Lecture 7

Two special discrete distributions

I. Bernoulli Trials

- Bernoulli Trials
  1. Each trial has only 2 outcomes: we call them success and failure
  2. For each trial, the probability of success $P(S)$ is the same and denoted by $p=P(S)$. The probability of failure is then $P(F)=1-p$ for each trial and is denoted by $q$, so that $p+q=1$.
  3. Trials are independent.

Example: Tossing a fair coin.

- The distribution of a Bernoulli r. v. is

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x)$</td>
<td>1-p</td>
<td>p</td>
</tr>
</tbody>
</table>

Here $X=1$ is called success; $X=0$ is called failure.

- $E(X)=p$, $Var(X)=p(1-p)$.

Example 7: Roll a die, we are interested in getting a 6, so getting a 6 is a success.

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x)$</td>
<td>5/6</td>
<td>1/6</td>
</tr>
</tbody>
</table>

$E(X)=1/6$, $Var(X)=(5/6)(1/6)=5/36$.

II. Binomial Distribution

Example 1: Roll a fair die 5 times. Let $X$ stands for the no. of 6 showed up.

Question: What is the probability distribution of $X$?

Let $S$ stand for “Success” (6 shows up). “F” for failure (6 not showing up).

- Binomial model:
  An experiment is called a Binomial Model $B(n,p)$, if
  1) A fixed number of trails, say $n$, interested in number of one type of outcome in $n$ trial.
  2) Each trial has two possible outcomes, success and failure.
  3) Trials are independent; probability of each outcomes doesn’t change from trial to trial. The probability of success is $p$.

Suppose the number of trials is $n$. Let $X=$the number of success. Then
\[ f(x) = P(X = x) = \binom{n}{x} p^x (1-p)^{n-x} \]

Here

\[ \binom{n}{x} = \frac{n!}{x!(n-x)!} \]

Example of factorials:

\[ 5! = \]
\[ 8! = \]
\[ 100! = \]

Example: Rolling a die 5 times.
Let X stands for the no. of 6 showed up. Find the probability distribution of X.

- **Mean, variance and standard deviation of Binomial distribution** are given by

\[ \mu = np , \sigma^2 = npq , \sigma = \sqrt{npq} \]

**Example 7:** Exercise 5.65 (P231)

For the binomial distribution with \( n=4, p=0.45 \), find the probability of
(a). Three or more successes.
(b). At most three success.
(c). Two or more failures.

**Example 8:** Example 16 (P230). \( n=3, p=0.5 \).
• How to use the Binomial Table

**Example 9:** Example 15(P228)
In Class Exercises

1. For the following probability distribution

<table>
<thead>
<tr>
<th>x</th>
<th>f(x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>.3</td>
</tr>
<tr>
<td>2</td>
<td>.3</td>
</tr>
<tr>
<td>3</td>
<td>.25</td>
</tr>
<tr>
<td>5</td>
<td>.15</td>
</tr>
</tbody>
</table>

a) Find E(X)

b) Find Var(X)

c) Find sd(X)

2. A box contains 10 red balls and 5 green balls.

a) Draw a ball randomly. Is this a Bernoulli experiment?

b) Consider four draws with replacement, find the probability that

1) All four trials result in red balls

2) All are green balls

3) There is at least one red ball.

4) There are exactly two red balls.
3. A box contains 7 red balls and 3 green balls.

a) Draw successively 3 balls with replacement. Is this a Binomial experiment?

b) Draw successively 3 balls without replacement. Is this a Binomial experiment?

c) In a) what is the probability that there are just 2 red balls in the 3 draws?

d) In a) what is the probability that more than 2 red balls were got in the 3 draws?

e) In a) what is the probability that there is no red ball in the 3 draws?

f) Using binomial table to check your results in part c) – part e)