For centuries investigators have been testing, confirming and disconfirming hypotheses by observation and experimentation. Yet the logic of the process is still far from being understood.

by Wesley C. Salmon

A scientific hypothesis is confirmed, one generally assumes, by finding that it has true consequences. Newton’s gravitational theory, for example, implied that any two material bodies experience a mutual attraction. The theory was confirmed by observations of the behavior of falling objects, the motions of the planets and the ebb and flow of the tides; it received direct experimental confirmation when Henry Cavendish devised a torsion balance with which the attraction between two objects could be demonstrated in the laboratory. Such findings do not verify a scientific generalization conclusively. In spite of the many different experimental confirmations of Newton’s theory, it is no longer considered totally acceptable. Disconfirming evidence turned up, and now Newtonian physics has been superseded by Einsteinian relativity. No matter how securely a theory seems to be established, one cannot be sure it will not suffer a similar fate. The most one can say is that experimental results tend to confirm a theory and that in some cases accumulated confirming evidence elevates a general hypothesis to the status of at least provisional acceptability. For example, the law of the conservation of energy is currently taken, on the basis of much confirming evidence and no compelling disconfirmations, to be a sound scientific generalization.

The foregoing characterization of scientific testing of hypotheses may seem straightforward and free of problems, but it is an oversimplification. In the

PARADOX OF THE RAVENS suggests the subtleties of confirmation theory. If all ravens are black, surely non-black things must be non-ravens. The generalizations are logically equivalent, so that any evidence that confirms one must tend to confirm the other. Hence the observation of a green vase seems to confirm the hypothesis that all ravens are black. Even a black raven finds it strange.
missing details there lurk a host of fundamental difficulties. A good way to expose some of the underlying problems is to consider a series of simple examples, each of which has some puzzling or counterintuitive feature. Bertrand Russell once remarked: "A logical theory may be tested by its capacity for dealing with puzzles, and it is a wholesome plan, in thinking about logic, to stock the mind with as many puzzles as possible, since these serve much the same purpose as is served by experiments in physical science." Although Russell’s statement was intended to apply primarily to deductive logic, in particular as it enters into pure mathematics, I think it is equally apt when applied to the logic of confirmation in the empirical sciences.

Logical puzzles are not new to confirmation theory. Ever since the 1940’s philosophers have been exercising their heads on two very famous ones. The first is Carl G. Hempel’s "paradox of the ravens," which can be described as follows:

Observations of black ravens (in the absence of any observations of ravens of other colors) would normally be taken as confirmation of the generalization “All ravens are black.” This statement is logically equivalent to a second generalization, “All non-black things are non-ravens.” The observation of non-black things that are non-ravens (in the absence of any observations of non-black ravens) would seem to confirm this second generalization. Since the two generalizations are logically equivalent to each other, whatever counts as evidence for one must also count as evidence for the other. Hence the observation of a green raven (a non-black non-raven) appears to be evidence for the hypothesis “All ravens are black.” There seems to be something wrong.

The second puzzle is Nelson Goodman’s "grue-bleen paradox." Two peculiar color terms, "grue" and "bleen," are defined. Consider an arbitrary future point in time $t_n$. For an object existing at any time $t$, we shall say that the object is grue at $t$ if the object is green and the time $t$ is earlier than $t_n$, but if the time $t$ is later than $t_n$ the object must be blue in order to qualify as grue. If we take midnight on December 31, A.D. 2000, as $t_n$, an object that exists during a period that extends into both the 20th and the 21st century is grue during the entire period if it is green during the 20th century but changes to blue at the beginning of the 21st century and remains blue thereafter. “Bleen” is defined analogously:

An object is bleen during the entire time span if it is blue prior to the end of the 20th century and green thereafter. Grue and bleen are strange terms, but they are perfectly well defined.

Now, we would normally say that observations of green emeralds (in the absence of observations of emeralds of other colors) tend to confirm the generalization “All emeralds are green.” Since the present date of writing is earlier than $t_n$ however, each emerald observed to be green is also observed to be grue—at least now—so that these same observations confirm the hypothesis “All emeralds are grue.” What, then, should we predict regarding 21st-century emeralds? Will they be green? Or will they be grue—which is to say blue? (As Henry E. Kyburg, Jr., has remarked, this is one of the most pressing problems in the entire logic of confirmation, since we have only 27 years in which to solve it.)

The usual immediate reaction to this grue-bleen terminology is that it is “positional”: it involves an arbitrary reference to a particular point in time. That certainly is true if we start with ordinary English words. As Goodman has observed, however, if we start with the grue-bleen terminology, then our ordinary color words turn out to be positional. “Green” means “grue prior to $t_n$ but bleen thereafter,” whereas “blue” means “bleen prior to $t_n$ but grue thereafter.” Is there any logical reason for preferring the English terms, or is our preference for them as opposed to Goodman’s grue-bleen terminology merely the result of historical accident?

These two puzzles have been a standard part of confirmation theory for the past 30 years. An enormous literature has grown up around them. By now every theorist believes he has the definitive answer to each of them, but there is conspicuous lack of agreement on what the correct answers are. Instead of adding to this discussion I shall deal with some other puzzles that are less familiar but to my mind no less fundamental.

In setting out to test a hypothesis an investigator uses the hypothesis to predict some phenomenon whose occurrence or nonoccurrence can be ascertained by observation. A general hypothesis by itself, however, entails no observable facts: the hypothesis must be applied to some particular situation whose description constitutes a set of "initial conditions." In order to predict an eclipse, for instance, the astronomer needs to know not only the laws of motion that govern the earth and its natural satellite but also the relative positions of
the earth, the moon and the sun at some particular time; from the laws of motion and the initial conditions together he can deduce the time and place of a total solar eclipse. Often it is necessary to manipulate circumstances in order to achieve a set of initial conditions suitable for the testing of a given hypothesis, that is what is involved in performing an experiment.

A puzzle proposed by Russell is directly pertinent to the kind of experimental scheme I have just outlined. Consider the hypothesis “Pigs have wings.” In conjunction with the observed fact (initial condition) that pork is good to eat, we deduce the consequence—we predict—that some winged creatures are good to eat. When we see that people enjoy eating ducks and turkeys, we observe that the consequence—or prediction—is true; we appear to have a confirmation of the original hypothesis.

By no stretch of the imagination can the verification of such a consequence be taken to lend any weight to the hypothesis in question. If we call such an outcome a “positive instance,” meaning that it agrees with the deduced prediction, we must conclude that positive instances do not necessarily lend the slightest credibility to the hypothesis. This apparently silly example points a profound and significant moral: There is more to scientific confirmation than merely finding true consequences. (This is a point that should be kept firmly in mind when evaluating such work as that of Immanuel Velikovsky on the basis of allegedly true predictions.)

If we reason deductively from the premises “All mammals have hair” and “Whales are mammals” to the conclusion “Whales have hair,” we can be sure that the conclusion will be true if the premises are true (as in fact they are); this is essentially the defining characteristic of valid deductive inference. In deduction, however, it is an elementary logical error (known as “the fallacy of affirming the consequent”) to argue backward from the truth of the conclusion to the truth of the premises. When it comes to scientific induction, on the other hand, it sounds quite respectable to suppose that the observation of hair in the embryonic whale (which is known to be a mammal) is evidence for the generalization that all mammals have hair. This double standard led Morris R. Cohen to the facetious characterization of logic texts as books that are divided into two parts: in the first part [on deduction] the fallacies are explained and in the second part [on induction or scientific method] they are committed (with apparent impunity).

Russell’s pigs-have-wings example shows that instances of the fallacy of affirming the consequent do not, however, automatically qualify as sound scientific confirmations.

The relation of logical entailment has the obvious and important property of transitivity: if A entails B and B entails C, then A entails C. If deducibility of true consequences were the whole story regarding confirmation, then it too would be a transitive relation. If C were to confirm B because it follows from B, and if B were to confirm A because it

“PIGS HAVE WINGS” is the hypothesis. The investigator knows that pork is good to eat. On the basis of the hypothesis he predicts that some winged creatures must be good to eat. Testing the hypothesis, he tries duck, and finding it good to eat, considers the hypothesis confirmed. (The pig, punning in French, knows this true consequence is irrelevant.)
follows from A, then C would have to confirm A, since C would follow from A by transitivity of deductive entailment. Indeed, it might seem intuitively that confirmation is a transitive relation. The following argument was actually cited as an example of sound inductive reasoning in a book published a dozen years ago: "Since, if there was smoke here there was very probably fire here, and if there is soot here there was very probably smoke here, and there is soot here, there was probably fire here." That argument, however, has essentially the same logical structure as the following one: Since it is very probable that any scientist who ever lived is alive today (it has been estimated that 90 percent of all scientists are still alive), and since it is very probable that any organism alive today is a microorganism, then, given that Smith is a scientist, it is likely that he is a microorganism.

This example shows unmistakably that A (being a scientist) can lend confirmation to B (being alive at present), and B can in turn lend weight to C (being a microorganism), whereas A not only fails miserably to confirm C but also is actually incompatible with it. The example illustrates a serious risk involved in thinking about confirmation: the danger of drawing unfounded analogies with deduction. It is easy to assume intuitively that properties of deductive relations apply more or less exactly to the logic of confirmation. That is not the case. Some of the most fundamental properties of deductive relations fail absolutely and completely when one deals with probabilities of confirmation. In order to emphasize the distinction let us confine the unqualified term "confirmed" to the incremental sense and always use a phrase such as "highly confirmed" for the absolute sense. We would say, then, that the special theory of relativity was confirmed by the clock-retardation experiment and that it is highly confirmed by the total body of evidence supporting it.

Although the distinction between these two senses of confirmation is obvious and has long been acknowledged, its implications have not always been clearly recognized. As a matter of fact, the incremental sense has some strange properties that are easily overlooked because they are not shared by the absolute concept. Consider the following example, which, although it is totally fictitious, nevertheless exhibits the logical possibilities.

Jones consults his physician about a respiratory ailment. After a preliminary examination the physician says that he thinks Jones has pneumonia but that he is not sure whether it is bacterial, viral or (a rare possibility) both types together. Further testing is required. Jones is given the prescribed test. The physician tells him the test has confirmed the hypothesis that he has bacterial pneumonia and has also confirmed the hypothesis that he has viral pneumonia but has disconfirmed the hypothesis that he has pneumonitis! Most people would find it understandable if at this point Jones sought the advice of a different physician.

Yet the physician may be on solid ground. Suppose that on the basis of the superficial examination he concludes there is a 96 percent chance that Jones has pneumonia, but he has no indication of whether it is bacterial or viral (assuming those are the only two types). Moreover, he decides there is a 2 percent chance that Jones has both types. Consequently the probability that Jones has bacterial pneumonia is 49 percent and the chance that he has viral pneumonia is also 49 percent. Suppose further that there is a test that quite reliably picks out the rare cases where both types are present together. When the test is administered to Jones, the result is positive, making it 89 percent certain that he has both types. Assume, moreover, that this test seldom comes out wrong for someone who has only one type of pneumonia; that is, if the result is positive and the individual does not have both types, he is very likely to have neither type. In
"ALL" AND "ALMOST ALL" can be very different. Given that all $A$ are $B$ and all $B$ are $e$, it must be true that all $A$ are $e$ (left). However, given that almost all $A$ are $B$ and almost all $B$ are $e$, it may happen that no $A$ are $e$ (right). In the example in the text, although most scientists ($A$) are living things ($B$) and most living things ($B$) are microorganisms ($C$), no scientists at all are microorganisms.

The fanciful example illustrates the possibility that evidence can confirm each of two hypotheses and yet disconfirm their disjunction (the one-or-the-other-or-both combination). In this case the result of the special test increased the probability that Jones had bacterial pneumonia and also increased the probability that he had viral pneumonia, and at the same time the result decreased the probability that he had one or the other—in other words, since there are only the two types, that he had pneumonia!

To see how this is possible we must look at the addition rule for probabilities [see illustration below]. In order to compute the probability for a disjunction of nonexclusive alternatives we add together the probabilities of each of the two alternatives taken separately and then subtract the probability of their joint occurrence. For example, the probability of getting on a draw from a standard bridge deck either an honor card (an ace, a king, a queen, a jack or a 10) or a spade (a spade, say) is equal to the probability of an honor card (20/52) plus the probability of a spade (13/52) minus the probability of a spade honor (5/52). The subtraction is necessary because the spade honors were, so to speak, counted twice: once as honors, once as spades. The answer is 28/52 [see illustration on page 82]. In the pneumonia example the probability of the disjunction decreases whereas the probability of each alternative increases by virtue of the high (89 percent) probability of the joint (or twice-counted) occurrence after the test in contrast with its low (2 percent) probability before the test.

Results such as these are characteristic of incremental confirmation, and they are disconcerting partly because there is a natural tendency to confuse the incremental concept with the absolute. Confirmation in the absolute sense means that a hypothesis has a high probability of being correct. If viral pneumonia is highly confirmed in the absolute sense, then pneumonia is at least as highly confirmed. That is because having pneumonia (of some kind) is a logical consequence of having viral pneumonia, and a basic rule of mathematical probability states that the probability conferred on a logical consequence of any proposition

<table>
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<tr>
<th><strong>ADDITION</strong></th>
<th>$\text{PROBABILITY (A OR B)} = \text{PROBABILITY (A)} + \text{PROBABILITY (B)} - \text{PROBABILITY (A AND B)}$.</th>
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<tr>
<td><strong>LOGICAL CONSEQUENCE</strong></td>
<td>IF $A$ entails $B$, then $\text{PROBABILITY (B)}$ is greater than or equal to $\text{PROBABILITY (A)}$.</td>
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<tr>
<td><strong>MULTIPLICATION</strong></td>
<td>$\text{PROBABILITY (A AND B)} = \text{PROBABILITY (A)} \times \text{PROBABILITY (B, GIVEN A)}$.</td>
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<tr>
<td><strong>BAYES’S THEOREM</strong></td>
<td>$\text{PROBABILITY (A, GIVEN B)} = \frac{\text{PROBABILITY (A)} \times \text{PROBABILITY (B, GIVEN A)}}{\text{PROBABILITY (B)}}$.</td>
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CONCEPT OF PROBABILITY does for empirical confirmation of hypotheses what logical deduction does for mathematical proof. Some of the probability rules referred to in the text are listed. Another, for transitivity, is given in illustration on the opposite page.
is as great as or greater than the probability of the proposition itself. I have not violated that condition in describing Jones's case: the probability of pneumonia is greater than that of viral pneumonia on the basis of the preliminary diagnosis and also on the basis of the special test. Nevertheless, when it comes to incremental confirmation (a change in confirmation), it turns out that new evidence can increase the probability of a proposition and yet decrease the probability of one of its consequences. This is one striking logical contrast between incremental and absolute confirmation.

Even stranger things can happen. Suppose two investigators set out to test a hypothesis. Each goes to his own laboratory to perform an experiment and each achieves a positive finding: a confirmation of the hypothesis. Could it ever happen that although each finding confirms a hypothesis, the conjunction of them disconfirms the same hypothesis? It is logically possible.

Let A and B be two atoms of an imaginary isotope that can decay radioactively in any one of three ways. Given that disintegration has occurred, there is a probability of .7 that an alpha particle was emitted, a probability of .2 that a negative electron was emitted and a probability of .1 that a positive electron (a positron) was emitted. Suppose both of these atoms have just disintegrated and the ejected particles are approaching each other. Consider the hypothesis that they will annihilate on meeting, an event that will occur if and only if one particle is a negative electron and the other is a positron. Given no further information, the probability of annihilation is .2 times .1 plus .1 times .2, or .04. Suppose one physicist finds that atom A has ejected an electron; on the basis of this evidence the probability that annihilation will occur is .1, since that is the probability that B emits a positron. Suppose another physicist finds that atom B has ejected an electron; on the basis of this evidence (but without the evidence obtained by the first physicist) there is likewise a probability of .1 that annihilation will occur. Yet on the basis of both pieces of evidence in conjunction, it is clear that annihilation is impossible.

Each of two pieces of evidence separately confirms the annihilation hypothesis (each raises its probability from .04 to .1). In conjunction, however, they not only disconfirm it, they actually refute it. Could such a thing ever happen in the course of an investigation? In an experiment on the Compton scattering of photons by electrons one physicist might measure the frequency of photons scattered at a particular angle while another measures the energy of the recoiling electrons. Even if each set of measurements confirms the hypothesis of Compton scattering, how can one be sure that taken together they do so? In the case of Compton scattering it happens that the conjunction of the findings does confirm the hypothesis, but this does not follow automatically from the mere fact that separately the findings are confirmations; it depends on more circumstances, including the fact that the conjunction itself is one of the predictions of the theory. The annihilation example shows, nevertheless, that there are broad and basic questions about the legitimacy of supposing that the accumulation of many confirming test results must inevitably enhance the credibility of scientific hypotheses.

So far we have spoken only of confirmation, or of the case in which the outcome of the test is positive. It remains to discuss the negative test result: the case in which the prediction of the hy-
PROBABILITY (PNEUMONIA) = PROBABILITY (BACTERIAL) + PROBABILITY (VIRAL) - PROBABILITY (BOTH), OR

.49 + .49 - .02 = .96

.90 + .90 - .89 = .91

WHAT CONFIRMS each of two hypotheses may at the same time disconfirm their disjunction. After a superficial examination the physician decides there is a 96 percent chance that Jones has pneumonia; he does not know if it is bacterial or viral, but guesses there is only a 2 percent chance that both types are present. The various probabilities are diagrammed by the overlapping bars (left). A test is ordered. The result greatly increases the probability that Jones has bacterial pneumonia and also that he has viral pneumonia (right). The reason lies in the addition of probabilities.

It has often been argued that there is a strong asymmetry between the positive and the negative cases because of a simple fact in logic: While the inference from a true conclusion to the truth of the premises is a case of the fallacy of affirming the consequent, the inference from the falsehood of a conclusion to the falsity of at least one premise is perfectly valid; it is known as *modus tollens*, or denying the consequent. Since a valid deduction is defined as one whose conclusion must be true if its true premises, we can indeed conclude that a valid deductive argument with a false conclusion cannot have premises that are all true. And so it seems that a negative outcome not only disconfirms a hypothesis but also actually refutes it conclusively.

The situation is not quite that simple. As Pierre Duhem has pointed out, in most, if not all, cases auxiliary hypotheses are involved when one sets out to test a scientific hypothesis. In the testing of an astronomical theory telescopes and other instruments are likely to be used, so that the laws of optics and other laws are involved in the effort to establish a connection between a spot on a photographic plate and a celestial body. In a sophisticated science the initial conditions required to test a hypothesis are hardly ever ascertainable by direct observation; auxiliary hypotheses are needed to relate what is actually observed to suitable initial conditions. Moreover, predicted outcomes may be somewhat remote from direct observation, and again auxiliary hypotheses come into play.

The net result of these complications is that the negative outcome of an experimental test of a hypothesis cannot be taken automatically as a refutation of that hypothesis. The negative test result shows only that something is wrong somewhere. It may be that the hypothesis being tested is false or it may be that some auxiliary hypothesis is false. Strictly speaking, the negative result only disproves the conjunction of the auxiliary hypotheses and the hypothesis being tested; it does not refute the tested hypothesis by itself. (As an interesting
historical example, consider the false predictions about the motions of Uranus that emerged from Newtonian mechanics in the 19th century. Instead of refuting Newtonian physics they led to the discovery of Neptune; the negative outcome was attributed to the auxiliary hypothesis or perhaps to the initial conditions themselves. Later, however, irregularities in the orbit of Mercury led not to the discovery of a new planet as predicted on the basis of Newtonian mechanics but rather to the eventual downfall of Newtonian mechanics: the precession of the perihelion of Mercury was a crucial early item of evidence for Einstein’s general theory of relativity.)

In view of Duhem’s fundamental insight what moral about confirmation and disconfirmation should be drawn regarding a negative test outcome? Surely either the hypothesis under test is disconfirmed or the auxiliaries are disconfirmed to some extent; surely neither the auxiliaries nor the main hypothesis can be confirmed by the negative outcome. Both assumptions sound plausible but they are wrong. It is logically possible for an experimental outcome that conclusively refutes a conjunction of two hypotheses to confirm each of them individually. This possibility is closely related to the radioactive-decay example.

Let us assume the same two-atom set-up as in that example: atoms A and B each have three possible decay modes, with the probabilities .7 for the alpha particle, .2 for the negative electron and .1 for the positron. This time it is the annihilation of the two particles that is the observed evidence. We consider the hypothesis that atom A emitted a negative electron. Since we know by virtue of the annihilation that one of the atoms emitted a negative electron, but we do not know which, the hypothesis that it was emitted by A has probability .5. The same goes for the hypothesis that B emitted a negative electron. The fact that annihilation occurred makes it impossible, however, that both of the atoms ejected negative electrons; it therefore refutes the conjunction of the two hypotheses. Nevertheless, this very evidence confirms each hypothesis separately, because in each case it raised the probability from .2 to .5.

Imagine what this kind of thing might mean to scientific methodology. Scientist Smith comes home late at night after a hard day at the laboratory. "How did your work go today, dear?" asks his wife. "You know the Smith hypothesis on which I’ve staked my entire reputation? Well, today I ran an experimental test, and the outcome was negative."

"Oh, dear, what a shame! Does that kill your favorite hypothesis and send your reputation down the drain?"

"Not at all. In order to carry out the test I made use of some auxiliary hypotheses."

"Oh, what a relief—saved by Duhem! Your hypothesis wasn’t refuted after all."

Mrs. Smith breathes a deep sigh.

"Better than that," Smith continues. "I actually confirmed the hypothesis."

"Why, that’s wonderful, dear," replies Mrs. Smith. "You must have found that by rejecting the auxiliary hypothesis you could show the test actually supported your hypothesis. How ingenious!"

"No," Smith continues, "it’s even better. I found I had confirmed the auxiliary as well!"

Such outlandish possibilities may seem to make a shambles of scientific methodology. The fact that actual scientists in actual practice do not get involved in such difficulties may be taken by many to show that confirmation is a matter of scientific intuition and resists all efforts at formalization. I do not believe such a conclusion is justified, and I shall argue my case by offering a parallel in the history of the concept of proof in mathematics.

The idea of mathematical demonstration had emerged by 600 B.C. Thales of Miletus is credited with bringing geometry from Egypt to Greece and in the process transforming it into a mathematical science. Although the Egyptians had applied geometry in surveying, there is no evidence that they actually proved any geometrical theorem; Thales is believed to have proved, among other theorems, that the base angles of an isosceles triangle are equal. By about 300 B.C. Euclid had recast geometry as an axiomatic system in which all theorems are to be deduced from a small number of axioms or postulates. Some elementary portions of deductive logic were developed in antiquity by Aristotle and the Stoic philosophers. Yet it was not until 1879 that Gottlob Frege developed a deductive logic that could begin to be adequate to characterize deduction in mathematics. At the very minimum, then, it took 2,500 years from the time mathematical proof was first employed for logicians to come to any clear understanding of its nature.

Mathematical logic has now flourished for 100 years, and many deep results have been established. The process has not been without its vicissitudes. For example, Russell found a famous contradiction in the very logic on which Frege had tried to base all mathematics. It arose from considering puzzles similar to the famous barber paradox: In a certain town there is a barber who shaves every man who does not shave himself. Who shaves the barber? [see "Paradox," by W. V. Quine; Scientific American, April, 1962]. The fact that working mathematicians were not constantly embroiled in contradictions did not prevent Russell’s paradox from constituting a crisis in the foundations of mathematics. Other astonishing developments, such as Kurt Gödel’s proof of the essential incompleteness of arithmetic [see “Gödel’s Proof,” by Ernest Nagel and James R. Newman; Scientific American, June, 1956], were disquieting to say the least, although they did not exhibit outright inconsistency.
Empirical scientists have been making observations and performing experiments in order to test sophisticated hypotheses ever since the rise of modern science in the 16th and 17th centuries. When it comes to drawing conclusions from the results of these observations and experiments, we are far from having a clear understanding of the kind of reasoning involved. We are now in a situation analogous to that of mathematics during the millenniums in which mathematical proof was used often and with good results while the logic behind it remained basically mysterious. Current work in confirmation theory and inductive logic is attempting to remedy the situation.

There is much difference of opinion as to the best course to follow in trying to deal with the puzzles of confirmation. Two resources seem to me to offer considerable promise of help. The first of these is Bayes's theorem, a simple theorem in the mathematical calculus of probability [see bottom illustration on page 79]. Bayes's theorem is often called the "rule of inverse probability." Given the probability that certain evidence would obtain if a particular hypothesis is true (and given some other probabilities as well), Bayes's theorem enables one to compute the probability that the hypothesis is true given that the aforementioned evidence is found. In at least certain cases it can be used to ascertain the probability that some particular cause was operative, given that a certain effect has occurred.

Bayes's theorem has been widely exploited in recent years by statisticians who called themselves Bayesians, notably the late L. J. Savage. Bayes's theorem contributed to confirmation theory a scheme that seems far more adequate to inference in science than the fallacy of affirming the consequent can ever hope to be. (It holds a key to puzzles arising from the pigs-have-wings and scientist-as-microorganism examples, which were based on an oversimplified notion of scientific confirmation.)

The second resource arises from a clear recognition of the incremental concept of confirmation as opposed to the absolute concept. Incremental confirmation involves change of probability, which is basically a concept of probabilistic relevance. The way is thus open to defining a measure of relevance based on the mathematical characteristics of probability, by means of which incremental confirmation can be studied in a precise and systematic fashion. Such a measure was defined and elaborated by Rudolf Carnap in 1950, but insufficient attention seems to have been paid to it. The pneumonia puzzle and the two annihilation puzzles were devised by relying on Carnap's treatment of relevance, and further careful attention to a formal concept of incremental confirmation will, I believe, strip such examples of their paradoxical air.

At the present stage of development studies in confirmation theory and inductive logic have produced more paradoxes and puzzles than convincing or widely accepted solutions of fundamental problems. Further work on such puzzles should, however, yield rich insights into the logic of the empirical sciences, much as studies in the foundations of mathematics have paid rich rewards in the understanding of that discipline's logic.

CONJUNCTION of two confirmations may refute a hypothesis. If atom \( A \) or \( B \) decays, the probability is .2 that it emits an electron and .1 that it emits a positron. The probability of annihilation, which will occur if one atom emits an electron and the other a positron, is therefore .2 \( \times \) .1 \( + \) .1 \( \times \) .2, or .04. If scientist \( A \) observes only that atom \( A \) has emitted an electron, he considers that the probability of annihilation is raised to .1 (the probability that atom \( B \) emits a positron). Scientist \( B \), observing only that atom \( B \) has emitted an electron, reports the same increase in probability. The conjunction of both observations (the emission of two electrons), however, means that there can be no annihilation.