Homework #2
Due date: 2/19/2013 (Tuesday)

1. What is BOLD? In your own words, fully explain the mechanism of BOLD fMRI (from stimulus input to image voxel signal). (20 points)

   Translate the slice “Two Main Paths” in Lecture “Part II: Neurophysiology and BOLD” to words.

2. What common imaging sequences are used in BOLD fMRI? Why do we normally use gradient-echo EPI instead of spin-echo EPI for BOLD fMRI? (5 points)

   EPI and spiral.

   We need to detect the effect of T2* change due to BOLD. Gradient-echo EPI is sensitive to T2* change but not spin-echo EPI.

3. Describe some common image artifacts found in fMRI EPI studies. (5 points)

   Head motion (Page 271), geometric distortion (Page 278), white pixels due to RF leakage (Page 268), susceptibility artifacts (signal loss) (Page 255), and N/2 ghost (misalignment of EPI images due to the back-and-forth trajectory in EPI k space) (page 149).

4. In your own words, what is linear system? What is impulse response? (10 points)

   Linear system needs to obey:

   If X1 (t) -> Y1 (t), X2(t) -> Y2(t),

   Then,

   aX1(t) -> aY1(t), bX2(t) -> bY2(t).

   aX1(t) + bX2(t) -> aY1(t) + bY2(t).

   X1(t-t0) -> Y1 (t-t0), the time shifts the same amount in input and output.

   Input response is the response due to one impulse (one event).

5. Explain event-related design. In traditional event-related design, we need to separate each event with a reasonable time, please explain why we don’t have to obey this rule in rapid event-related design. (10 points)

   Present one event (stimulus) at a time.
In rapid event-related design, we randomize the order and ISI. With the assumption of linear system, we can run deconvolution to resolve the IRFs.

6. What is the mathematical relationship between block and event-related designs? (5 points)

   The response of a block of stimuli
   = the convolution of impulse response and the block of the stimulus input.

   The height of the plateau of the response due to a block of stimuli = the area under the curve of the impulse response, which is the response due to one event (impulse).

7. In your own words, please explain the equation “$Z = X \beta + \varepsilon$” that has been used in fMRI design and data analysis. (10 points)

   $Z$ = the time course of image signal intensity at one voxel.
   $X$ = the design matrix.
   $\beta$ = baseline + linear trend + other trends if modeled + IRF.
   $\varepsilon$ = error terms used to model noise.

8. In your own words, what is block-related design? How the general equation “$Z = X \beta + \varepsilon$” is modified in block-related design? (5 points)

   Block-related design is characterized by the pattern of blocks of stimulus inputs of ON-OFF-ON-OFF ....

   The “$X$” and “$\beta$” are modified.

   $X$ = the modified design matrix or the reference functions = the original design matrix convolves with an impulse response function model (commonly a gamma function)
   $\beta$ = baseline + linear trend + other trends if modeled + k (the scaling factor comparing to X, instead of a detailed IRF)

9. Please compare the advantages and disadvantages of block and event-related designs. (5 points)

   Block Design
   Advantages: Best for detection, easy to implement the paradigm, relative immune to timing error.
   Disadvantage: Poor in characterizing IRF.

   Traditional (Slow) Event Related Design:
   Advantage: No assumption of linear system,
Zhu: Intro fMRI

able to characterize IRF.
Disadvantages: Time inefficient,
not as good in detection comparing to block-design.

Rapid Event Related Design:
Advantages: more life like,
more time efficient comparing to slow event-related design,
flexible design,
able to characterize IRF.
Disadvantage: Not as good in detection comparing to block-design.

10. How % BOLD signal change is calculated in block-related design? (5 points)

11. During the lecture on Feb 10, we walked through the matrix operation of
“Z = Xβ + ε” with two examples. But we did not have enough time to fill up the
matrices for all time points in the class. In this homework, you need to complete
these two examples. Plot the outputs and explain what the outputs mean. (20
points)

Answer from a student (with minor edits):

**Example 1**

<table>
<thead>
<tr>
<th></th>
<th>x0</th>
<th>x1</th>
<th>x2</th>
<th>x3</th>
<th>x4</th>
<th>x5</th>
<th>x6</th>
<th>x7</th>
<th>x8</th>
<th>x9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Z =</td>
<td>1 0 1 0 0 0 0 0 0 0</td>
<td>1 1 0 1 0 0 0 0 0 0</td>
<td>1 2 0 0 1 0 0 0 0 0</td>
<td>1 3 0 0 0 1 0 0 0 0</td>
<td>1 4 0 0 0 1 0 0 0 0</td>
<td>1 5 0 0 0 0 0 1 0 0</td>
<td>1 6 0 0 0 0 0 0 1 0</td>
<td>1 7 0 0 0 0 0 0 0 0</td>
<td>1 8 0 0 0 0 0 0 0 0</td>
<td>1 9 0 0 0 0 0 0 0 0</td>
</tr>
<tr>
<td>X =</td>
<td>1 1 0 1 0 0 0 0 0 0</td>
<td>1 1 0 1 0 0 0 0 0 0</td>
<td>1 2 0 0 1 0 0 0 0 0</td>
<td>1 3 0 0 0 1 0 0 0 0</td>
<td>1 4 0 0 0 1 0 0 0 0</td>
<td>1 5 0 0 0 0 0 1 0 0</td>
<td>1 6 0 0 0 0 0 0 1 0</td>
<td>1 7 0 0 0 0 0 0 0 0</td>
<td>1 8 0 0 0 0 0 0 0 0</td>
<td>1 9 0 0 0 0 0 0 0 0</td>
</tr>
<tr>
<td>β =</td>
<td>100 (β0)</td>
<td>1 (β1)</td>
<td>0 (h0)</td>
<td>1 (h1)</td>
<td>3 (h2)</td>
<td>4 (h3)</td>
<td>3 (h4)</td>
<td>-1 (h5)</td>
<td>0 (h6)</td>
<td></td>
</tr>
</tbody>
</table>
\[ Z = X\beta + \varepsilon \quad \rightarrow \varepsilon = (\varepsilon_0, \varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4, \varepsilon_5, \varepsilon_6, \varepsilon_7, \varepsilon_8, \varepsilon_9) \]

\[ \hat{Z} \text{ (estimated)} = X\beta = \]

\[
\begin{array}{l}
100 = 100 + 0 + 0 \\
102 = 100 + 1 + 1 \\
105 = 100 + 2 + 3 \\
107 = 100 + 3 + 4 \\
107 = 100 + 4 + 3 \\
104 = 100 + 5 + (-1) \\
106 = 100 + 6 + 0 \\
107 = 100 + 7 + 0 \\
\text{(return to baseline)} \\
108 = 100 + 8 + 0 \\
109 = 100 + 9 + 0 \\
\end{array}
\]

**Time & Signal Intensity**

\[ Z = X\beta + \varepsilon \quad \rightarrow \varepsilon = (\varepsilon_0, \varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4, \varepsilon_5, \varepsilon_6, \varepsilon_7, \varepsilon_8, \varepsilon_9) \]

\[ \hat{Z} \text{ (estimated)} = X\beta = \]

\[
\begin{array}{l}
100 = 100 + 0 + 0 \\
102 = 100 + 1 + 1 \\
105 = 100 + 2 + 3 \\
107 = 100 + 3 + 4 \\
107 = 100 + 4 + 3 \\
104 = 100 + 5 + (-1) \\
106 = 100 + 6 + 0 \\
107 = 100 + 7 + 0 \\
\text{(return to baseline)} \\
108 = 100 + 8 + 0 \\
109 = 100 + 9 + 0 \\
\end{array}
\]

**Note:** The time axis should be from 0 to 22.5 sec with intervals of 2.5 sec.

**Example 2**

\[
\begin{array}{c|cccccccccc}
Z & z_0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
 & z_1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
 & z_2 & 1 & 2 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\
 & z_3 & 1 & 3 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\
 & z_4 & 1 & 4 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\
 & z_5 & 1 & 5 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\
 & z_6 & 1 & 6 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\
 & z_7 & 1 & 7 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
 & z_8 & 1 & 8 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\
 & z_9 & 1 & 9 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
\end{array}
\]

**Example 2**

\[
\begin{array}{c|cccccccccc}
Z & z_0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
 & z_1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
 & z_2 & 1 & 2 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\
 & z_3 & 1 & 3 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\
 & z_4 & 1 & 4 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\
 & z_5 & 1 & 5 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\
 & z_6 & 1 & 6 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\
 & z_7 & 1 & 7 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
 & z_8 & 1 & 8 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\
 & z_9 & 1 & 9 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
\end{array}
\]
\[ Z = X\beta + \varepsilon \quad \Rightarrow \quad \varepsilon = (\varepsilon_0 \ \varepsilon_1 \ \varepsilon_2 \ \varepsilon_3 \ \varepsilon_4 \ \varepsilon_5 \ \varepsilon_6 \ \varepsilon_7 \ \varepsilon_8 \ \varepsilon_9) \]

\( \hat{Z} \) (estimated) = \( X\beta = \)

- \( 100 = 100 + 0 + 0 \) (affected by the 1st stimulus only)
- \( 102 = 100 + 1 + 1 \) (affected by the 1st stimulus only)
- \( 105 = 100 + 2 + 3 \) (affected by both stimuli)
- \( 108 = 100 + 3 + 1 + 4 \) (affected by both stimuli)
- \( 110 = 100 + 4 + 3 + 3 \) (affected by both stimuli)
- \( 108 = 100 + 5 + 4 + (-1) \) (affected by both stimuli)
- \( 109 = 100 + 6 + 3 \) (affected by both stimuli)
- \( 106 = 100 + 7 + (-1) \) (affected by the 2nd stimulus only)
- \( 108 = 100 + 8 + 0 \)
- \( 110 = 100 + 9 + 1 \) (affected by the 3rd stimulus only)

Note: The time axis should be from 0 to 22.5 sec with intervals of 2.5 sec.