omega \wedge (dx_\omega + i_x d) \wedge \eta \]
\[ = (dx + i_x d) \wedge \eta + \omega \wedge (dx + i_x d) \wedge \eta. \quad \checkmark \]
Proposition B \[ L_x = d_i f + i_x d. \]

Proof. By 31.2 it will suffice to show that they agree on 0-forms and exact 1-forms. Let \( f \) be a function. Then
\[
(d_i f + i_x d) f = d_i (f) d + i_x df = d_i df = d_i f(X) = Xf = L_x f.
\]

So they agree on 0-forms. Let \( \omega \) be an exact 1-form, that is \( \omega = df \) for some function \( f \). Then
\[
L_x \omega = d(L_x f) = d(x f) = d(x df) = d_i df + i_x d f = (d_i f + i_x d) df.
\]

So they agree on exact 1-forms and we are done. \( \square \)

4) Proposition The following are equivalent for a 2-tensor \( \omega \) on a finite-dimensional vector space \( V \):

(a) \( \omega \) is nondegenerate.

(b) The matrix \( (\omega_{ij}) \) of \( \omega \) with respect to any basis is nonsingular.

(c) The linear map \( \tilde{\omega}: V \to V^\ast \) defined by \( \tilde{\omega}(X)(Y) = \omega(X, Y) \) is invertible.

Proof. (b) \( \iff \) (c). It is a fact from linear algebra that a nonsingular matrix remains nonsingular under any change of basis. So in (b) we need only consider a single basis. Since \( \omega \) is a bilinear map, we have \( \omega(X, Y) = X^T W Y \). Now \( X^T W Y \) is a \( 1 \times n \) matrix and so represents an element of \( V^\ast \); in fact, we can see that \( X^T W \) corresponds to \( \tilde{\omega}(X) \) because \( \tilde{\omega}(X)(Y) = X^T W Y = \omega(X, Y) \). Thus \( X^T W \) is also the matrix of \( \tilde{\omega} \) and so \( (\omega_{ij}) \) is nonsingular if and only if \( \tilde{\omega} \) is invertible.

(a) \( \implies \) (c). Suppose \( \omega \) is nondegenerate and \( X \) be arbitrary. Since \( \tilde{\omega}(X) \) is the zero function in \( V^\ast \), \( \tilde{\omega}(X)(Y) = 0 \) for all \( Y \in V \). Hence \( \tilde{\omega}(X, Y) = 0 \) for all \( Y \) which implies \( X = 0 \). So ker \( \tilde{\omega} \) is trivial and \( \tilde{\omega} \) is one-one. Since \( \dim V = \dim V^\ast \) we know \( \tilde{\omega} \) is invertible. \( \checkmark \)

(c) \( \implies \) (a). Suppose \( \tilde{\omega} \) is invertible. Then it has trivial kernel, so if \( \tilde{\omega}(X) \) sends all vectors to 0 then \( X = 0 \). If \( \omega(X, Y) = 0 \) for all \( Y \in V \) then also \( \tilde{\omega}(X)(Y) = \omega(X, Y) = 0 \) for all \( Y \). But by hypothesis this means \( X = 0 \). So \( \omega \) is nondegenerate. \( \square \)
Let $M$ be a manifold with $\dim M = 2n$. Then $\omega \in \Omega^2(M)$ is a symplectic form if and only if $d\omega = 0$ and $\omega \wedge \ldots \wedge \omega$ is nowhere $0$.

Proof. ($\Rightarrow$) Assume $\omega$ is symplectic. Then by definition $\omega$ is closed so $d\omega = 0$. Also for each $p \in M$, $\omega_p$ is a symplectic tensor. By the seminar talk, there exists a basis $\{\alpha^i, \beta^j\}$ of $T_p^*M$ such that $\omega_p = \sum \alpha^i \wedge \beta^i$ and also

$\omega_p \wedge \ldots \wedge \omega_p = \alpha^1 \wedge \beta^1 \wedge \ldots \wedge \alpha^n \wedge \beta^n$.

Since the $\alpha^i, \beta^i$ are linearly independent the right-hand side is not $0$. Since $p$ was arbitrary this holds over all of $M$ and $\omega \wedge \ldots \wedge \omega$ is nowhere $0$.

($\Leftarrow$) Contrapositive. Assume $\omega$ is not symplectic. If $\omega$ is not closed then $d\omega \neq 0$ and we are done. So suppose $\omega$ is degenerate. Hence there is some $p \in M$ and some $0 \neq X \in T_pM$ such that $\omega_p(X, Y) = 0$ for all $Y \in T_pM$.

Since $X$ is not $0$ there is a basis $\{X, X_i\}$ for $T_pM$. But then

$$(\omega_p \wedge \ldots \wedge \omega_p)(X, X_i, \ldots, X_{i-1}) = 0$$

because $\omega_p(X, X_i) = 0$ for all $i$. This implies that $\omega_p \wedge \ldots \wedge \omega_p$ is the zero map and so $\omega \wedge \ldots \wedge \omega$ is nowhere $0$. \hfill $\Box$

---

Let $(V, \omega)$ be a symplectic vector space of dimension $2n$. We will show in each case that there exists a symplectic basis $\{A_i, B_i\}$ of $V$ with the appropriate properties.

1. Suppose $S$ is a symplectic subspace of $V$. Then $W_S$ is nondegenerate and by 12.22 $(S, \omega_S)$ has a symplectic basis $\{A_i, B_i\}_{i=1}^k$. Clearly $S = \text{span}\{A_i, B_i, \ldots, A_k, B_k\}$ so we must now expand this to a symplectic basis for $V$. 

Note that \( \dim S = 2k \) and \( \dim S + \dim S^\perp = \dim V = 2n \), so \( \dim S^\perp = 2(n-k) \). Now \( S \) is symplectic so \( S \cap S^\perp = \{0\} \). Thus also \( S^\perp \) is symplectic because \( S^\perp \cap S^\perp = S \cap S^\perp = \{0\} \). By 12.22, \( (S^\perp, \omega|_{S^\perp}) \) has a symplectic basis \( \{A_{1n}, B_{1n}, \ldots, A_n, B_n\} \). Since \( S \cap S^\perp = \{0\} \), we have \( \{A, B, \ldots, A_n, B_n\} \) as the desired basis for \( V \).

b. Suppose \( S \) is isotropic. Then \( \omega|_S = 0 \). Let \( \{A_1, \ldots, A_k\} \) be any basis for \( S \). Note that \( \omega(A_i, A_j) = \omega|_S(A_i, A_j) = 0 \) for all \( i \neq j \). By the linear independence of the \( A_i \) and the existence of a function \( \tilde{f}_i : V \to \mathbb{R} \) such that \( \tilde{f}_i(A_i) = \delta_{ij} \). Since \( \omega \) is nondegenerate, by problem (4), \( \tilde{\omega} : V \to V^* \) is an isomorphism giving \( -B_i \in V \) such that \( \tilde{\omega}(B_i) = \tilde{f}_i \). This means \( \omega(B_i, X) = \omega(-B_i, X) = \tilde{\omega}(B_i)(X) = \tilde{f}_i(X) = f_i(X) \).

Particularly for the \( A_i \), we have \( \omega(A_i, B_i) = f_i(A_i, B_i) = \delta_{ij} \).

Note that \( \{A_1, B_1, A_2, \ldots, A_k\} \) is linearly independent so there exists \( \tilde{f}_2 : V \to \mathbb{R} \) such that \( \tilde{f}_2(A_2) = \delta_{ij} \) and \( \tilde{f}_2(B_i) = 0 \). Again, \( \tilde{\omega} \) gives us a \( B_2 \in V \) such that \( \omega(A_2, B_2) = \tilde{f}_2(A_2) = \delta_{ij} \) and \( \omega(B_2, B_2) = \tilde{f}_2(B_2) = 0 \). We add \( B_2 \) to the set and continue. Until we have \( \{A_1, B_1, A_2, B_2, \ldots, A_k, B_k\} \).

Put \( T = \text{span} \{A_1, B_1\} \). Then by construction \( \{A_1, B_1\} \) is a symplectic basis for \( T \) and \( S \) is a symplectic subspace of \( V \). By part (a) we can expand the basis of \( T \) to \( \{A_1, B_1\} \) a symplectic basis for \( V \). But if we look at where we began, \( S = \text{span} \{A_1, \ldots, A_k\} \) and we are done.

C. Suppose \( S \) is coisotropic. Then \( S \supset S^\perp \) and so \( S^\perp \) is isotropic because \( S^\perp \subset S = \{0\} \). Put \( \dim S = n+k \). Then \( \dim S^\perp = n-k \). By part (b) there exists a symplectic basis \( \{A_1, B_1\} \) for \( V \) such that \( S^\perp = \text{span} \{A_1, \ldots, A_n\} \) where we have rotationally interchanged \( A \) and \( A_{k+1} \) without loss.

By def of \( (S^\perp)^\perp = S^\perp \) for \( i \neq j \), hence \( \omega(A_i, A_j) = 0 \) for all \( i \neq j \).

Similarly, \( B_i \in (S^\perp)^\perp = S^\perp \) for \( i \neq j \) because \( \omega(B_i, A_j) = -\omega(A_j, B_i) = 0 \) for all \( i \neq j \) since \( i \neq j \) in this case. Thus \( S = \text{span} \{A_1, \ldots, A_n, B_1, \ldots, B_n\} \).
1. Suppose $S$ is Lagrangian. Then $S=S_{\perp}^{\perp}, w_{S} = 0$, and $\dim S = n$.

The fact that $w_{S} = 0$ implies $S$ is isotropic. So by part (b) there is a symplectic basis $\{A_{1}, B_{1}, \ldots, A_{n}, B_{n}\}$ such that $S = \text{span} \{A_{1}, \ldots, A_{n}\}$.

But $k = \dim S = n$, so $S = \text{span} \{A_{1}, \ldots, A_{n}\}$.

$$3 \frac{1}{2} \subset S$$

7) Let $\omega = \sum_{i=1}^{n} dx^{i} \wedge dy^{i}$ be the standard symplectic form on $\mathbb{R}^{2n}$. The real symplectic group is defined to be

$$\text{Sp}(n, \mathbb{R}) = \{ A \in \text{GL}(2n, \mathbb{R}) \mid A^{*}w = w \}.$$

a. Let $A \in \text{Sp}(n, \mathbb{R})$. Since $\omega$ is fixed, $\Delta dx^{i} \wedge dy^{i} = \Delta dx^{i} \wedge dy^{i}$ so the standard basis must be mapped by $A$ to a basis where $\omega(Ax^{i}, Ax^{j}) = \omega(Ay^{i}, Ay^{j}) = 0$ for all $i, j$ and the mixed terms $\omega(Ax^{i}, Ay^{j})$ must equal 0 except when $i = j$, otherwise $\Delta dx^{i} \wedge dy^{i} \neq 0$. Thus $A$ takes the standard basis to a symplectic basis.

If $A$ maps the standard basis to a symplectic basis then all the appropriate terms go to zero and $A^{*}w = w$. So $A \in \text{Sp}(n, \mathbb{R})$.

b. Let $J = \left( \begin{array}{cc} 0 & I_{n} \\ -I_{n} & 0 \end{array} \right)$. For $A \in \text{GL}(2n, \mathbb{R})$, $A^{T}JA = J$ if and only if $A$ maps $(I_{n}, 0_{n})$ to a symplectic basis, so by part (a) if and only if $A \in \text{Sp}(n, \mathbb{R})$.

c. $\text{Sp}(n, \mathbb{R})$ is clearly a group. It is also a manifold by the preimage theorem applied to $E : \text{GL}(2n, \mathbb{R}) \rightarrow \text{Sp}(n, \mathbb{R}).$ Specifically, $\text{Sp}(n, \mathbb{R}) = \{ A \mid \omega(Ax^{i}, Ay^{j}) = 0 \}$.

Thus $\text{Sp}(n, \mathbb{R})$ is a Lie subgroup. Its dimension is $n(2n+1)$.

Is $\mathbb{R}$ a submanifold?

D. We know the Lie algebra of $\text{GL}(2n, \mathbb{R})$, $\text{Sp}(n, \mathbb{R})$ is the subalgebra given by $JA + A^{T}J = 0$. It remains to be shown that the bracket operation is closed in $\text{Sp}(n, \mathbb{R})$. Incomplete.

e. Is $\text{Sp}(n, \mathbb{R})$ compact? No. Example for (e)

For each $n \in \mathbb{N}$, set $A_{n} = \left( \begin{array}{cc} 0 & -n \\ \frac{1}{n} & 0 \end{array} \right)$.

Check: $A_{n}^{T}JA_{n} = J$ \forall n \in \mathbb{N}.

$\iff A_{n} \in \text{Sp}(1, \mathbb{R})$. But $\{\text{integer} \}$ contains no $\text{Sp}(1, \mathbb{R})$ in (e).