Lecture 8

A random variable is said to be continuous if it takes uncountable infinite number of values within an interval.

If X is continuous, then \( P( X = x ) = 0 \) for every \( x \)

1. Normal Random Variables. The normal random variable is a very important r.v. in our real life. Exam scores, Heights of students, Weights of baseballs are good examples of "normal random variables". Two quantities must be specified when we talk about normal random variable: mean \( \mu \), which indicates the expectation (center) of the r.v., and standard deviation \( \sigma \), which indicates the spreadness of the r.v.  

   \[ X \sim N(\mu, \sigma) \]

2. The characteristics of the normal distribution.

   a. The normal curve is bell shaped.  
   b. The curve is symmetric about \( \mu \).  
   c. \( \sigma \) determines the spread of the curve.  
   d. The area under the whole curve is 1.  
   e. 3\( \sigma \) rules
Example: The SAT scores have a normal distribution with \( \mu = 500, \sigma = 100, (N(500, 100)) \). Suppose you got 600, where do you stand among all students who took the SAT.

3. Standard Normal Distribution: \( N(0,1) \)

4. How to find probability from the standard normal distribution?

   **Draw a picture of the area you want to find.**

   (1). \( P(Z < b) \)

   Go to table directly. The most left column and top row give you the b value, and inside the table are probabilities.

   Example: \( Z \sim N(0,1) \). Find \( P(Z < 0), P(Z < -2.5), P(Z < 2.55) \).

   (2). \( P(Z > a) \)

   \( P(Z > a) = 1 - P(Z < a) \), then use table to find \( P(Z < a) \).

   Example: \( P(Z > 0), P(Z > 1.23), P(Z > -2.55) \)

   (3) \( P(a < Z < b) \)

   \( P(a < Z < b) = P(Z < b) - P(Z < a) \). then use table to find \( P(Z < b) \) and \( P(Z < a) \).

   Example: \( P(0 < Z < 1), P(-1 < Z < 1.454) \)

5. Given probability, how to find the cut point.
For example,
\[ P(Z < z) = 0.5, \text{ then } z = ? \]
\[ P(Z < z) = 0.975, \text{ then } z = ?, \]
\[ P(Z > z) = 0.2346, \text{ then } z = ? \]

6. How to find the probability from a general normal distribution \( N(\mu, \sigma) \).

(1). \( P(X < b) \)

- Compute the z-score of \( b \): 
  \[ z = \frac{b - \mu}{\sigma}, \]
- Go to the table, find the probability \( P(Z < z) \).
- \( P(X < b) = P(Z < z) \).

Example: \( X \sim N(1, 2) \). Find \( P(X < 2) \).

(2). \( P(X > a) \)

- Compute the z-score of \( a \): 
  \[ z = \frac{a - \mu}{\sigma}, \]
- Go to the table, find the probability \( P(Z < z) \).
- \( P(X > a) = 1 - P(Z < z) \). Use the table to find \( P(Z < z) \).

Example: \( X \sim N(1, 2) \). Find \( P(X > 1.23) \).
(3) \( P(a<X<b) \)

- Compute the z-scores of a and b: 
  \[ z_a = \frac{a - \mu}{\sigma} \quad z_b = \frac{b - \mu}{\sigma}, \]
- Go to the table, find the probability \( P(Z<z_a) \) and \( P(Z<z_b) \).
- \( P(a<X<b) = P(Z<z_b) - P(Z<z_a) \), then use table to find \( P(Z<b) \) and \( P(Z<a) \).

Example: \( X \sim N(1,2) \). Find \( P(0<Z<1) \).

*Example (Example 7, P269):* The number of calories in a salad on the lunch menu is normally distributed with mean = 200 and standard deviation 5. Find the probability that the salad you select will contain
  (a) More than 208 calories.
  (b) Between 190 and 200 calories.

Suggested Exercises.

2.103,  Example 9 on page 152,  4.42,  4.56,  4.118,  5.1, 5.12,  5.65,  5.73 (a), (b), 5.89, 5.97,  6.25