Connecting Fractions and Number Lines through Length Measurement Models

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Strengthening Tomorrow’s Education in Measurement (STEM) Project

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What is STEM?
Strengthening Tomorrow’s Education in Measurement

Elementary Curriculum Analysis of Spatial Measurement – Length, Area, Volume

Lesson Study Work Elementary Math Methods Courses

Virtual Manipulatives

Professional Development work with 33 facilitators in 23 regions

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What we are going to be doing...

We will explore what happens if we use length as a quantity to enter the study of fractions via partitioning.
What we are going to be doing...

This will be an active session, with opportunities to talk with a small group and the whole room. We encourage questions and thoughts as we go.
What this Session is about

• CCSSM Grade 3 - Understand fraction
  – Understand a fraction $1/b$ as the quantity formed by 1 part when a whole is partitioned into $b$ equal parts; understand a fraction $a/b$ as the quantity formed by $a$ parts of size $1/b$.
  
  – Understand a fraction as a number on the number line; represent fractions on a number line diagram.
Why we are interested in this

• More can be done with students’ knowledge of length to build understandings of fractions

• Length as a quantity has nice affordances for partitioning (the CCSSM’s entry point for fractions)
Our goals for this session...

1. Walk out with a better understanding of one (possibly new) way of thinking about developing fractions from measurement.

2. Consider the benefits and limitations of using lengths to model fractions and their operations.
Some Framing

“If I’m having to remember...then I’m not working on mathematics”

- Dave Hewitt (1999)

Arbitrary and Necessary Part 1:
A way of viewing the mathematics curriculum
Journal: For the Learning of Mathematics 19(3)
Arbitrary and Necessary

Arbitrarily True (conventional)
  – Order of Operations
  – Measurement Conversions

Necessarily True
  – Operations on numbers (including fractions)
Fraction Content Foci in CCSSM

- **Grades 1&2**: Intuitive understanding of fraction
- **Grade 3**: Meaning & representation of fraction
- **Grade 4**: Fraction operations (+, −, x)
- **Grade 5**: Fraction operations (+, −, x, ÷)
- **Grade 6**: Fraction operation (÷)

**For more details on CCSSM fraction standards, check out:**

- [http://msu.edu/~stemproj](http://msu.edu/~stemproj)
- [http://commoncoretools.me/tools/](http://commoncoretools.me/tools/)
- [http://turnonccmath.net/](http://turnonccmath.net/)
Reflection Questions

• What did the length quantity (the strip) help with and what did it make more difficult?

• Does your work change if you presume that you don’t know the rules for the four operations?

• Are there important “breaking points” for this length model?
Your First Task

• Break into groups of 3
• In your groups you will be representing fractions as lengths using the long side of the sentence strip.
Your First Task

• Using a length unit (sentence strip), represent the following quantities:
  – **Yellow**: \(\frac{2}{3}, \frac{4}{3}, \text{ and } 1 \frac{2}{3}\)
  – **Orange/Pink**: \(\frac{3}{4}, \frac{5}{4}, \text{ and } 1 \frac{3}{4}\)
  – **White**: \(\frac{4}{5}, \frac{6}{5}, \text{ and } 1 \frac{4}{5}\)

• Choose one representation to share out

Note: We will see these fractions again in the next few tasks.
Fraction Operations (Addition)

1) Determine the sum as a combined length on your strip(s).
2) Read off the sum as best you can from the strip(s).

A. $\frac{1}{3} + \frac{1}{4}$  
B. $\frac{1}{4} + \frac{1}{5}$

C. $\frac{1}{5} + \frac{1}{3}$  
D. $\frac{1}{5} + \frac{1}{4} + \frac{1}{3}$
Fraction Operations (Multiplication)

1) Determine the product using your strip(s).
2) Read off the product as best you can from the strip(s).

A. $6 \cdot \frac{1}{3}$

B. $1 \frac{1}{2} \cdot \frac{2}{3}$

C. $\frac{3}{4} \cdot 2 \frac{1}{3}$

D. $\frac{2}{3} \cdot \frac{9}{10}$
Fraction Operations (Division)

1) Determine the quotient using your strip(s).
2) Read off the quotient as best you can from the strip(s).

A. \( \frac{6}{5} \div \frac{2}{5} \)  
B. \( \frac{6}{4} \div \frac{2}{4} \)

3) What changes if we replace 4 by 10 in part B?
4) What changes if we replace 6 by 7 in both questions?
Fraction Operations (Division)

Using your length unit, represent the following quotients as lengths based on the unit:

1) Determine the quotient using your strip.
2) Read off the quotient as best you can from the strip.

C. $\frac{7}{5} \div \frac{1}{10}$  
D. $\frac{7}{10} \div \frac{1}{20}$
Reflection Questions

• What did the length quantity (the strip) help with and what did it make more difficult?

• Does your work change if you presume that you don’t know the rules for the four operations?

• Are there important breaking points for this model?

• Would you use this approach with teachers or choose a different approach?

• Would more choice about using different manipulatives be good or not?
We want to thank the National Science Foundation for funding this work.

Thank you for coming!

For more information: http://www.msu.edu/~stemproj

If you have any questions please e-mail us at: stemproj@msu.edu