CONCLUSIONS WITHIN MATHEMATICAL TASK ENACTMENTS: A NEW PHASE OF ANALYSIS

Samuel Otten
Michigan State University
ottensam@msu.edu

This study builds on the mathematical tasks framework developed during the QUASAR project by considering the ways in which enacted mathematical tasks are concluded. Though the framework originated from a cognitive tradition, this study takes a sociocultural perspective and reinterprets task phases through the lens of activity structure. Based on observations of 84 middle-school class periods and 4 detailed task conclusion analyses, a modification of the mathematical tasks framework to include a separate conclusion phase is proposed. Implications for analysis of cognitive demand are discussed, as is the relationship between task conclusions and characterizations of the nature of mathematics.

Introduction

Students’ engagement with mathematical tasks is widely recognized as a central component of mathematics education at all levels. For the middle school level in particular, Stein and her colleagues (Stein, Grover, & Henningsen, 1996) built on the work of Doyle (1988) and earlier cognitive science to develop the mathematical tasks framework (see Figure 1). Within this framework, the definition of a mathematical task is based on the earlier concept of an academic task but is broadened in duration to align with the mathematical idea under consideration. In other words, a new mathematical task has not begun until the mathematical idea being focused on has shifted, which means that a single mathematical task may contain several academic tasks. The mathematical tasks framework utilizes the key notion of cognitive demand, the kind of thinking processes entailed in solving the task, which has been found to correlate with student performance (Stein & Lane, 1996). The framework becomes valuable as a research tool because its phases allow the level of cognitive demand to be tracked throughout the enactment of the task. For example, during the set-up phase, teachers may cast a task written to be procedures-with-connections as procedures-without-connections; or during the implementation phase, a doing-mathematics task may descend into unsystematic exploration or nonmathematical activity. Overall, we know that it is difficult for teachers to enact mathematical tasks in ways that authentically engage students in the thought processes of mathematics, even when a task is written with a high level of cognitive demand (Henningsen & Stein, 1997).

Figure 1. The mathematical tasks framework (adapted from Stein & Smith, 1998)
More recent work in mathematics education has brought to the foreground the sociocultural aspects of mathematics teaching and learning. This view implies that mathematical task enactments involve not only students’ cognitive processes but also their participation in the activities and discourses of mathematics (Chapman, 2003; O’Connor, 2001; Otten & Herbel-Eisenmann, 2009). The current study views task enactments through the lens of classroom activity structures for the purpose of examining the alignment between the existing mathematical tasks framework and the nature of the classroom activity. In particular, I examine middle school level task enactments, as did the framework developers, and argue that this activity lens reveals a distinct phase of task enactments—the task conclusion—that has thus far been unrecognized by the framework. As is shown below, this conclusion phase is significant from an activity perspective but also has implications with respect to trajectories of cognitive demand. Thus a revised framework is proposed.

**Theoretical Perspective**

Within the broad realm of sociocultural theory, I employed Halliday’s (Halliday & Matthiessen, 2003) theory of systemic functional linguistics (SFL) to examine discourse practices in middle school mathematics classrooms. SFL recognizes three metafunctions of language—ideational, interpersonal, and textual. Language is used to make sense of experience and in so doing serves the **ideational** metafunction; that is, it provides cues and clues regarding the meaning of what is being talked about. Language is also a means for acting out the social relationships of those who are using the language, thus serving the **interpersonal** metafunction. The **textual** metafunction refers to aspects of the organization of the language itself. Any use of language involves all three metafunctions, though not always to the same degree.

Within SFL, language is viewed as a system in which various sets of options exist with different meaning-making potentials. When someone is producing a text—written, verbal, or otherwise—they make many (possibly unconscious) choices about their language use, each of which influences subsequent choices and the potential ways in which the text will be construed. This language-in-use simultaneously shapes and is shaped by the context of the interaction. For example, a teacher closing the door and addressing the class in a full voice is serving the **textual** metafunction of marking the beginning of a lesson, but the local context (e.g., the bell ringing, students taking their seats and expecting a lesson to begin) are simultaneously influencing the teacher’s discourse. This is closely related to Lemke’s (1990) discussion of **activity structures**:

> [A classroom lesson] has a pattern of organization, a structure… In real life you never know for sure what is coming next, but if you can recognize that you are in the midst of a patterned, organized kind of social activity, like a lesson, you know the probabilities for what is likely to come next. (pp. 2–3)

Thus the various language options noted by SFL are restricted in a patterned way when the language is occurring within an activity structure such as lecture, going over homework, or small-group work. Furthermore, activity structures can be recognized by analyzing discursive cues and their textual functions marking the activity structure.

**Method**

Data for the present study consisted of task enactments drawn from a larger project involving eight middle grades (6–10) mathematics teachers engaged in literature study groups and cycles.

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of action research around classroom discourse. The videotaped class periods in which these task enactments occurred come from the baseline year of the larger project, before any interventions with the teachers had taken place. The present study is focused on the middle-school level, so two of the eight teachers were excluded because they taught high-school content. This left 84 classroom observations from six middle school mathematics teachers as the full corpus for the present study. (Note that there were less mathematical task enactments than class periods because many tasks continued over multiple class periods, and I excluded any tasks whose enactments were not entirely captured within the observed class periods.)

I reviewed all transcripts and watched video clips from the 84 selected observations. I took notes on discursive markings of the task phases (e.g., “teacher addresses class from the front of the room”), general features of the classroom events (e.g., "students worked in pairs on a handout for 20 minutes"), mathematical content of the tasks (e.g., "area of triangles"), and general impressions of what I perceived to be the task conclusion (e.g., "short conclusion with the teacher emphasizing main points"). Using these notes, I then selected four tasks to undergo detailed analysis. The selections were made based on several criteria (i.e., varied teachers, varied mathematical content, varied curriculum materials, varied features of the task conclusion), all intended to lead to a variety of task conclusions in the detailed analysis phase.

The four selected task enactments were analyzed using three different tools from SFL. First, discourse markers and their textual functions were coded with respect to the activity structures of the task enactment. This analysis is the centerpiece of the current study, which focuses on the alignment between the activity structures and the original enactment phases. Second, lexical chains (Christie, 2002) were created to map the semantic progressions of the task conclusions. Third, various linguistic analyses (e.g., theme analysis, amplification analysis, agency analysis; Martin, 2009) were performed with each of the four task conclusions to better understand their discursive nature. These latter analyses are only peripherally reported on here.

Results

Within the full corpus of examined task enactments, a strong alignment existed between the set-up phase of the mathematical tasks framework and the activity structure of “setting up” in-class work. This phase was characterized by the teacher addressing the entire class, usually from the front of the room, and directing student attention to the written task (often in a textbook, but also appearing on hand-outs, overhead transparencies, or the board). Examples of teacher talk marking the beginning of this phase are the following: “Take a look at the three triangles on the top of page fifty-eight…”; “OK, the first part of the assignment I’m handing out to you says…”; and “When you get this worksheet, I would expect to see…”. This phase continued with teachers maintaining the floor (Edelsky, 1993) of the interaction as they describe or clarify the written task and communicate what the students will be expected to do. The students during this activity are typically expected to sit quietly, listen, and possibly ask clarifying questions (e.g., “Are we going to turn this in today?”).

There was also a strong alignment between the start of the implementation phase and a shift in activity structure. In particular, the shift from the set-up phase to the implementation phase was cued discursively by the teacher, who indicated that the students should begin working on the task (e.g., “So go ahead and get started”), and by the students, who began working (often after physically rearranging themselves into pairs or small groups). The teacher relinquished the floor and the discourse separated into several smaller interactions (in the case of partner or group work) or relatively few interactions at all (in the case of individual seatwork). Within this activity
structure, the teachers typically circulated the room, listening and observing students as they worked and also talking quietly with students if they had questions or if the teacher had a question for them. Students were expected to be actively engaging with the task and, when appropriate, interacting with one another around the mathematical ideas.

These results show that, as was expected, the set-up phase of the mathematical tasks framework is strongly aligned with the classroom activity structure at that point in the task enactment. Moreover, the beginning of the implementation phase corresponds with a marked shift in activity structure. Further analysis reveals, however, that an additional activity structure shift takes place later in the task enactment that does not correspond with a phase of the mathematical tasks framework. Specifically, an activity structure that may be described as a conclusion phase of the task enactment occurred throughout the corpus. I now turn to the results regarding this phase, first describing general trends before turning to the detailed findings of the four selected task enactments.

The beginning of a task conclusion is generally marked by the teacher’s reclamation of the floor. Physically, teachers who have been circulating the room or working at their desk return to the front of the room. They also may turn on the overhead projector or write something on the board, drawing student attention from their own work back to the front of the room. Discursively, teachers raise their voice and address the entire class, typically communicating that work time on the task is complete, either explicitly (e.g., “OK, it’s almost time to go, so please pass your papers to the left”) or implicitly by indicating that something else is about to happen (e.g., “We’re going to go over a few of them now”). The activity structure is teacher-led (though students may be called on to give extended explanations) but is distinct from the activity of the set-up phase since students are expected to answer questions, share solutions, or ask questions about mathematical ideas (rather than just about classroom procedures or task expectations).

The functions of the task conclusion and the relationship of the conclusion to the written task are also distinct from both the set-up phase and the implementation phase. The full corpus examination revealed that, within the conclusion activity structure, many different instructional functions existed. In many task conclusions, the teacher presented the answers to the written task, either verbally or on the board, and solicited questions from the students over any part that was not clear. Alternatively, teachers called on students to present their answers. In addition to this answer-presenting function, task conclusions often involved the sharing of various solutions. In some cases, this activity went beyond mere sharing into comparing or connecting different strategies. The task conclusion was also a place where the teachers highlighted or summarized main ideas and possibly asked students to reflect back on their work. There were instances of teachers prompting for generalizations of the solution strategies and of teachers using the task conclusion as an opportunity to practice previously developed skills. In a small number of cases, the task conclusion was the site of concept formalization or mathematical conjecture. In other cases (typically at the end of a class period), the task conclusion was brief, consisting only of the teacher asking the students to write their name on their work and “pass it forward.”

This overview of functions of the conclusion phase points to differing relationships between the enactment phases and the written task. In the set-up phase, the teacher is typically clarifying the written task and describing what the students are expected to do in the implementation phase. In other words, the set-up phase consists of talk about what will be happening in the implementation phase. In the implementation phase, the students are actually working on the mathematical task, that is, the work is happening. In the conclusion phase, the teacher is typically leading the class in a consideration of what happened during the implementation phase.

There were cases of students continuing to work solving the written task during the conclusion phase, but generally such activity ceased in favor of sharing, reflecting on, comparing, or connecting solutions.

To provide more insight into the nature of task conclusions, I now turn to a more detailed examination of four task enactments. I focus elsewhere (Otten, 2010) on the mathematical and discursive functions and content of these four enactments, but focus here particularly on the discursive evidence with respect to activity structure pointing to the separability of the task conclusion as a phase.

Mr. Nicks, an eighth grade teacher in an urban school district, enacted a task over the course of two class periods involving the determination of perimeters and areas of various polygons. The set-up phase and implementation phase occurred on the first day of the task enactment. The following day of class began with Mr. Nicks asking five students to write on the front blackboard their work for one of the task items each. Thus the students were no longer working to solve the task items but were beginning the task conclusion. After the students at the board had finished writing and everyone had taken their seats, Mr. Nicks addressed the class, “So you should have a pen out; you should have that green worksheet out.” Mr. Nicks then called out item numbers (e.g., “Number one. Number one comes out to be…”) to focus the students’ attention as they went over the handout. When Mr. Nicks arrived at items that had been written on the board, he explained the solutions, gesturing to the student work, and asked for any questions from the class. The activity structure of this conclusion phase was distinct from the implementation phase in that Mr. Nicks took a leading role from the front of the classroom and the students were expected to be sitting quietly, unless called upon, with their desks arranged in separate rows rather than together in pairs. The task conclusion lasted approximately 20 minutes and 30 seconds, slightly longer than the set up phase and implementation phase combined (or 53% of the entire enacted task). The conclusion phase ended when the students passed in their work and Mr. Nicks asked them to open their textbooks to a new section.

Ms. Doss, a sixth grade teacher in a rural school district using a reform textbook series, enacted a task over the course of three class periods involving the use of fraction strips to determine the filled portions of fundraising thermometers. On the first day, Ms. Doss set up the task and had the students work on it, and on the second day she reprised her set-up and had the students continue to work. Ms. Doss then used the entirety of the third day for the task conclusion. She began the task conclusion by addressing the class, “OK, then get out problem one-point-three from yesterday. (…) You should have problem one-point-three out, ready to go. That’s the one we did with thermometers, OK?” She then said that they would “get started summarizing problem one-point-three.” Ms. Doss continued, “We’re going to work through each of those thermometers.” She then asked for student volunteers to come up to the overhead projector, where the thermometers were displayed, to “explain how they found” the fractions for each thermometer. Ms. Doss pushed not only for the students’ answers but for descriptions of how they arrived at those answered. She also used the task conclusion to emphasize the main ideas of numerators and denominators and what they meant in the context of the task. This conclusion phase was distinct from the implementation phase in that Ms. Doss was positioned near the overhead projector, where the thermometers were displayed, to explain how they found the fractions for each thermometer. Ms. Doss pushed not only for the students’ answers but for descriptions of how they arrived at those answered. She also used the task conclusion to emphasize the main ideas of numerators and denominators and what they meant in the context of the task. This conclusion phase lasted approximately 31 minutes, nearly an entire class period (37% of the entire enacted task). The conclusion phase ended when Ms. Doss asked the students to put their work into their mathematics notebook and take out their homework assignment from the previous day.

Mr. Ewing, a sixth grade teacher from an urban school district, enacted a task over the course of two class periods involving the naming and ordering of decimal numbers. The context of the task was that the students were serving as baseball managers in developing a batting order based on fictional players’ statistics. The task conclusion took place during the latter part of the second day of the task enactment. Mr. Ewing, who had been circulating amongst the groups, returned to the front of the room and gathered whole-class attention by asking the students to move their desks back into rows. He said that they would “take some time and talk about our batting orders.” He further signaled the beginning of the conclusion by turning on the overhead projector and displaying a blank transparency. Mr. Ewing proceeded to solicit batting orders from the groups, recording them on the transparency. He then asked the students which players were easiest to place in the order and which portions of the lineup were open to managerial discretion. The task conclusion lasted approximately 12 minutes and 35 seconds (26% of the entire enacted task). The conclusion phase ended when the dismissal tone sounded and Mr. Ewing asked the students to “pass [their] papers up, please.”

Ms. Tibilar, a seventh grade teacher in the same school district as Ms. Doss, enacted a task over the course of two class periods involving the determination of various triangles with the same area. Relative to the previous three cases in this study, the task conclusion in Ms. Tibilar’s case was more difficult to analyze. The difficulty was not because the task conclusion lacked a clear beginning or ending, but because it contained the set-up and implementation of a sub-task within itself and thus had a complex activity structure. During the second day of the task enactment, Ms. Tibilar began the task conclusion by addressing the class from the front of the room, “OK, so what we have to do then is just go ahead and talk about a couple of them on there, and see if everybody’s looks the same or if we’ve got some different ones.” She continued by stating that she did not want to go over every part of the written task, only to “talk about a couple of things that I think are important from it.” Ms. Tibilar then led a whole-class discussion focusing on the students’ work creating different triangles with an area of 15 square centimeters. The relationship between the (fixed) area and the (variable) perimeter was uncovered and explored. This phase was distinct from the implementation phase because the students were no longer actively working on the written task but were instead discussing the work they had done during the previous class period and using it as a basis for making sense of the mathematical ideas of area and perimeter. The task conclusion ended with Ms. Tibilar evaluating the discussion (“Absolutely amazing discussion”) and directing the students to place their written work in their journals. The task conclusion lasted approximately 60 minutes and 25 seconds (78% of the entire enacted task), though approximately 21 minutes of the task conclusion were spent working on a sub-task. (The sub-task was a refinement of the original task that was designed to answer a concern that had arisen in the conclusion discussion. Since it focused on the same mathematical idea as the broader mathematical task and fed back into the original task conclusion, I have characterized it as a sub-task rather than a separate task.)

**Discussion**

This study examined mathematical task enactments at the middle school level using the sociocultural lens of activity structure (Lemke, 1990) as a means of validating and extending the mathematical tasks framework of Stein and her colleagues (Stein, Grover, & Henningsen, 1996). The set-up phase of task enactments were found to align strongly with a “setting up” activity structure, and the shift to the implementation phase corresponded with a shift in activity structure as the students began working to solve the written task. This analysis revealed, however, an
additional conclusion phase in which the teacher reclaims the discursive floor and leads the class in a “looking back” or “sharing out” of work done during the implementation phase. The phase is also distinguished by its relationship to the task itself which is typically no longer being worked on but instead is being looked back upon, forming a sort of symmetry with the set-up phase wherein the work on the task lies ahead. This identification of the conclusion as a separate phase is corroborated by other research such as that of Shimizu (2006) who found that teachers in countries across the globe engage in the activity of “summing up” the main ideas of their lessons or connecting between mathematical concepts, utilizing public talk for this activity. (It should be noted that Shimizu’s work is on the lesson-level rather than the task-level and does not connect to the mathematical tasks framework.) Figure 2 presents the revised mathematical tasks framework, including the conclusion phase.

![Figure 2. The mathematical tasks framework with a conclusion phase](image)

Although it is clear from an activity structure perspective that the task conclusion is distinct from the implementation, the question remains of whether it is necessary to make this distinction with respect to cognition. I argue that the inclusion of the conclusion phase in the framework will also add value in this respect for at least two reasons. First, the original mathematical tasks framework was deeply concerned with both the thinking processes that the students engaged in and the ways in which those processes characterized the students’ mathematical experiences. As noted by Henningsen and Stein (1997), “the nature of tasks can potentially influence and structure the way students think and can serve to limit or broaden their views of the subject matter with which they are engaged” (p. 525). Thus we must consider not only the processes themselves but how the students’ interpret and reflect upon those processes, forming a notion of what it means to “do mathematics.” It is reasonable to suppose that reflection and characterization of the thinking processes from the implementation phase takes place during the conclusion phase, perhaps not exclusively, but substantially nonetheless.

Second, though space did not allow me to report it here, there were instances in this data corpus of what seemed to the level of cognitive demand shifting in the conclusion phase. For instance, Mr. Ewing’s task enactment mentioned above involved procedures-without-connections during its implementation but descended into nonmathematical activity in the task conclusion as the baseball context became the sole focus rather than the decimal numbers (Otten, 2010). In another task conclusion, the level of cognitive demand could be argued to have descended from procedures-with-connections to procedures-without-connections as the teacher used the conclusion to emphasize only the answers rather than the variety of solution strategies. Alternatively, cognitive demand may increase during the conclusion phase. Another task enactment examined from this data corpus exhibited an implementation phase that seemed to be at the procedures-without-connections level but a conclusion phase that consisted of a rich discussion increasing the cognitive demand to procedures-with-connections. Furthermore, although I did not see it in the course of this study, it is conceivable that a task set-up and
implemented at the unsystematic exploration or procedures-without-connections level of cognitive demand could be modified by a masterful teacher into the doing-mathematics level as the students use their previous work as a basis for conjecturing or generalization in the conclusion phase. Therefore, since one of the primary benefits of the mathematical tasks framework is its capacity to illuminate cognitive demand trajectories, it seems that a fuller understanding of these trajectories would be possible by considering task enactments through to their conclusions.

**Endnote**
This data was collected as part of an NSF grant (#0347906; Herbel-Eisenmann, PI) focusing on mathematics classroom discourse. Any findings or recommendations expressed in this article are those of the author and do not necessarily reflect the views of NSF. I thank Beth Herbel-Eisenmann and the teachers, without whom this study would not have been possible.

**References**


