Session 1

Finding Halves of Crazy Cakes

Materials

- Scissors (1 per pair)
- Transparency of Student Sheet 1
- Student Sheet 1 (2 per student)
- Glue sticks (1 per pair)
- Crayons or markers
- Rulers (1 per pair)
- Overhead projector

What Happens

Students each divide a set of “crazy cakes” in half so two people sharing them would receive the same amount of cake. Since these shapes cannot be readily divided in half based on lines of symmetry, the activity requires that students look for parts of each shape that are equal in area and use visual relationships to make their divisions. Their work focuses on:

- understanding that equal fractions of a whole have the same area
- understanding that cutting and pasting shapes conserves their area
- finding halves of a shape

Activity

Crazy Cakes for Two

Draw a copy of example A from Student Sheet 1, Crazy Cakes for Two, on the board, or display it on a transparency if you are using an overhead. You can project the image of example A on the board so that you or a student can draw directly on the board. You may want to make several copies of example A by moving the projector and tracing the image on the board. Tell students that they will find different ways to divide shapes into equal parts in this investigation as a way of learning about fractions such as halves and fourths. You can begin this way:

If this were a cake you had to share evenly between two people, how could you cut it? You need to be able to explain how you know that each person would get the same amount of cake.

Draw or have students draw their solutions on the board. For each idea, ask students to tell you how they know each person would get the same amount of cake. Welcome students’ questions and challenges as part of this introduction. Your job is to help students look at the area of each of their “halves” and determine whether they are in fact equal and to help them verbalize the strategies they are using for finding these solutions.

Hand out Student Sheet 1, Crazy Cakes for Two. Encourage students to use cutting, folding, or any other means that helps them prove to themselves and to one another that they have made halves for example A. See the Teacher Note, Strategies for Dividing Crazy Cakes (p. 6).
Dividing Shapes in Half  Students work in pairs to find ways to divide fairly each of the crazy cakes on Student Sheet 1. Point out to students that Crazy Cake I is more difficult than the rest. If students would like to have something to show their families as they begin a new unit, each student can create a finished version of Student Sheet 1 that shows with pencils or markers how to "cut" each cake. Make available extra copies of Student Sheet 1 in case they want to cut out the shapes to help them find and prove halves.

Visit student groups as they work and ask them to show you how they know both people will get the same amount of cake. They must convince you the two parts are equal. If a student thinks "there's no way" to divide a given shape in half, ask what it is about the shape that makes it "impossible." This can help students think about the areas of parts of the shape in relation to each other.

Proving Halves Are Equal  Once students have had enough time to create "fair shares" of most of the examples on Student Sheet 1, gather the whole class for a discussion of their solutions or, if you prefer, suggest that two or three pairs get together to compare their work and explain their strategies. There are two main goals for this discussion: for students to talk about the thinking that went into their solutions, and for students to see the variety of strategies they can use to help them compare the area in each of their halves for a given shape. If students compare their work in small groups, bring the whole class together briefly to share strategies they learned from one another and to discuss any unresolved disputes. For the whole-class discussion, use an overhead projector or a chalkboard so students can diagram their halves for the class. See the sample strategies and discussion in the Dialogue Box, How Do You Know It's Half? (p. 7).

Don't be too concerned about getting through each example on Student Sheet 1 during the discussion. It is more important for students to learn strategies that work in a variety of situations than to learn a "right" way of dividing each particular shape. Take whatever time is needed for students to communicate the different strategies they used and to try out strategies they have learned from others. You might start the discussion by asking a student to show a strategy learned from someone else. Emphasize matching areas, not accuracy in drawing lines or cutting out shapes.

---

Session 1 Follow-Up

Inventing Crazy Cakes  Students make up their own Crazy Cakes with which they can challenge one another. Before asking another student to cut their cakes into halves, students must be able to do so themselves; they should be able to make halves with a straight line.
The crazy cakes on Student Sheet 1 will initially challenge many students, but there are ways to approach these examples that will help students build their understanding of area. Example A, for example, can be divided into halves fairly in several ways by focusing on the "bump" and the indentation.

Two correct divisions of example A

An incorrect division

In the solutions that work, the bump and the indentation (which are the same area) are in the same half. Some students may need to cut out the "bump" and put it in the indentation to convince themselves that they are the same size.

Other general strategies that students might use for dividing crazy cakes are the following:

- When a shape is not symmetrical, it is useful to cut off and move part of the shape to make the whole figure symmetrical before dividing the shape in half (this could have been done in example D). This is commonly referred to as "cut and paste," shown below for example H.

- A more unusual method is to fold or cut and paste noncongruent halves so they equal the same shape, as in example B.

To see or to show that two parts of a shape are congruent (the same shape), students may cut out the pieces and flip, turn, and slide them until they are exactly on top of one another, as in this division of example D.
How Do You Know It’s Half?

Dividing crazy cakes in half presents a good opportunity for students to work with area without using any numbers. It is an important basis for the rest of the unit, in which students work with fractional concepts by dividing areas into equivalent pieces. Being able to visualize how two pieces might have the same area even though they look different is an important part of what students do in the following sessions.

Does anyone have another way to “cut” cake A in half?

I see that both Kumiko and Sarah have cut cake B by separating the triangle from the square. Can you explain to us how you know that the triangle and the square are equal pieces?

Kumiko: Rikki said before it looked like a rocket and all it needed was a top, so that gave me an idea. I put the square on top of the triangle. Then I cut off the extra pieces [marked a] and put them in here [marked b].

Sarah, what did you do with cake B?

Sarah: I cut this part off [the triangle marked a] and moved it up to the top [the triangular space marked b].
Crazy Cakes for Two

Divide each of the "strange cakes" below into two equal halves. The two halves do not need to have the same shape.