1. Introduction

When faced with a gap in mean outcomes between two groups, researchers frequently examine how much of the gap can be explained by differences in observable characteristics. A common approach to distinguish between explained and unexplained components follows the seminal papers of Oaxaca (1973) and Blinder (1973), with the original “Oaxaca–Blinder” (O–B) decomposition based on separate linear regressions for the two groups. Letting $d$ be an indicator variable for group membership, $\bar{y}$ be the scalar outcome of interest for a member of group $\bar{1}$, $X^d$ be a row vector of observable characteristics (including a constant), $\hat{\beta}^d$ be the column vector of coefficients from a linear regression of $y^d$ on $X^d$, and overbars denote means, it is straightforward to show that

$$\bar{y} - y^0 = (\bar{X}^1 - \bar{X}^0)\hat{\beta}^1 + \bar{X}^0(\bar{\hat{\beta}}^1 - \hat{\beta}^0).$$

(1)

In this expression, the first and second terms on the right hand side represent the explained and unexplained components of the difference in mean outcomes, respectively.

Both seminal articles pointed out that the decomposition in Eq. (1) is not unique because an equally compelling alternative decomposition exists:

$$\bar{y} - y^0 = (\bar{X}^1 - \bar{X}^0)\hat{\beta}^0 + \bar{X}^1(\bar{\hat{\beta}}^1 - \hat{\beta}^0).$$

(2)

We analyze four methods to measure unexplained gaps in mean outcomes: three decompositions based on the seminal work of Oaxaca (1973) and Blinder (1973) and an approach involving a seemingly naïve regression that includes a group indicator variable. Our analysis yields two principal findings. We show that the coefficient on a group indicator variable from an OLS regression is an attractive approach for obtaining a single measure of the unexplained gap. We also show that a commonly-used pooling decomposition systematically overstates the contribution of observable characteristics to mean outcome differences when compared to OLS regression, therefore understating unexplained differences. We then provide three empirical examples that explore the practical importance of our analytic results.

While the first term on the right hand side of Eq. (2) is still interpreted as the explained component, this alternative calculation generally will yield different values from Eq. (1), and there is often little reason to prefer one to the other. Many papers acknowledge this ambiguity by simply reporting both decompositions.

Several papers have proposed alternative O–B decompositions, with perhaps the most widely adopted alternative proposed by Neumark (1988). That paper develops a decomposition based on a pooled regression without group-specific intercepts. It is important to emphasize that Neumark (1988) does not analyze the measurement issue of whether his pooled decomposition or those based on Eqs. (1) and (2) distinguish between explained and unexplained gaps. Rather, he analyzes what fraction of an unexplained wage gap, already purged of productivity differences, represents discrimination, demonstrating that different assumptions regarding employer behavior can lead to each of the three decompositions. Despite this difference in motivation, the pooled decomposition he proposed has been adopted...
as the primary approach to measure explained and unexplained gaps in a number of empirical studies.3

Researchers also routinely use an even simpler approach to measure unexplained gaps. They estimate the pooled regression including an indicator variable for group membership as well as the other observable characteristics, interpreting the coefficient on the group indicator as the unexplained component. For example, this method has been applied to the measurement of union wage premiums (e.g., Lewis, 1986), racial test score gaps (e.g., Fryer and Levitt, 2004), and racial wage gaps (e.g., Neal and Johnson, 1996).

In this paper, we compare these various methods for assessing the unexplained gap in mean outcomes between two groups. Our analysis yields two principal findings. First, we show that the coefficient on the group indicator from a pooled OLS regression is an attractive approach for obtaining a single measure of the unexplained gap. Second, we show that the pooled O–B strategy systematically overstates the role of observables in explaining mean outcome as compared to OLS with a group indicator, thereby understating unexplained differences.4 The intuition for this result is straightforward: the pooled regression coefficients on observable covariates are biased due to the omission of group–specific intercepts, which in turn causes the role of observables to be overstated. We then provide three empirical examples that explore the practical importance of our analytic results, two based on wage gaps and one based on test score gaps.

2. The relationship among four measures of the unexplained gap

As in the Introduction, let y be the scalar outcome of interest, d be an indicator variable equal to 1 for an individual in group 1 and 0 otherwise, X be the vector of observable characteristics (including a constant but not d), and overbars denote means. We study four different measures of the unexplained gap in y between groups 0 and 1. The first two measures come from the standard O–B decompositions listed in Eqs. (1) and (2): define Gap1 to be \( \bar{y} \) – \( \bar{X} \bar{\beta} \), the final term in Eq. (1), and similarly define Gap0 to be the final term in Eq. (2). The third measure, Gap*, is the unexplained component from Neumark’s (1988) proposed decomposition,

\[
\bar{y} = \bar{X} \hat{\beta} + \bar{\epsilon}, \tag{3}
\]

where \( \bar{\beta} \) is defined to be the coefficient vector from the pooled regression of \( y \) on \( X \). The first term on the right hand side of Eq. (3) is again interpreted as the explained component, and the sum of the final two terms is the unexplained component, Gap*. If y denotes a wage, for example, then these two terms correspond to each group’s advantage or disadvantage relative to the pooled wage structure. Finally, the fourth unexplained gap measure, GapOLS, is the coefficient on d from the pooled OLS regression of \( y \) on \( d \) and \( X \). We compare these gaps by specifying a population data generating process and then deriving what each of the gaps measure.

2.1. The case in which coefficients are equal across groups

We begin by assuming that the mean outcomes between groups 0 and 1 differ only by a constant and that the outcome is influenced by only one observable characteristic x. These assumptions simplify the exposition substantially, but as we describe below, all of the results in this section extend to the case in which the outcome depends on a vector of characteristics X. We relax the assumption of equal coefficients across groups in the next subsection.

Specifically, suppose the population relationship between \( y, d \), and \( x \) is

\[
y = \alpha_0 + \alpha_d d + \alpha_x x + \epsilon, \tag{4}
\]

with \( \epsilon \) orthogonal to \( d \) and to \( x \) conditional on \( d \). Under these strong assumptions, a sensible definition of the population unexplained gap is \( \hat{\alpha}_d \). Moreover, under these assumptions, the probability limit of GapOLS is \( \hat{\alpha}_d \).

To derive probability limits of the other estimates of the unexplained gap, we introduce some additional notation. An O–B unexplained gap can always be written as the difference in overall mean outcomes minus the difference in predicted mean outcomes, and both of these differences can be denoted by linear projections. Letting \( b(z|w) \) denote the slope from a linear projection of \( z \) on \( w \) and a constant, a general expression for an O–B unexplained gap is

\[
\text{Gap} = \left[ \bar{y}_1 - \bar{y}_0 \right] - \left[ \bar{y}_1 - \bar{y}_0 \right] \hat{\alpha}_d \tag{5}
\]

\[
= b(y|d) - b(\hat{\theta}x|d),
\]

where \( \hat{\theta} \) is a coefficient computed from sample data. The choice of \( \hat{\theta} \) is what distinguishes different O–B decompositions from each other. For example, \( \text{Gap}_1 \) is obtained when \( \hat{\theta} \) is the OLS slope coefficient from a regression of \( y \) on \( x \) and a constant using data from group 1, while \( \text{Gap}_0 \) is obtained when \( \hat{\theta} \) is the OLS slope coefficient using data from group 0.

Consider the probability limit of an O–B gap under the data generating process described by Eq. (4):

\[
\text{plim Gap} = \text{plim} \left[ b(y|d) - b(\hat{\theta}x|d) \right]
\]

\[
= \frac{\text{cov}(d, y) - \text{cov}(d, x) \text{plim} \hat{\theta}}{\text{var}(d)} \tag{6}
\]

\[
= \frac{\text{cov}(d, \alpha_0 + \alpha_d d + \alpha_x x + \epsilon) - \text{cov}(d, x) \text{plim} \hat{\theta}}{\text{var}(d)}
\]

\[
= \alpha_d + \frac{\text{cov}(d, x) \text{plim} \hat{\theta}_{\alpha_x} - \text{cov}(d, x) \text{plim} \hat{\theta}_{\alpha_x}}{\text{var}(d)}.
\]

Thus, the estimated gap converges to \( \alpha_d \) whenever \( \text{plim} \hat{\theta} = \alpha_x \). Because \( \text{plim} \hat{\theta} = \alpha_x \), in both group–specific regressions, the probability limits of Gap0 and Gap1 are \( \alpha_d \), implying that Gap0, Gap1, and GapOLS are asymptotically equivalent.

In contrast, Gap* generally will not converge to \( \alpha_d \). The difference arises because, in a pooled regression that does not include the

---


4 To our knowledge, the only references to this issue in the literature are in Fortin (2006) and Jann (2008). Both papers discuss the idea that omitting a group indicator can transfer some of the unexplained gap to the explained portion. However, neither paper develops a general expression for the bias. In addition, both discuss the issue in a context where the group with better outcomes also has larger characteristics positively associated with outcome (e.g., males having more education in a model of male–female wage gaps). It has not previously been shown that the bias relative to pooled OLS with a group indicator exists whenever the groups have different mean characteristics (i.e., whenever O–B decompositions are used).

5 In regressions with the scalar x, the constant will be denoted separately, while in the more general case X will denote a vector of characteristics including a constant.
group-specific intercept \( d \), \( \text{plim} \hat{\theta} \) typically does not equal \( \theta \), due to omitted variables bias. To see this formally, consider the probability limit of \( \text{Gap}^p \).

\[
\text{plim} \text{Gap}^p = \text{plim} b(y|d) - b(xb(y|x)|d)
\]

\[
= \frac{\text{cov}(d,y)}{\text{var}(d)} - \frac{\text{cov}(d,x\text{cov}(x,y) / \text{var}(x))}{\text{var}(d)}
\]

\[
= \frac{\text{cov}(d,y)}{\text{var}(d)} - \frac{\text{cov}(d,x)}{\text{var}(d)} \times \frac{\text{cov}(x,y)}{\text{var}(x)}
\]

\[
= \frac{1}{\text{var}(d)} \left( \frac{\text{cov}(d,y)}{\text{var}(y)} - \frac{\text{cov}(d,x)\text{cov}(x,y)}{\text{var}(y)} \right).
\]

It is useful to compare this expression to an alternative representation of the probability limit of \( \text{Gap}^{OLS} \). Defining \( \bar{z}(w) \) to be the component of \( z \) that is orthogonal to \( w \) in the population (so that \( \bar{z}(w) = z - wb(z|w) \)), then

\[
\text{plim} \text{Gap}^{OLS} = \text{plim} \hat{\theta}_d
\]

\[
= \frac{\text{cov}(\hat{d}(x), \hat{y}(x))}{\text{var}(\hat{d}(x))}
\]

\[
= \frac{\text{cov}(d, \hat{y}(x))}{\text{var}(d)(x))} - \frac{\text{cov}(x, \hat{y}(x))}{\text{var}(d)(x))} \times \frac{\text{cov}(d, x)}{\text{var}(d)}
\]

\[
= \frac{1}{\text{var}(\hat{d}(x))} \left( \frac{\text{cov}(d, y) - \text{cov}(d, x)\text{cov}(x, y)}{\text{var}(y)} \right).
\]

where the fourth equality follows because \( \text{cov}(x, \hat{y}(x)) = 0 \) by the definition of \( \hat{y}(x) \). Comparing Eqs. (7) and (8),

\[
\text{plim} \text{Gap}^p = \frac{\text{var}(\hat{d}(x))}{\text{var}(d)} \text{plim} \text{Gap}^{OLS}.
\]

The intuition for this result is straightforward. The omission of \( d \) from a pooled regression leads to omitted variables bias in the estimated coefficient on \( x \). Because the coefficient on \( x \) captures both the direct effect of \( x \) on \( y \) and the effect of \( d \) on \( y \) indirectly through the correlation between \( d \) and \( x \), it tends to explain “too much” of the gap in outcomes, leading the unexplained gap to be too small. We illustrate this effect in Fig. 1 for the case in which \( y^1 > y^0, \theta^1 > \theta^0, \) and \( \delta_d > 0 \). The total gap in mean outcomes is \( y^1 - y^0 \), and based on the group 1 regression line (the top line in the figure), the explained gap is \( y^1 - \bar{A} \) and the unexplained gap is \( \bar{A} - y^0 \). Note that the steepness of the line determines the magnitudes of the explained and unexplained gaps, so \( \text{Gap}^1 \) and \( \text{Gap}^0 \) are identical because the group 1 and group 0 lines are parallel. In contrast, the regression line for the pooled regression (denoted as the dashed line in the figure) must be steeper than either group line due to omitted variables bias. As a result, \( \text{Gap}^p \) must be less than the other three unexplained gap measures.\(^6\)

\(^6\) Neumark (1988), p. 293, makes a similar point about the case illustrated in Fig. 1. We note, however, that our finding that \( \text{Gap}^p \) is smaller in absolute value than \( \text{Gap}^{OLS} \) does not require that \( \delta_d > 0 \), or that either measure is bounded between zero and the overall difference in mean outcomes.

Finally, in Appendix A1 we show that the asymptotic relationship given in Eq. (9) is also an exact result that holds in finite samples. Further, although we have assumed \( x \) is a scalar for notional convenience, the relationship between \( \text{Gap}^p \) and \( \text{Gap}^{OLS} \) holds when \( x \) is vector-valued and regardless of whether model (4) is correct: in all circumstances, \( \text{Gap}^p \) is exactly equal to \( \text{Gap}^{OLS} \) multiplied by \( (1-R^2) \) from the auxiliary regression of \( d \) on all observable covariates.\(^7\)

\[^7\] An implication of these results is that, while \( \text{Gap}^{OLS} \) and \( \text{Gap}^p \) will always have the same sign, the sign of the explained component can differ depending on which approach is used. If \( \gamma_0 > 0, \delta_d > 0, \) and \( \theta^1 > \theta^0 \), then \( \gamma_0 > 0 \) will be smaller than \( \text{Gap}^{OLS} \) and the associated explained component will be negative. In this situation, \( \gamma_1 > \gamma_0 \) may be larger than \( \text{Gap}^{OLS} \) multiplied by \( (1-R^2) \) from the auxiliary regression of \( d \) on \( x \). If so, \( \gamma_1 > \gamma_0 \) will be larger than \( \text{Gap}^p \), so that the explained component will be positive. The use of \( \text{Gap}^p \) would therefore imply that observable characteristics explain a positive fraction of an outcome gap, despite the fact that the group with “better” outcomes has “worse” observable characteristics.

2.2. The case in which coefficients vary across groups

The relationship between \( \text{Gap}^p \) and \( \text{Gap}^{OLS} \) presented above is exact and general (see Appendix A1). Thus, in the varying coefficients case, \( \text{Gap}^p \) is still systematically less than \( \text{Gap}^{OLS} \) whenever the averages of observable characteristics differ between the two groups. Turning to the relationship between \( \text{Gap}^{OLS}, \text{Gap}^1 \) and \( \text{Gap}^0 \), we once again begin by assuming that the outcome is influenced by only one observable characteristic \( x \). We assume again that \( \epsilon \) is orthogonal to \( d \) and to \( x \) conditional on \( d \), but now we allow the coefficient on \( x \) to vary between the two groups,

\[
y = \lambda_0 + \lambda_d d + \lambda_x x + \lambda_{dx} dx + \epsilon.
\]

Eq. (6) showed that the probability limit for an O-B unexplained gap based on \( \theta \) can be written as

\[
\text{plim} \text{Gap} = \frac{\text{cov}(d,y)}{\text{var}(d)} - \frac{\text{cov}(d,x)\text{cov}(x,y)}{\text{var}(d)\text{var}(x)} \text{plim} \hat{\theta}.
\]

Based on this expression, it is straightforward to see that

\[
\text{plim} \text{Gap}^1 = \frac{\text{cov}(d,y)}{\text{var}(d)} - \frac{\text{cov}(d,x)\text{cov}(x,y|d=1)}{\text{var}(d)\text{var}(x|d=1)}
\]

and

\[
\text{plim} \text{Gap}^0 = \frac{\text{cov}(d,y)}{\text{var}(d)} - \frac{\text{cov}(d,x)\text{cov}(x,y|d=0)}{\text{var}(d)\text{var}(x|d=0)}
\]
As we show in the Appendix A, \( \text{Gap}^{\text{OLS}} \) is a weighted average of \( \text{Gap}^1 \) and \( \text{Gap}^0 \),
\[
\text{Gap}^{\text{OLS}} = w^1 \text{Gap}^1 + w^0 \text{Gap}^0,
\]
(13)
with the weights given by sample analogs of the following:
\[
w^1 \text{plim} w^1 = \frac{\Pr(d = 1 | \text{var}(x) = d = 1)}{\Pr(d = 1 | \text{var}(x) = d = 1) + \Pr(d = 0 | \text{var}(x) = d = 0)},
\]
(14a)
\[
w^0 \text{plim} w^0 = \frac{\Pr(d = 1 | \text{var}(x) = d = 1) + \Pr(d = 0 | \text{var}(x) = d = 0)}{\Pr(d = 1 | \text{var}(x) = d = 1) + \Pr(d = 0 | \text{var}(x) = d = 0)},
\]
(14b)

It is straightforward to show that these weights are bounded by 0 and 1, implying that \( \text{Gap}^{\text{OLS}} \) is always bounded by \( \text{Gap}^1 \) and \( \text{Gap}^0 \). In addition, the structure of these weights is intuitively appealing, with \( \text{Gap}^{\text{OLS}} \) approaching \( \text{Gap}^1 \) for large values of \( \text{var}(x) \) and \( \text{Gap}^{\text{OLS}} \) approaching \( \text{Gap}^1 \) for large values of \( \text{var}(y) \) and for values of \( \Pr(d = 1) \) close to 1. When \( \text{var}(x) \) does not vary across groups, the weights are the sample analogs of \( \text{Pr}(d = 1) \) and \( \text{Pr}(d = 0) \) so that \( \text{Gap}^{\text{OLS}} \) is simply the group-size weighted average of \( \text{Gap}^1 \) and \( \text{Gap}^1 \), identical to the O–B decomposition proposed by Cotton (1988).

Although Eqs. (14a) and (14b) hold exactly when \( x \) is a scalar, they are only suggestive of the relationships among the gaps when \( x \) is a vector of characteristics. In the vector case, \( \text{Gap}^{\text{OLS}} \) is not necessarily bounded between \( \text{Gap}^0 \) and \( \text{Gap}^1 \). However, the relationship among the three measures depends on similar quantities as in the scalar case, making it unlikely that \( \text{Gap}^{\text{OLS}} \) will deviate far from the bounds; the extent it deviates from the bounds is an empirical issue that we return to below. In the vector case, \( \text{Gap}^{\text{OLS}} \) will still tend to \( \text{Gap}^0 \) or \( \text{Gap}^1 \) depending on the relative group sample size and the variance of observables. To see why, recall that Eq. (5) implies that all three gaps can be written as the total gap minus a gap-specific explained portion, \( \hat{X}_1 - \hat{X}_0 \), with the choice of \( \hat{\beta} \) uniquely determining the gap, \( \text{Gap}^0 \) is based on \( \hat{\beta}_0 \), \( \text{Gap}^1 \) is based on \( \hat{\beta}_1 \), and \( \text{Gap}^{\text{OLS}} \) is based on \( \hat{\beta}^{\text{OLS}} \), the slope coefficient on \( x \) from a pooled regression of \( y \) on \( X \) and \( d \). The coefficient vector \( \hat{\beta}^{\text{OLS}} \) will be closer to \( \hat{\beta}_1 \) (and therefore \( \text{Gap}^{\text{OLS}} \) will be closer to \( \text{Gap}^1 \)) when group 1 is a larger share of the sample and when the variances of the \( X \)’s are larger in group 1 than in group 0. Our empirical results in the next section will demonstrate the extent to which \( \text{Gap}^{\text{OLS}} \) deviates from \( \text{Gap}^1 \) and \( \text{Gap}^1 \) in three different contexts, as well as the extent to which \( \text{Gap}^0 \) deviates from \( \text{Gap}^{\text{OLS}} \).

### 3. Empirical examples

We demonstrate the practical importance of the analytic results shown above by presenting three empirical examples: the male–female wage gap among full-time, full-year workers using Current Population Survey (CPS) data; and the white–black wage gap among full-time, full-year working males using CPS data; and the white–black test score gap among the Kindergarten Cohort (ECLS-K).

First, the two outcomes we first show results for 1985 and 2001, and for the last we present separate results for reading and math test scores.

For both sets of wage gap results, we use a relatively sparse set of regressors, controlling for age, education, and occupation.8 We define full-time, full-year workers as those who are at least 18 years old and are working more than 30 h a week and 40 weeks a year. The hourly wage is measured as annual earnings divided by annual hours, and all models examine the gap in the log hourly wage. For the male–female results, we include all men (group 1) and women (group 0) and control for whether an individual is black. For the white–black results, we only include males who report being black (group 0) or white (group 1). In analyzing white–black test score differentials, we follow the specifications of Fryer and Levitt (2004), who show that seven covariates are sufficient to explain the entire gap in kindergarten test scores between whites and blacks based on \( \text{Gap}^{\text{OLS}} \).

We provide the results in Table 1. For each example, we list the sample size, the total gap between the two groups, the four measures of unexplained gaps discussed in the previous section (\( \text{Gap}^1 \), \( \text{Gap}^0 \), and \( \text{Gap}^{\text{OLS}} \)), and the \( R^2 \) from the auxiliary regression of group status on the other regressors.

<table>
<thead>
<tr>
<th>Table 1: Empirical results.</th>
<th>White–black log wage gap</th>
<th>Male–female log wage gap</th>
<th>White–black test score gap</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>28,163</td>
<td>40,949</td>
<td>48,499</td>
</tr>
<tr>
<td>Total gap</td>
<td>0.254</td>
<td>0.216</td>
<td>0.372</td>
</tr>
<tr>
<td>Share in group 1</td>
<td>0.527</td>
<td>0.902</td>
<td>0.598</td>
</tr>
<tr>
<td>( \text{Gap}^1 )</td>
<td>0.130</td>
<td>0.105</td>
<td>0.346</td>
</tr>
<tr>
<td>( \text{Gap}^0 )</td>
<td>0.126</td>
<td>0.129</td>
<td>0.388</td>
</tr>
<tr>
<td>( \text{Gap}^{\text{OLS}} )</td>
<td>0.127</td>
<td>0.105</td>
<td>0.276</td>
</tr>
<tr>
<td>Auxiliary ( R^2 )</td>
<td>0.034</td>
<td>0.028</td>
<td>0.238</td>
</tr>
</tbody>
</table>

8 We include a quartic in age, 4 education categories (less than high school, high school, some college, and completed college), and 14 occupation categories (the complete “Major Occupation” codes listed in the CPS for these years).

10 Specifically, we include indicators for whether the mother’s age at first birth was over 30 or less than 20, an indicator for whether the mother received WIC payments, a quadratic in the number of books in the home, the child’s birthweight in ounces, and an NCES-created summary measure of the family’s SES. See Fryer and Levitt (2004) for more details on these measures, and see Appendix Tables 1 and 2 for summary statistics for the estimation samples we use.
differential in 1985 and both test score differences), but in the other three it does not. Gap$^f$ is substantially outside the Gap$^1$ and Gap$^0$ estimates for both male–female wage gaps because of the high $R^2$ of the auxiliary regression and the associated attenuation of Gap$^f$ relative to Gap$^{OLS}$.\footnote{Oaxaca and Ransom (1994) found a similar result in their male–female wage example (see their Table 3, column 2), but they did not comment that this result was to be expected.}

As further illustration of the relationships among the four Gap measures, Figs. 2 and 3 show the white–black and male–female wage gaps for each year between 1985 and 2007. In the male–female case shown in Fig. 2, the plots of Gap$^{OLS}$, Gap$^1$, and Gap$^0$ are quite similar. Gap$^f$ is substantially lower in all years, due to the relatively high power of the covariates in explaining group membership, i.e., men and women are substantially different on observable dimensions. In the white–black case shown in Fig. 3, Gap$^f$ is consistently larger than Gap$^1$, but the plots of Gap$^{OLS}$ and Gap$^f$ are essentially identical to Gap$^1$ because the white group represents a large fraction of the population and because the explanatory variables do not predict group membership. In both graphs, Gap$^{OLS}$ lies in between or nearly in between Gap$^1$ and Gap$^f$ for every year.

### 4. Discussion and conclusion

We analyze four methods to measure unexplained gaps in mean outcomes, three based on the decomposition methods of Oaxaca (1973) and Blinder (1973) and one based on a pooled regression with a group indicator variable. Our analysis yields two principal findings. We show that, in the case of a single observable characteristic, the coefficient on the group indicator from a pooled OLS regression is a weighted average of the unexplained gaps from the two standard O–B approaches, with intuitively sensible weights that are bounded between 0 and 1 and sum to 1. The strict bounding result on these weights remains our own, of course. Haider gratefully acknowledges the financial support of the Australian National University as a Gruen Fellow.

To explore the practical significance of our results, we provide empirical examples involving white–black and male–female wage differentials and the white–black kindergarten test score gap. These examples demonstrate that Gap$^{OLS}$ is typically close to the standard Oaxaca–Blinder unexplained gaps but systematically larger than Gap$^f$. Gap$^f$ will deviate from Gap$^{OLS}$ to the extent that there are differences in the means of observable characteristics between the two groups. This deviation can be large enough to drive Gap$^f$ substantially below both standard O–B measures, as is the case with the male–female wage differentials.

Taken together, our analytical and empirical results suggest that the pooling O–B decomposition without a group-specific indicator should not be used to distinguish between explained and unexplained gaps, although this method may be useful to assess how much of an unexplained gap represents discrimination if specific assumptions are met. In contrast, Gap$^{OLS}$ provides an attractive summary approach to separate between-group mean differences into explained and unexplained components.

**Acknowledgment**

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**Appendix A**

### A1. The exact relationship between Gap$^{OLS}$ and Gap$^f$

Consider a sample of observations on $y$, a scalar outcome of interest, $d$, an indicator variable for group membership, and $X$, a vector of observed characteristics. For this appendix section define each to be the vector or matrix of deviations from its respective sample mean. Further, define $P = X(X'X)^{-1}X'$ to be the projection matrix onto $X$ and $M = I - P$ to be its complement. Note that we need make no assumptions about relationships in the population.

Gap$^{OLS}$ is then given by

\[
\text{Gap}^{OLS} = (d'Md)^{-1}(d'My).
\] (A1)

Similar to Eq. (7) in the text, Gap$^f$ can be expressed as the difference of two regression coefficients, one that equals the total gap between the two groups and one that equals the predicted gap, which is constructed using fitted values from a pooled regression of $y$ on $X$.  

**Fig. 2.** Unexplained male–female log wage differentials by year, CPS.

**Fig. 3.** Unexplained black–white log wage differentials by year, CPS.
Therefore,

\[
\text{Gap}^p = (d'd)^{-1}d'y - (d'd)^{-1}d'Py \\
= (d'd)^{-1}d'My \\
= (d'd)^{-1}(d'Md) \times \text{Gap}^{\text{OLS}} \\
= (1 - R_{X\pi}^2) \times \text{Gap}^{\text{OLS}},
\]

where \( R_{X\pi}^2 \) is the \( R^2 \) from the auxiliary regression of \( d \) on \( X \). As a result, \( \text{Gap}^p \) will always be smaller than \( \text{Gap}^{\text{OLS}} \) except when \( d \) is orthogonal to \( X \), which corresponds to the case in which covariates can explain none of the difference across groups in average outcomes.

**A2. The relationship between \( \text{Gap}^p \), \( \text{Gap}^1 \), and \( \text{Gap}^{\text{OLS}} \) in the scalar \( x \) case**

We first derive two expressions that will be useful in the final result. Defining \( \pi = \Pr(d = 1) \), note that

\[
\begin{align*}
\text{var}(x) &= (1 - \pi) \times \text{var}(x|d = 0) + \{E[x|d = 0] - E(x)\}^2 \\
&\quad \times \pi \times \text{var}(x|d = 1) + \{E[x|d = 1] - E(x)\}^2 \\
&= (1 - \pi) \times \text{var}(x|d = 0) + \{E[x|d = 1] - E(x)\}^2 (1 - \pi)^2 \\
&\quad + \pi \times \text{var}(x|d = 1) + \{E[x|d = 1] - E(x)\}^2 (1 - \pi)^2 \\
&= (1 - \pi) \times \text{var}(x|d = 0) + \pi \times \text{var}(x|d = 1) + \frac{\text{cov}(x, d)^2}{\text{var}(d)} \\
&\quad \times [\{1 - \pi\} \text{var}(d) + \{1 - \pi\} \text{var}(d)] \\
&= (1 - \pi) \times \text{var}(x|d = 0) + \pi \times \text{var}(x|d = 1) + \frac{\text{cov}(x, d)^2}{\text{var}(d)}. \quad (A3)
\end{align*}
\]

The first equality is the decomposition of the variance of \( x \) into "within group" and "between group" components, the second equality follows from applying the law of iterated expectations to \( E(x) \), the third follows because \( E(x|d = 1) - E(x|d = 0) = \text{cov}(x, d) / \text{var}(d) \) for any binary variable \( d \), and the fourth because \( \text{var}(d) = \pi \{1 - \pi\} \) for any binary variable \( d \). Similar logic implies that

\[
\text{cov}(x, y) = (1 - \pi) \times \text{cov}(x, y|d = 0) + \pi \times \text{cov}(x, y|d = 1) + \frac{\text{cov}(x, d) \text{cov}(y, d)}{\text{var}(d)}. \quad (A4)
\]

Beginning with the result in Eq. (13), we combine Eqs. (11), (12), (14a), and (14b):

\[
\begin{align*}
\text{plim}[w^1 \text{Gap}^1 + w^0 \text{Gap}^0] &= w^1 \left[ \frac{\text{cov}(d, y)}{\text{var}(d)} \right] \text{cov}(d, x), \text{cov}(y|d = 1) \\
&\quad + w^0 \left[ \frac{\text{cov}(d, y)}{\text{var}(d)} \right], \text{cov}(x|d = 0) \\
&= \frac{\text{cov}(d, y)}{\text{var}(d)} - \frac{\text{cov}(d, x) \text{cov}(y, d)}{\text{var}(d)}, \text{cov}(x|d = 0) \quad (A5)
\end{align*}
\]

where

\[
\begin{align*}
\Pi &= \frac{\pi \times \text{var}(x|d = 1)}{\pi \times \text{var}(x|d = 1) + (1 - \pi) \times \text{var}(x|d = 0)} \times \text{var}(x|d = 1) \\
&\quad + \frac{(1 - \pi) \times \text{var}(x|d = 0)}{\pi \times \text{var}(x|d = 1) + (1 - \pi) \times \text{var}(x|d = 0)} \times \text{var}(x|d = 0).
\end{align*}
\]

Simplifying \( \Pi \),

\[
\begin{align*}
\Pi &= \frac{\pi \times \text{var}(x|d = 1)}{\pi \times \text{var}(x|d = 1) + (1 - \pi) \times \text{var}(x|d = 0)} \times \text{var}(x|d = 1) \\
&\quad + \frac{(1 - \pi) \times \text{var}(x|d = 0)}{\pi \times \text{var}(x|d = 1) + (1 - \pi) \times \text{var}(x|d = 0)} \times \text{var}(x|d = 0). \quad (A6)
\end{align*}
\]

The equality on the last line follows from using Eqs. (A3) and (A4) to simplify the preceding line. As a result,

\[
\begin{align*}
\text{plim}[w^1 \text{Gap}^1 + w^0 \text{Gap}^0] &= \frac{\text{cov}(d, x) \text{cov}(d, y)}{\text{var}(d)} \left[ \frac{\text{cov}(x, y) - \text{cov}(d, x) \text{cov}(y, x)}{\text{var}(x)} \right] \\
&= \frac{\text{cov}(d, x) \text{cov}(y, x)}{\text{var}(d) - \text{cov}(d, x)^2}. \quad (A7)
\end{align*}
\]

Recall from the text that

\[
\text{plim} \text{Gap}^{\text{OLS}} = \frac{1}{\text{var}(d(x))} \left( \frac{\text{cov}(d, y)}{\text{var}(x)} \right). \quad (A8)
\]

Since \( d(x) \) represents the residuals from a population regression of \( d \) on \( x \),

\[
\begin{align*}
\text{var}(d(x)) &= \text{var} \left[ d - \frac{\text{cov}(d, x)}{\text{var}(x)} \right] \\
&= \text{var}(d) - \text{cov}(d, x)^2 / \text{var}(x). \quad (A9)
\end{align*}
\]

This implies that Eq. (A8) can be rewritten as follows:

\[
\begin{align*}
\text{plim} \text{Gap}^{\text{OLS}} &= \frac{1}{\text{var}(d(x))} \left( \frac{\text{cov}(d, y)}{\text{var}(x)} \right) \\
&= \frac{\text{var}(x)}{\text{var}(d(x)) - \text{cov}(d, x)^2} \left( \frac{\text{cov}(d, y) \text{var}(x) - \text{cov}(d, x) \text{cov}(y, x)}{\text{var}(x)} \right) \\
&= \frac{\text{cov}(d, y) \text{var}(x) - \text{cov}(d, x) \text{cov}(y, x)}{\text{var}(x) \text{var}(d(x)) - \text{cov}(d, x)^2}. \quad (A10)
\end{align*}
\]

Comparing Eqs. (A7) and (A10) gives the desired result.

**Appendix Table 1**

Descriptive statistics for CPS.

<table>
<thead>
<tr>
<th>Year</th>
<th>Wage</th>
<th>Black</th>
<th>Female</th>
<th>Age</th>
<th>HS+</th>
</tr>
</thead>
<tbody>
<tr>
<td>1983</td>
<td>45637</td>
<td>8.79</td>
<td>0.08</td>
<td>38.72</td>
<td>0.84</td>
</tr>
<tr>
<td>1984</td>
<td>46196</td>
<td>9.10</td>
<td>0.08</td>
<td>38.61</td>
<td>0.85</td>
</tr>
<tr>
<td>1985</td>
<td>48499</td>
<td>9.62</td>
<td>0.09</td>
<td>38.52</td>
<td>0.85</td>
</tr>
<tr>
<td>1986</td>
<td>48365</td>
<td>10.09</td>
<td>0.09</td>
<td>38.44</td>
<td>0.86</td>
</tr>
<tr>
<td>1987</td>
<td>48402</td>
<td>10.44</td>
<td>0.09</td>
<td>38.47</td>
<td>0.86</td>
</tr>
<tr>
<td>1988</td>
<td>49495</td>
<td>10.80</td>
<td>0.09</td>
<td>38.58</td>
<td>0.87</td>
</tr>
</tbody>
</table>

(continued on next page)
### Appendix Table 1 (continued)

<table>
<thead>
<tr>
<th>Year</th>
<th>N</th>
<th>Wage</th>
<th>Black</th>
<th>Female</th>
<th>Age</th>
<th>HS+</th>
</tr>
</thead>
<tbody>
<tr>
<td>1989</td>
<td>46,741</td>
<td>11.12</td>
<td>0.09</td>
<td>0.41</td>
<td>38.68</td>
<td>0.87</td>
</tr>
<tr>
<td>1990</td>
<td>52,015</td>
<td>11.71</td>
<td>0.09</td>
<td>0.41</td>
<td>38.89</td>
<td>0.88</td>
</tr>
<tr>
<td>1991</td>
<td>51,402</td>
<td>11.99</td>
<td>0.09</td>
<td>0.43</td>
<td>38.89</td>
<td>0.88</td>
</tr>
<tr>
<td>1992</td>
<td>50,018</td>
<td>12.40</td>
<td>0.09</td>
<td>0.43</td>
<td>39.08</td>
<td>0.89</td>
</tr>
<tr>
<td>1993</td>
<td>48,839</td>
<td>13.67</td>
<td>0.09</td>
<td>0.42</td>
<td>39.66</td>
<td>0.90</td>
</tr>
<tr>
<td>1994</td>
<td>43,719</td>
<td>14.55</td>
<td>0.09</td>
<td>0.42</td>
<td>39.86</td>
<td>0.89</td>
</tr>
<tr>
<td>1995</td>
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<td>15.14</td>
<td>0.09</td>
<td>0.42</td>
<td>39.66</td>
<td>0.90</td>
</tr>
<tr>
<td>1996</td>
<td>44,941</td>
<td>15.82</td>
<td>0.09</td>
<td>0.43</td>
<td>40.15</td>
<td>0.90</td>
</tr>
<tr>
<td>1997</td>
<td>46,314</td>
<td>16.41</td>
<td>0.09</td>
<td>0.43</td>
<td>40.27</td>
<td>0.89</td>
</tr>
<tr>
<td>1998</td>
<td>47,551</td>
<td>16.61</td>
<td>0.09</td>
<td>0.43</td>
<td>40.45</td>
<td>0.89</td>
</tr>
<tr>
<td>1999</td>
<td>76,647</td>
<td>18.12</td>
<td>0.11</td>
<td>0.43</td>
<td>40.25</td>
<td>0.90</td>
</tr>
<tr>
<td>2000</td>
<td>75,429</td>
<td>19.02</td>
<td>0.11</td>
<td>0.43</td>
<td>40.63</td>
<td>0.90</td>
</tr>
<tr>
<td>2001</td>
<td>73,809</td>
<td>19.48</td>
<td>0.11</td>
<td>0.43</td>
<td>40.98</td>
<td>0.90</td>
</tr>
<tr>
<td>2002</td>
<td>72,531</td>
<td>19.88</td>
<td>0.11</td>
<td>0.43</td>
<td>41.34</td>
<td>0.90</td>
</tr>
<tr>
<td>2003</td>
<td>71,711</td>
<td>20.39</td>
<td>0.11</td>
<td>0.43</td>
<td>41.40</td>
<td>0.90</td>
</tr>
<tr>
<td>2004</td>
<td>72,170</td>
<td>20.97</td>
<td>0.10</td>
<td>0.43</td>
<td>41.48</td>
<td>0.90</td>
</tr>
<tr>
<td>2005</td>
<td>72,500</td>
<td>21.93</td>
<td>0.11</td>
<td>0.43</td>
<td>41.69</td>
<td>0.90</td>
</tr>
</tbody>
</table>

Note: Entries are unweighted means of the variables listed in the column headings, listed by survey year. Everyone who worked full-time and full-year is included (at least 30 h per week and 40 weeks per year). Wages are in nominal dollars.

### Appendix Table 2

Descriptive statistics for ECLS-K.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Full sample</th>
<th>Blacks</th>
<th>Whites</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>13,040</td>
<td>1708</td>
<td>11,332</td>
</tr>
<tr>
<td>SES composite</td>
<td>0.10</td>
<td>0.0 (0.78)</td>
<td>0.77</td>
</tr>
<tr>
<td># books in home</td>
<td>80.78</td>
<td>40.44</td>
<td>87.09</td>
</tr>
<tr>
<td></td>
<td>(59.70)</td>
<td>(39.97)</td>
<td>(59.82)</td>
</tr>
<tr>
<td>Mother’s age at first birth</td>
<td>24.20</td>
<td>20.93</td>
<td>24.71</td>
</tr>
<tr>
<td></td>
<td>(5.45)</td>
<td>(4.76)</td>
<td>(5.37)</td>
</tr>
<tr>
<td>Child’s birthweight in pounds</td>
<td>7.42</td>
<td>6.97</td>
<td>7.49</td>
</tr>
<tr>
<td></td>
<td>(1.30)</td>
<td>(1.37)</td>
<td>(1.28)</td>
</tr>
<tr>
<td>WIC participation</td>
<td>0.35</td>
<td>0.71</td>
<td>0.29</td>
</tr>
<tr>
<td></td>
<td>(0.48)</td>
<td>(0.45)</td>
<td>(0.46)</td>
</tr>
<tr>
<td>Fall K math</td>
<td>53.64</td>
<td>39.21</td>
<td>55.90</td>
</tr>
<tr>
<td></td>
<td>(28.17)</td>
<td>(25.39)</td>
<td>(27.92)</td>
</tr>
<tr>
<td>Fall K reading</td>
<td>51.78</td>
<td>41.96</td>
<td>53.31</td>
</tr>
<tr>
<td></td>
<td>(28.72)</td>
<td>(26.95)</td>
<td>(28.69)</td>
</tr>
</tbody>
</table>

Note: Cell entries are unweighted means of the variable listed in the row headings, with standard deviations reported in parentheses. The “Full sample” column includes both black and white kindergarten students in ECLS-K. All covariates are measured as described in Fryer and Levitt (2004).

### References


