Math assignment: Crazy Cakes

Divide each of the "strange cakes" below into two parts with equal area. The two parts need not be congruent.

Note: Lettering is discontinuous because these are just five of the nine figures (A through I) on the "Crazy Cakes for Two" student sheet in Different Shapes, Equal Pieces.
**Sharing homework (Crazy Cakes)**  
(15 minutes)

Have participants return to the same small groups in which they first discussed cases 7–10. The rest of the session will be devoted to crazy cakes. First, small groups will share and discuss their crazy-cakes homework. Then they will view a video and examine the thinking of some fourth-grade students working on these same problems.

For this first 15-minute discussion, tell participants to talk about only the first two diagrams (A and C). They will turn to diagrams D, H, and I when you view the video cases. Suggest that participants continue to think about the actions of decomposing and recomposing as they discuss their work.

As you listen in on small groups, ask people to justify and explain their methods. Ask, “How do you know that will make two parts of equal area?” to help people to think about the reasons behind their approaches. If appropriate for your group, you can introduce such geometric terms as rotation (turn), translation (slide), reflection (flip), and symmetry.

**Video discussion (Crazy Cakes)**  
(45 minutes)

This video segment (15 minutes) presents fourth-grade students explaining their approaches to making equal shares of cakes D, H, and I. Show the video in three parts, stopping for discussion after each one. The discussion in each case should include an explanation of what the student in the video did, and also one or two of the seminar participants’ approaches on their homework. Make available extra copies of the crazy-cakes diagrams so that participants can work through the students’ sometimes complicated approaches.

The first video segment, based on cake D, is very short (1 minute). Ask the participants to describe the student’s approach.

The next video segment, for cake H, runs 6 minutes. When you break for discussion, ask the group to explain the method and what part of it seemed wrong to one student. Use the cake H transparency on the overhead for reference in this discussion.

Be sure to allow enough time to show and discuss the segment for cake I, which lasts 8 minutes and shows the work of two different pairs. During discussion, ask participants to explain the students’ methods by demonstrating with cake I on the overhead.

Wrap up this activity with a more general discussion of the questions “What mathematical ideas do you see in the work of these students?” or “How is what these students are doing related to the idea of composing and decomposing?”
This portion of the video for Session 2 is to be shown in three segments, in which students show how they divided equally the crazy cakes D, H, and I.

**Cake D**

In this first segment (1 minute), Kevin shows where he divides the cake. He explains that the pieces "are congruent. Rotate it and fold it and they'll be the same pieces."
**Cake H**

In this second segment (6 minutes), Aneschka and Kayla describe how they split cake H. Aneschka marks four cuts (first diagram, below) and labels each piece as 1 or 2, for person 1 or person 2.

![First Diagram](image1)

The class doesn’t understand. Kayla, Aneschka’s partner, says there’s a problem because the two triangles labeled 1 are different sizes. At the teacher’s urging, Aneschka starts over. Following the teacher’s suggestion, she shades each person’s portion in different colors (second diagram, above). The class now can see Aneschka’s solution; however, several classmates are concerned that Kayla says she still does not agree.

**Cake I**

The third segment (8 minutes) shows two different approaches to cake I. First Andy and Kevin offer their complex solution.

Andy and Kevin cut off several triangles and move them around to create a symmetric shape which they then divide in half. The first time through, they confuse everyone. The second time, the teacher helps them keep track of their moves, for themselves and for the class, and the class can follow. The teacher congratulates them on their persistence.

![Andy and Kevin](image2)

Then Joanna and Jose offer a second solution. They have the same general strategy—to create a symmetric shape—but they do it in fewer steps. After Joanna and Jose complete their demonstration, the class talks about some vocabulary: *congruence, symmetry, one-line symmetry,* and *two-line symmetry.*
Crazy Cakes for Video Discussion

Copy this image on transparency film for use with an overhead projector.
Crazy Cakes for Video Discussion

Copy this image on transparency film for use with an overhead projector.
Crazy cakes

Phoebe

Grade 4, February

My fourth-grade students just started a fractions unit that uses an area model of fractions. In the introductory activity, students divide paper “crazy cakes” (irregular polygonal shapes) for two people to share equally.* Working with partners, they had to divide each cake and be able to prove that the resulting pieces were halves.

After students had finished dividing a number of cakes, we began a discussion. I picked what I expected was a fairly easy cake to divide and put it on the overhead. Elsa drew a line to show how she split it. I then turned the question to the class, “Can you prove or disprove that these are halves?”

Neil said, “I know they are halves because they are symmetrical. I didn’t actually fold it, but it’s obvious that if I fold it on the line, the two sides would match exactly.”

I remembered seeing Matthew with a different proof, so I asked him to share his. Matthew explained, “I divided it in the same place, but I have a different reason. I split the ‘extra pieces’ off each side to make a

Phoebe

Grade 4, February

square. I knew I could split a square into halves with this line [corner to corner]. Then I just have these two extra pieces that match. One goes with each half."

Matthew's split  Mackenzie's split

Mackenzie said, "I proved it kind of like Matthew, but I split it in a different place. I thought about the square, like Matthew did, and I split it like this [top to bottom]. Then I had the extra pieces on each half."

I summarized what we had so far. "We have two different ways of splitting the cake, and two different ways of proving one of those splits. But I see one way those are all the same: You are matching regions of one half with regions of the other half. Neil exactly matched one half with the other. They are the same size and shape and fit exactly on top of each other. Matthew matched the two halves of the square with each other, and then the two extra pieces. Mackenzie did not split her cake symmetrically, but she still matched regions. She matched the two halves of the square with each other, and then the two extra pieces. If you can match the areas of the two sides, you have halves."

We moved on to another shape, and I asked Marie to show where to cut it. Marie usually lacks confidence in her mathematical ability, often prefacing any comments with, "This is probably wrong, but...." She had obviously enjoyed the crazy cake activity, though, and I had overheard her explaining it clearly to her partner during the work period. She had been excited as she found and proved her solutions. This seemed like a good opportunity for her to share something she was sure of.
Marie drew a single line and told us, “When I did this before, I cut the shape out. When I cut on this line, I could put the two pieces together. They were just the same.” Most students had the same solution, although only a few had actually cut the pieces.

![Marie's split](image)

We worked with several other cake shapes and, as it turned out, all the solutions presented were correct, although it wasn’t always obvious when they were first presented. When students solved these problems for themselves, they found only one solution and then moved on to the next problem. In our discussion, when we saw a variety of solutions for the same cake, students actually said out loud, “I never would have thought of doing it that way.” This also happened when we saw a variety of proofs for the same solution. I sensed that everyone was seeing something (not necessarily the same thing) for the first time. The solutions and the strategies for proving them held a degree of novelty that kept everyone engaged.