Lecture 11

Material Covered in This Lecture:

- Chapter 5, Section 5.1: Sampling Distribution for Counts and Proportions
- Chapter 5, Section 5.2: The Sampling Distribution of a Sample Mean

1. Sample Proportion

   (1). Let \( X \sim B(n, p) \). Then \( p \) is called the population proportion; and \( \hat{p} = \frac{X}{n} \) is called the sample proportion. We use the sample proportion to estimate the population proportion.

   (2). Mean and Standard deviation of the sample proportion \( \hat{p} \):

   \[
   \mu_\hat{p} = p, \quad \sigma_\hat{p} = \sqrt{\frac{p(1-p)}{n}}.
   \]

   **Example 1 (Example 5.8, p.342):** A sample survey asks a nationwide random sample of 2500 adults if they agree or disagree that "I like buying new clothes, but shopping is often frustrating and time-consuming." Suppose that 60% of all adults would agree if asked this question.

   (a). What is the probability that the sample proportion who agree is at least 58%? (Using Minitab).

   (b). Find the mean and the standard deviation of the sample proportion.

2. Normal Approximation for Counts and Proportions

   Let \( X \sim B(n, p) \). If \( np \geq 10, \, n(1-p) \geq 10 \), then
(a). $X$ is approximately $N(np, \sqrt{np(1-p)})$.

(b). $\hat{p}$ is approximately $N\left(p, \frac{p(1-p)}{n}\right)$.

**Example 2. (Example 5.10, p.345):** Find $P(\hat{p} \geq 0.58)$.

**Example 3. (Example 5.3, p.337; Example 5.11, p.345):** The financial records of businesses may be audited by state tax authorities to test compliance with tax laws. It is too time consuming to examine, for example, all sales and purchases made by a company during the period covered by the audit. The auditor will examine samples, sometimes chosen by judgment but now often chosen at random. Random sampling is fast and easy when records are kept in electronic form.

An audit examines an SRS (Simple Random Sample)\(^1\) of 150 sales records out of 10,000 available. One issue is whether each sale was correctly classified as subject to state sales tax or not. Suppose that in fact 800 out of the 10,000 sales are incorrectly classified.

Let $X$ by the number of bad records in the sample. Find $P(X \leq 10)$.

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3. **The Sampling Distribution of a Sample Mean**

Let $X_1, X_2, \ldots, X_n$ be a sample from a population $X$ with mean $\mu$ and standard deviation $\sigma$. Let $\bar{X}$ be the sample mean.

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\(^{1}\) Simple Random Sample (SRS): A simple random sample of size $n$ consists of $n$ individuals from the population chosen in such a way that every set of $n$ individuals has an equal chance to be the sample actually selected.
(a). $\overline{X}$ is a random variable.

(b). Mean and Standard Deviation of the sample mean.

$$\mu_{\overline{X}} = \mu, \quad \sigma_{\overline{X}} = \frac{\sigma}{\sqrt{n}}.$$ 

(c). The distribution of the sample mean.

- If the sample is from the Normal distribution $N(\mu, \sigma)$, then $\overline{X} \sim N\left(\mu, \frac{\sigma}{\sqrt{n}}\right)$.

- (Central Limit Theorem) If the sample is NOT from the Normal distribution, then the sample mean $\overline{X}$ is approximately normal $N\left(\mu, \frac{\sigma}{\sqrt{n}}\right)$.

**Example 4 (Example 5.18, p.364):** The time $X$ that a technician requires to perform preventive maintenance on an air-conditioning unit is governed by the exponential distribution whose density curve appears below. The exponential distribution has $\mu = 1$, and $\sigma = 1$.

![Density Curve of E(1)](image)

The company operates 70 of these units. What is the probability that their average maintenance time exceeds 50 minutes.